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INDIAN AGRICULTURAL
RESEARCH INSTITUTE, NEW DELHI.

PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

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PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

VOL. LI. 1930–31.

I.—On the Pregnancy Rate in the Lactating Mouse and the Effect of Suckling on the Duration of Pregnancy.

By L. Mirskaia and F. A. E. Crew (Department of Animal Genetics, University of Edinburgh).

(MS. received November 12, 1930. Read December 1, 1930.)

THE pregnancy rate amongst our adult breeding stock is 85 per cent., and the length of gestation nineteen days. Though the matter has not been critically examined, it is a common observation amongst mouse breeders that a suckling mother only rarely becomes pregnant to proceed to a normal parturition. Kirkham (1916), for example, states that "white mice are able to become pregnant whilst lactating, but when suckling a litter only about one female in five undergoes a complete pregnancy, either failing to ovulate (the majority of cases) or the fertilised eggs developing normally until shortly after implantation in the uterus, when they die and are absorbed (the minority of cases)." Several investigations have shown, moreover, that if the suckling mother does become pregnant, this second pregnancy is prolonged beyond its normal duration. Daniel (1910), indeed, formulated a law concerning this fact. "The period of gestation, in lactating mothers, varies directly with the number of young suckled." His records allowed him to think that there existed an almost exact relation of one day added to the gestation period for each animal suckled. Kirkham (1916), in repeating and extending these experiments, obtained results which showed no correlation between the duration of gestation and either the number of young suckled or the number of embryos within the uterus. This work showed, however, that when more

than two young were being suckled the implantation of embryos did not occur until the fourteenth day instead of on the fifth day after fertilisation, and that during this time the blastulæ lay free in the lumen of the uterus. This same author (1918), as a result of further experimentation, concluded that the full activity of the mammary glands was responsible for this delayed implantation, and that this influence of the mammary glands was subject to marked individual variation, probably referable to metabolic idiosyncrasies.

For the purpose of examining the frequency of pregnancy following postpartum mating in the suckling mother, the mouse is an ideal material, for within twenty-four hours after parturition the mother is available for mating, and in the majority of cases, in our experience, mating occurs. We have previously shown (Crew and Mirskaia, 1930, *a*), that when, in our stock, the litters are removed at birth, the rate of fertile to infertile matings, *i.e.* the pregnancy rate, immediately following parturition is not less than that which is characteristic of the stock as a whole. It follows, therefore, that the suggestion offered by Kirkham and relating to the frequency of ovulation cannot explain a lowered pregnancy rate in the case of the lactating mother. If, then, this is less than 85 per cent., some other explanation than failure to ovulate must be found.

Two different groups of mice were used: both of them were from our own mouse colony and were line-bred albinos of known history. The first group consisted of young females pregnant for the first time; the mean age at first parturition being 62·5 days. In order to obtain these, 150 young virginal females were taken at the time of the first œstrous cycle and placed with males. Those which quickly became pregnant and which mated after parturition were selected for the present experiment. Twenty-nine individuals were thus chosen. The second group consisted of older, multiparous females, known to be good breeders. The two groups were kept in an electrically heated, thermostatically controlled mousery, and received daily milk and lettuce in addition to the usual cereals.

Table I. records the results obtained. It is seen that the pregnancy rate amongst the primiparæ was 24·1 per cent., whilst amongst the multiparæ it was 50 per cent. That of the multiparæ is twice that of the primiparæ, but is much lower than the 85 per cent. which is characteristic of our ordinary adult stock.

In a previous paper (Mirskaia and Crew, 1930) we have shown that amongst animals recently having attained puberty, and mating for the first time, the pregnancy rate is about 50 per cent. Amongst the young lactating mothers of the present experimental group receiving similar

TABLE I.

Primiparæ.

No. of Animal.	Age at first parturition.	Date of first parturition.	Litter size.	Date of first post-partum mating.	Date of subsequent parturition.	Length of interval.	Litter size.	Prolongation of pregnancy.
	days.					days.		days.
1	60	19.5	5	19.5				
2	55	19.5	9	20.5				
3	63	22.5	5	23.5				
4	62	22.5	4	22.5				
5	66	26.5	5	26.5				
6	71	30.5	9	31.5				
7	53	31.5	8	31.5	29.6	29	7	10
8	53	31.5	8	31.5				
9	71	1.6	5	1.6				
10	52	1.6	3	1.6				
11	52	1.6	3	1.6	2.7	31	6	12
12	60	7.6	4	7.6				
13	59	7.6	4	7.6	9.7	32	4	13
14	55	8.6	7	8.6				
15	60	9.6	4	10.6				
16	62	11.6	8	12.6				
17	58	11.6	4	12.6				
18	57	12.6	5	12.6				
19	63	12.6	4	13.6				
20	58	19.6	6	19.6				
21	59	19.6	6	20.6	22.7	32	3	13
22	71	20.6	6	20.6	22.7	32	8	13
23	59	20.6	6	20.6	17.7	27	10	8
24	68	21.6	7	22.6				
25	60	21.6	8	22.6				
26	73	24.6	4	24.6	24.7	30	7	11
27	77	24.6	4	24.6				
28	79	26.6	10	26.6				
29	77	28.6	5	28.6				

Pregnancy rate 7/29 = 24·1 per cent.

Multiparæ.

No. of Animal.	Date of first parturition.	Litter size.	Date of first post-partum mating.	Date of subsequent parturition.	Interval.	Prolongation of pregnancy.	Litter size.
					days.	days.	
1	22.4	6	22.4	18.5	26	7	9
2	9.6	7	9.6	10.7	31	12	3
3	14.8	6	14.8	14.9	31	12	9
4	14.8	6	14.8	14.9	31	12	9
5	25.8	6	25.8	27.9	32	13	4
6	25.8	6	25.8	27.9	33	14	4
7	26.8	4	26.8				
8	27.8	7	27.8				
9	28.8	6	29.8	27.9	29	10	10
10	28.8	8	28.8				
11	29.8	7	30.8	3.10	34	15	4
12	31.8	7	31.8	3.10	33	14	7
13	1.9	10	1.9				
14	5.9	4	5.9				
15	6.9	5	6.9	3.10	27	8	9
16	6.9	8	6.9	11.9	35	16	7
17	9.9	6	9.9				
18	9.9	6	9.9				
19	11.9	4	11.9	6.10	25	6	11
20	17.9	6	17.9				
21	17.9	8	17.9				
22	19.9	8	20.9				
23	21.9	5	24.9				
24	25.9	8	25.9				

Pregnancy rate 12/24 = 50 per cent.

treatment the rate of pregnancy during such lactation is only 24·1 per cent. This discrepancy becomes noteworthy when it is remembered that the pregnancy rate increases with the age of the individual up to a maximum, and that the experimental animals of the present series were at least 19 to 20 days older (*i.e.* by the duration of pregnancy) than are similar animals at the time of first conception.

The difference between the pregnancy rates of the primiparæ and the multiparæ respectively is surely referable to some inherent difference between the two groups since they were living under exactly the same conditions and receiving the same treatment. The only difference between the groups is in the age of the animals. We have shown that the pregnancy rate and reproductive rate of young females in relation to first mating are distinctly lower than are those of a related adult group, and that this difference is to be explained by the fact that for full reproductive activity a certain level of somatic maturity, not yet reached by the puberal female, is essential. The results obtained in the present experiment completely support this conclusion. It has to be assumed that the difference in the pregnancy rates of the primiparæ and of multiparæ in this experiment is a reflection of the fact that the primiparæ had not yet achieved that degree of somatic maturity which permits the exhibition of full reproductive activity. The pregnancy rate of the multiparæ in this experiment is about twice as high as is that of the primiparæ. The pregnancy rate among our adult stock is nearly twice as high as is that of puberal animals at the time of their first pregnancy. The present study, therefore, provides further support for the distinction which we have endeavoured to make between puberty and maturity.

Turning now to the question of the cause of the prolongation of the period of gestation, it will be seen in Table II (which includes all the lactating mothers which, during the course of this study, gave birth to litters) that the figures do not offer any support for Daniel's law, but are in agreement with the findings of Kirkham.

There would appear to be no correlation between the number of young within and/or without and the degree of prolongation of gestation. For example, a total number of offspring of 15, as in cases Multiparæ 1, 3, 16, 19, can be associated with prolongations of 7, 12, 16, or 6 days respectively. All that can be stated from an examination of our figures, which include no litters of two or less, is that in every case the duration of gestation is prolonged. As has been stated, Kirkham suggested that the delayed implantation of the fertilised ova during suckling was due to the inhibiting action of the mammary gland upon the uterus. If this were indeed the

TABLE II.

Primiparæ.	No. of offspring without.	No. of offspring within.	Total No. of offspring.	Prolongation in days of gestation beyond 19.
7	8	7	15	10
11	3	6	9	12
13	4	4	8	13
21	6	8	14	13
22	6	3	9	13
23	6	10	16	8
26	4	7	11	11
Multiparæ.				
1	6	9	15	7
2	7	3	10	12
3	6	9	15	12
4	6	9	15	12
5	6	4	10	13
6	6	4	10	13
9	6	10	16	10
11	7	4	11	15
12	7	7	14	14
15	5	9	14	8
16	8	7	15	16
19	4	11	15	6

case, it would become a matter of very considerable difficulty to explain the differences exhibited, for example, by cases Primiparæ 13, 26 and Multiparæ 19, in each of which four young were being suckled, and yet the prolongation in these cases was 13, 11, and 6 days respectively. Manifestly, Kirkham's explanation is not sufficient—in fact, he himself recognised this, for he was forced to suggest that to account for such variation it was necessary to invoke individual metabolic idiosyncrasies. To us it seems more reasonable to hold the view that prolongation of gestation in the suckling mother is due to an insufficiency of the hormone (or hormones) responsible for the inception and maintenance of the state of pregnancy, and that the variations in the degree of prolongation are reflections of individual differences in respect of production of these hormones. In a previous paper (Crew and Mirskaia, 1930, *b*) we have produced evidence to show that, in the mouse, the hormonic conditions characteristic of pregnancy are very similar to those which obtain during lactation, and that in both states the corpus luteum is responsible for their inception and maintenance. If this is so, then it follows that when the mother conceives during lactation the corpus luteum, in its functioning, must be called upon to do more than usually is demanded of it.

It is established that the corpus luteum is concerned in the production of those reactions on the part of the uterus which make implantation of the ova possible (Corner, 1929; Clauberg, 1930), and also in the production of those reactions on the part of the mammary gland which lead to the elaboration of milk (Grüter, 1930). If, for the exhibition of these reactions, a certain minimal stimulus is required, then it is reasonable to assume that commonly, in the case of the suckling and pregnant mother, one of these reactions will proceed at the expense of the other; for example, lactation will proceed whilst implantation will not occur until the demands of the suckling young are being partly met by supplies from sources other than the maternal body (on or about the 14th day of lactation), when the hormonic stimulus can then be diverted to other purposes. Thus delayed implantation would occur, and in consequence gestation would be prolonged.

The degree of such prolongation will be variable if in different individuals the quality and quantity of corpus luteum functioning is variable. The figures presented in this paper suggest that there is a relation between the number of corpora lutea present and the degree of prolongation of pregnancy, the more corpora lutea the shorter the prolongation. It can be assumed that, in the case of litters of 9, 10, 11, there are as many corpora lutea in the ovary as there are offspring in the litter. In the case of litters of smaller size, it cannot be told, without further examination, whether or not prenatal death has depleted the litter.

If this explanation of prolonged gestation be correct, then it should be possible to obliterate this prolongation by the administration of appropriate corpus luteum extract to the mother during the earliest stages of pregnancy. It is noteworthy that offspring born after a gestation prolonged by 6 days are indistinguishable from others born after a prolongation of 16 days. It follows that either the rate of intrauterine fœtal development varies from case to case under these conditions, or else the length of prolongation in days is an indication of the delay in implantation in days. This matter deserves further investigation.

One last point of interest concerns the definition of pregnancy. An animal is said to be pregnant when the genital tract exhibits certain structural modifications which are regarded as being characteristic of the condition of pregnancy, and when in the uterus there are fertilised ova. If there are no fertilised ova, but the genital tract exhibits these pregnancy changes, then the female is said to be pseudopregnant. Can it then be said of an animal which does not exhibit these characteristic uterine modifications, but in the lumen of whose uterus fertilised ova are

lying free, that she is pregnant? Surely she does not become pregnant, according to the accepted definition of pregnancy, until her uterus is prepared to accommodate the fertilised ova.

Summary.

1. The pregnancy rate of primiparæ, suckling their young, was, in this experiment, 24·1 per cent.; that of multiparæ suckling their young, 50 per cent. It is shown that this difference is not due to differences in the incidence of ovulation associated with postpartum œstrus. The suggested explanation of this difference between puberal and adult groups is that a certain level of somatic maturity is a prerequisite for full reproductive activity.

2. In all cases the duration of pregnancy was prolonged. The degree of prolongation was variable and could not be related to the number of young in the uterus or suckling. The results provide no support for the suggestion that this prolongation, due to delayed implantation of the fertilised ova, is to be referred to an inhibitory action on the part of the mammary gland. The suggestion is made that the delayed implantation and prolonged pregnancy are due to inability on the part of the corpus luteum to cater adequately for implantation and lactation synchronously.

BIBLIOGRAPHY.

Clauberg, C. (1930), "Experimentelle Untersuchungen zur Frage eines Mäusetestes für des Hormon des Corpus luteum," *Zentralbl. für Gynaekologie*, **19**.

Corner, G. W., and W. M. Allen (1929), "Physiology of the Corpus Luteum. II. Production of a Special Uterine Reaction (progestational proliferation) by Extracts of the Corpus Luteum," *Amer. Jour. Phys.*, **88**, 326.

Crew, F. A. E., and L. Mirskaia, (*a*) (1930), "On the Effect of the Removal of the Litter upon the Reproductive Rate of the Female Mouse," *Quart. Jour. Exp. Phys.*, **20**, 3; (*b*) (1930), "The Lactation Interval in the Mouse," *Quart. Jour. Exp. Phys.*, **20**, 2.

Daniel, J. Frank (1910), "Observations on the Period of Gestation in White Mice," *Jour. Exp. Zool.*, **9**, 1.

Grüter, F. (1930), "Experiments upon the Effect of Hormones upon Milk Secretion with Special Reference to the Cow and the Goat," *Proc. Second Internat. Cong.*, Sex Res. (in press).

Kirkham, W. B. (1916), "The Prolonged Gestation Period in Suckling Mice," *Anat. Rec.*, **2**, 2; (1918); "Observations of the Relation between Suckling and the Rate of Embryonic Development in Mice," *Jour. Exp. Zool.*, **27**.

Mirskaia, L., and F. A. E. Crew (1930), "On the Genetic Nature of the Time of Attainment of Puberty in the Female Mouse," *Quart. Jour. Exp. Phys.*, **20**, 4.

(*Issued separately March* 19.)

II.—Observations on the Relative Rate of Growth of the Nails of the Right and Left Hands respectively: on Seasonal Variations in the Rate, and on the Influence of Nerve-section upon it. By Sir E. Sharpey-Schafer, F.R.S., P.R.S.E.

(MS. received January 21, 1931. Read February 2, 1931.)

In a paper published in the *Quarterly Journal of Experimental Physiology* in 1928, vol. xix, in which a description of the effects produced on sensation by de-nervation and subsequent regeneration of a cutaneous area was given, certain observations were incidentally made regarding the effect of de-nervation on the growth of the finger nails. The nerves severed were the branches of the ulnar to the little finger of the left hand, and the result of severance upon the rate of growth of the nails on the two sides was investigated and a comparison drawn. The results of the comparison were to show "a considerable falling off in the rate of growth of the nail of the de-nervated finger. The nail on the right hand (normal finger) took five months to grow from base to apex, that on the left (operated hand) took six months. Besides this, the nail has grown flatter and shows irregular markings which are absent on the normal side." The observation was made in 1927 during the period from the end of February to the end of August.

Since then two further comparative observations at different times of the year upon the rate of growth of the same nails have been made. One of these observations was in the winter period. It began on October 1, 1929, when both nails were marked with lunar caustic at the base. On the right (normal) side the nail had completed its growth in 184 days, whereas that on the left (operated) side took 205 days: *i.e.*, about 6 (calendar) months, and 6 months 3 weeks respectively. The other observation was in the summer period. It began on March 5, 1930, and continued until September. The nail of the right (normal) side took 157 days, that of the left (operated) side 182 days: *i.e.*, 5 calendar months and 8 days, and 6 months and 2 days respectively. This nearly corresponds with the original observation recorded in the above-mentioned paper, which was made at the same season of the year. On both sides the rate of growth was slower in winter than in summer, and in all three observations it was slower on the de-nervated than on the normal finger.

These remarks apply only to the little-finger nails, and from them it would seem that the effect of de-nervation has been markedly to diminish the rate of growth of the nail in both summer and winter. It is also obvious that, *ceteris paribus*, the nails have grown faster in summer than in winter. But it remained to ascertain whether the rate of growth may not be in every case faster on the right than on the left hand, for if it were so, the diminished rapidity of growth on the de-nervated finger might be accounted for otherwise than by the removal of a trophic effect exercised through the cutaneous nerves.

Accordingly, two complete series of observations have been made upon all the finger nails of both hands, one series occupying the winter of 1929–30, the other the summer of 1930. The results are tabulated as follows:—

	Length of Nail in mm.	Number of Days taken to grow from Base to Free Edge.		Rate of Growth per Week expressed in millimetres.	
		Winter.	Summer.	Winter.	Summer.
Right thumb	14·5	194	166	0·523	0·600
Left thumb	14	165	144	0·593	0·680
Right fore finger . . .	12	145	144	0·580	0·580
Left fore finger . . .	13	173	157	0·522	0·580
Right middle finger . .	12	143	132	0·588	0·680
Left middle finger . . .	12	154	157	0·545	0·535
Right ring finger . . .	12	171	144	0·490	0·580
Left ring finger . . .	12	198	171	0·424	0·490
Right little finger . . .	12	184	157	0·456	0·535
Left little finger (de-nervated)	11	205	182	0·376	0·423
Average	12·45	173	155	0·510	0·568

The average number of days taken by each finger nail to grow from base to free border was, on the right hand, in winter 167 days, in summer 149 days; on the left hand, in winter 179 days, in summer 162 days. The average rate in millimetres per week for all the nails was, in the right hand, in winter 0·528, in summer 0·595; in the left hand, in winter 0·492, in summer 0·542. In nearly every case the rate of growth was faster in summer than in winter; in most cases the difference was well marked. In nearly every case the rate was faster on the right hand than on the left, but the thumb nails offered a conspicuous exception, for both in winter and summer the growth was faster on the left hand than on the right, and was in fact the fastest observed.

The difference in the rate of growth showed itself markedly in the little-finger nails, the growth on the left (de-nervated) side being much

slower than on the right (normal) side. Thus in winter the nail on the (de-nervated) left side took 205 days to grow as against 184 of the normal right, and in the summer 182 days as against 157: the rate of growth being 0·376 mm. per week in winter and 0·423 mm. in summer, as against 0·456 mm. in winter and 0·535 mm. in summer. These differences are greater than those noticed between the other nails of the two sides, and suggest that the de-nervation plays a part in delaying the growth of the little-finger nail of the operated side—in other words, that the cutaneous nerves have a trophic influence on the growth of the nail. That such a trophic influence is exercised is rendered probable from the fact that the little-finger nail on the left or de-nervated side is more flattened in form, rougher on the surface, and more brittle in texture than the corresponding nail of the right hand. But as the differences in the rate of growth of the several nails on the hands are considerable, an extended series of observations on a number of individuals is required. The experiment, although a simple one, is spread over several months, and the silver marks on the nails are disfiguring. Few investigations of the kind have been systematically made on normal individuals, but it would be well if someone, who has both the time and opportunity of doing so, would undertake it and extend it to embrace a considerable number of persons of both sexes and of all ages. It should also include left-handed individuals. I am myself right-handed and 80 years old.

Summary.

The growth of the finger nails has been investigated, and the rate on the right side as compared with the left, and in summer as compared with winter, determined for each finger. In nearly every case the rate is faster in summer than in winter.

In nearly every case also the rate is faster on the right hand than on the left. A notable exception is presented by the thumb nails, which have grown faster on the left hand than on the right both in summer and winter. The slowest rate of growth occurred in the nail of the little finger of the left side. The cutaneous nerves (ulnar) of this finger had been severed, and, with the exception of those which subserve pain, had not shown functional recovery. It is therefore probable that a trophic influence is exercised through the nerves upon the growth of the nail; the probability being supported by the fact that the little-finger nail on the left or de-nervated side not only exhibits a slower rate of growth but is distinguished from the corresponding nail on the normal side by being more flattened in form, rougher on the surface, and more brittle in texture.

Literature.

There does not appear to be any work on the growth of the nails more recent than 1881. The earliest accurate notes on the subject I have been able to find are those of Berthold, of Göttingen, who, in a short paper in *Müller's Archiv* for 1850, narrates observations made on the finger nails of his own hands. He first noted the time for the nail of his middle finger to grow from the edge of the lunula to the proximal edge of the free border—a distance of 11 mm. He found this to be 132 days (about 4½ months). This was in the winter, November to March. He repeated the observation in summer and found it to be 116 days (not quite 4 months).*

Lastly, he determined the time taken by each finger nail in both hands. He does not give the time of year—presumably it was winter. The rates found were as follows: †—

	Right hand.	Left hand.
Middle finger	108	116
Ring finger	134	141
Fore finger	136	143
Little finger	147	152
Thumb	155	161

These figures show the rate to have been faster in all cases on the right hand than on the left, that it was fastest in the middle finger and slowest in the thumb. It is assumed that Berthold was right-handed, as he makes no statement to the contrary. He does not give his age, but Moleschott mentions that it was 46. Without giving any details he states that he has observed that the rate of growth differs with age, being fastest in children and slowest in old age.

The figures given by F. Pradier (*Gazette des Hôpitaux*, xxxiv, 1861) are strictly comparable with mine, Pradier having measured the rate of growth of each finger nail in both hands in the same way that I have done. He gives the measurements of the nails in millimetres as follows:—

Thumb	16
Fore finger	11
Middle finger	13
Ring finger	13
Little finger	11

* These figures are for 11 mm. only, and therefore not the full length of the nail.

† Berthold seems to have assumed that the measurement for the nails is the same in all (11 mm.) from the lunula to the proximal border of the free part, but this is by no means the case.

The number of days occupied by complete growth from root to free border was:—

	Right hand.	Left hand.
Thumb	112	115
Fore finger . . .	83	96
Middle finger . . .	111	134
Ring finger . . .	114	134
Little finger . . .	112	114

Pradier was 37 years old and right-handed. The experiment was begun in September 1860, and finished in February 1861.

Moleschott ("Wachsthum der Horngebilde, etc.," *Untersuchungen zur Naturlehre des Menschen*, xii, 1881), who determined the rate of growth by weighing his nail-parings every month, noticed a difference in rate between winter and summer in favour of the latter. He made two determinations on himself at different ages, one at 37 and the other at 53. The differences between winter and summer were rather greater at 37, but were not striking, and were greater in Berthold at 46 than in Moleschott at 37. He is, therefore, not inclined to attach importance to the question of age. I have not noticed in myself any age difference in the intervals of nail-paring.

It is clear that there is great variation in the rate of growth both in individuals and in the several finger nails.

As regards the effect of injury to nerves, the best account is furnished by Weir Mitchell (*Injuries of Nerves*, 1872), who found that such injury may affect both the rate of growth and the nutrition of the nails. He states that when the nerve to a finger has been (accidentally) cut, the growth of the nail is slower and that various changes may show themselves in its appearance and texture, *e.g.* it becomes clubbed; or dry, scaly, and fragile; or very thin; or with transverse markings.

M. Bernhardt (*Virchow's Archiv*, lxxxvi, 1861), on the other hand, was unable to substantiate any effect on the rate of growth from nerve injuries. But he gives only a few cases, and the method of measurement he adopted was not one calculated to show small differences in rate.

Several cases of accidental nerve-section, chiefly of the ulnar nerve, are related by Jonathan Hutchinson in the *London Hospital Reports* for 1866. Hutchinson does not record any observation on the rate of growth of the nails, but states that in one of his cases, a girl of 14, in which the ulnar nerve was severed at the elbow, leading to complete and permanent loss of sensation of all kinds in the little finger and ulnar side of the ring

finger, the nail of the little finger came away entirely about 2 months after the accident, but when seen by Mr Hutchinson at 20 months was completely restored and was to all appearances normal, except that it was smaller than the corresponding nail on the normal side.

I am indebted to my colleague Professor James Kendall for the following interesting note relating to the subject discussed in this paper.

Extract from one of Samuel Johnson's manuscript Diaries:—

"July 26, 1768.—I have shaved my nail by accident in whetting the knife, about an eighth of an inch from the bottom, and about a fourth from the top. This I measure that I may know the growth of nails." *

* *Everybody's Boswell*, Bell & Sons, 1930, p. 342.

(*Issued separately March* 19, 1931.)

III.—On some Problems involving the Persymmetric Determinants.* By J. Geronimus. *Communicated by* The General Secretary.

(MS. received December 8, 1930. Read January 12, 1931.)

§ 1.

Consider a system of "orthogonal polynomials"

$$P_0, \quad P_1(x), \quad P_2(x), \ldots$$

possessing the property

$$(1) \quad . \quad . \quad \sum_{i=1}^{r} p(x_i)\,P_k(x_i)\,P_s(x_i) = 0, \quad k \neq s, \quad k, s \leqslant r,$$

$x_1, x_2, \ldots, x_r$ being real given numbers, and $p(x)$ a given "weight" function.

It is clear that the polynomial $P_k(x)$ is determined, except for a constant factor by means of the relations

$$(2) \quad . \quad \sum_{i=1}^{r} p(x_i)\,P_k(x_i)\,x_i^{\nu} = 0, \quad (\nu = 0, 1, 2, \ldots, k-1)$$

and therefore we may write the known formula

$$(3) \quad . \quad . \quad P_k(x) = \begin{vmatrix} c_0 & c_1 & \cdots & c_k \\ c_1 & c_2 & \cdots & c_{k+1} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ c_{k-1} & c_k & \cdots & c_{2k-1} \\ 1 & x & \cdots & x^k \end{vmatrix} . \text{ Const.,}$$

supposing of course that

$$\begin{vmatrix} c_0 & c_1 & \cdots & c_{k-1} \\ c_1 & c_2 & \cdots & c_k \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ c_{k-1} & c_k & \cdots & c_{2k-2} \end{vmatrix} \neq 0.$$

In these formulæ the c's are the "moments" of the function $p(x)$, i.e.

$$(4) \quad . \quad . \quad . \quad c_k = \sum_{i=1}^{r} p(x_i)\,x_i^k, \quad (k = 0, 1, 2, \ldots).$$

* *Proc. R.S.E.*, vol. L, 1930, pp. 304–309.

It should be pointed out that

(5) . . . $P_r(x) = \text{Const}(x-x_1)(x-x_2)\ldots(x-x_r)$.

Consider also the "functions of the second kind"

(6) . $Q_k(y) = \sum_{i=1}^{r} \frac{p(x_i)P_k(x_i)}{y-x_i}, \quad (k = 0, 1, 2, \ldots, r-1)$.

From the evident relation

(7) . $Q_k(y)y^\nu = \sum_{i=1}^{r} \frac{p(x_i)P_k(x_i)x_i^\nu}{y-x_i}, \quad (\nu = 0, 1, 2, \ldots, k)$,

by putting

(8) . . . $R_s = \sum_{i=1}^{r} \frac{p(x_i)x_i^s}{y-x_i}, \quad (s = 0, 1, 2, \ldots)$,

we get easily the following formula

(9) . . .
$$\begin{vmatrix} R_0 & R_1 & \ldots & R_k & 1 \\ R_1 & R_2 & \ldots & R_{k+1} & y \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ R_k & R_{k+1} & \ldots & R_{2k} & y^k \\ 1 & y & \ldots & y^k & \dfrac{P_k(y)}{Q_k(y)} \end{vmatrix} = 0,$$

or otherwise

$$\frac{Q_k(y)}{P_k(y)} = -\begin{vmatrix} R_0 & R_1 & \ldots & R_k \\ R_1 & R_2 & \ldots & R_{k+1} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ R_k & R_{k+1} & \ldots & R_{2k} \end{vmatrix} : \begin{vmatrix} R_0 & R_1 & \ldots & R_k & 1 \\ R_1 & R_2 & \ldots & R_{k+1} & y \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ R_k & R_{k+1} & \ldots & R_{2k} & y^k \\ 1 & y & \ldots & y^k & 0 \end{vmatrix}.$$

On putting

(10) . . $w_s = R_{s+2} - 2yR_{s+1} + y^2R_s, \quad (s = 0, 1, 2, \ldots)$

we get the following result

(11) .
$$\frac{Q_k(y)}{P_k(y)} = \begin{vmatrix} R_0 & R_1 & \ldots & R_k \\ R_1 & R_2 & \ldots & R_{k+1} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ R_k & R_{k+1} & \ldots & R_{2k} \end{vmatrix} : \begin{vmatrix} w_0 & w_1 & \ldots & w_{k-1} \\ w_1 & w_2 & \ldots & w_k \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ w_{k-1} & w_k & \ldots & w_{2k-2} \end{vmatrix}.$$

Lastly we shall introduce the "Appell polynomials" corresponding to the given system of orthogonal polynomials, defined as follows:*

(12) $A_k(\eta) = \sum_{i=1}^{r} p(x_i)(x_i-\eta)^k = (c-\eta)^{(k)}, \quad (k = 0, 1, 2, \ldots)$,

* P. Appell, "Sur une classe de polynomes," *Annales de l'École Normale*, t. 9 (1880), pp. 119–144.

where the bracketed exponent means that after expansion c^s is to be replaced by c_s.

In the following paragraphs we show how this simple theory of orthogonal polynomials enables us to resolve some problems concerning persymmetric determinants.

§ 2.

Consider the problem of Jacobi:

Find a function

$$u(x) = \frac{N(x)}{M(x)} \tag{13}$$

from the conditions

$$u(x_i) = U_i, \quad (i = 1, 2, \ldots, n+m+1),$$

*it being understood that N and M are respectively of the n-th and m-th degrees in x.**

Let

$$p(x) = \frac{N(x)}{M(x) f'(x)}, \qquad f(x) = \prod_{s=1}^{n+m+1} (x - x_s). \tag{14}$$

It is clear then that

$$\sum_{i=1}^{n+m+1} p(x_i) M(x_i) x_i^\nu = \sum_{i=1}^{n+m+1} \frac{N(x_i) x_i^\nu}{f'(x_i)} = \frac{1}{2\pi i}\int_\Gamma \frac{N(x) x^\nu}{f(x)} dx,$$

where all roots of $f(x)$ lie within the contour Γ.

Hence we see that

$$\sum_{i=1}^{n+m+1} p(x_i) M(x_i) x_i^\nu = 0, \qquad (\nu = 0, 1, 2, \ldots, m-1),$$

and consequently

$$M(x) = \text{Const.}\, P_m(x).$$

Further, from the formula

$$\frac{N(x)}{f(x)} = \sum_{i=1}^{n+m+1} \frac{N(x_i)}{(x - x_i) f'(x_i)}$$

we conclude that

$$\frac{N(x)}{f(x)} = c\, Q_m(x),$$

and finally we get from (11) the required result of Jacobi,

$$\frac{N(x)}{f(x) M(x)} = \begin{vmatrix} R_0 & R_1 & \ldots & R_m \\ R_1 & R_2 & \ldots & R_{m+1} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ R_m & R_{m+1} & \ldots & R_{2m} \end{vmatrix} : \begin{vmatrix} w_0 & w_1 & \ldots & w_{m-1} \\ w_1 & w_2 & \ldots & w_m \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ w_{m-1} & w_m & \ldots & w_{2m-2} \end{vmatrix}. \tag{15}$$

* C. Jacobi, "Ueber die Darstellung einer Reihe gegebener Werthe durch eine gebrochene rationale Funktion."—*Gesammelte Werke*, t. iii, SS. 479–511.

§ 3.

Put now

(16) . $p(x) = \dfrac{\phi(x)}{\psi'(x)}, \quad c_\nu = \sum_{i=1}^{n} p(x_i) x_i^\nu, \quad (\nu = 0, 1, 2, \ldots),$

where

(17) $\phi(x) = \sum_{s=0}^{m} b_s x^s, \quad \psi(x) = \sum_{k=0}^{n} a_k x^k = a_n \prod_{i=1}^{n} (x - x_i), \quad m < n.$

It is clear that

$$\psi(x) = c P_n(x).$$

Consider the polynomial

(18) . $Q_{n-1}^{(i)}(x) = \dfrac{P_n(x)}{x - x_i} = \sum_{s=0}^{n-1} q_s^{(i)} x^s, \quad (i = 1, 2, \ldots, n).$

It is very easy to find that

(19) . . . . . $q_s^{(i)} = \dfrac{1}{c} \sum_{j=0}^{n-s-1} a_{s+j+1} x_i^j.$

Consider now the sum

(20) . . . . $L_r = \sum_{\nu=0}^{r} a_{n-\nu} c_{r-\nu}, \quad (r < n).$

We have

$$L_r = \sum_{\nu=0}^{r} a_{n-\nu} \sum_{i=1}^{n} p(x_i) x_i^{r-\nu} = c \sum_{i=1}^{n} p(x_i) q_{n-r-1}^{(i)}.$$

Thus we see that L_r is the coefficient of x^{n-r-1} in the polynomial

$$c \sum_{i=1}^{n} \frac{p(x_i) P_n(x)}{x - x_i} = \sum_{i=1}^{n} \frac{\phi(x_i) \psi(x)}{\psi'(x_i)\ (x - x_i)} = \phi(x).$$

Thus finally we obtain the theorem of Brioschi *

(21) . . . . $L_r = \sum_{\nu=0}^{r} a_{n-\nu} c_{r-\nu} = b_{n-r-1}.$

§ 4.

Consider now the problem of Sylvester: †
Determine the p's and q's in

(22) . . . . . . $\sum_{i=1}^{n} (p_i x + q_i y)^{2n-1}$

* F. Brioschi, "Intorno ad alcune questioni d'algebra superiore," *Opere Matematiche*, t. i, pp. 127–142.

† J. Sylvester, "On a remarkable discovery in the theory of canonical forms and of hyperdeterminants," *Collected Mathematical Papers*, t. i, pp. 265–283.

so as to make this expression identical with

$$(23) \quad . \quad . \quad . \quad . \qquad \sum_{s=0}^{2n-1} \binom{2n-1}{s} c_s x^{2n-s-1} y^s.$$

Dividing by y^{2n-1}, putting

$$(24) \quad . \quad p_i^{2n-1} = p(x_i), \quad \frac{x}{y} = -\eta, \quad \frac{q_i}{p_i} = x_i, \quad (i = 1, 2, \ldots, n),$$

we see that we are to satisfy the identity

$$(25) \quad . \quad . \quad . \qquad \sum_{i=1}^{n} p(x_i)(x_i - \eta)^{2n-1} = (c - \eta)^{(2n-1)}.$$

Thus we see that Sylvester's problem is reduced to the problem of determining the abscissae and the weight function when the Appell polynomial is given, or, what is the same thing, to the problem of finding the abscissae x_i and the weight function $p(x_i), (i = 1, 2, \ldots, n)$ knowing the first $2n$ moments.

$$\sum_{i=1}^{n} p(x_i) x_i^{\nu} = c_\nu, \quad (\nu = 0, 1, 2, \ldots, 2n-1).$$

Using (5) we get at once that $x_1, \ldots, x_n$ are the roots of the equation

$$\begin{vmatrix} c_0 & c_1 & \ldots & c_n \\ c_1 & c_2 & \ldots & c_{n+1} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ c_{n-1} & c_n & \ldots & c_{2n-1} \\ 1 & x & \ldots & x^n \end{vmatrix} = 0.$$

It is not difficult to find that *

$$(26) \quad . \quad . \quad \begin{matrix} p(x_i) = \\ (i = 1, 2, \ldots, n) \end{matrix} \begin{vmatrix} c_0 & c_1 & \ldots & c_n \\ c_1 & c_2 & \ldots & c_{n+1} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ c_{n-1} & c_n & \ldots & c_{2n-1} \\ 0 & K_0 & \ldots & K_{n-1}(x_i) \end{vmatrix} : \begin{vmatrix} c_0 & c_1 & \ldots & c_n \\ c_1 & c_2 & \ldots & c_{n+1} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ c_{n-1} & c_n & \ldots & c_{2n-1} \\ 0 & 1 & \ldots & n x_i^{n-1} \end{vmatrix},$$

where for the sake of brevity we have put

$$(27) \quad . \quad . \quad K_s(x) = \sum_{\nu=0}^{s} c_\nu x^{s-\nu}, \quad (s = 0, 1, 2, \ldots, n-1).$$

* J. Geronimus, "Sur un problème d'Hermite," *Bulletin de la Classe des Sciences Physiques et Mathématiques de l'Académie des Sciences d'Ukraine*, t. iv, fasc. 5 (1930), pp. 295–298.

(Issued separately April 6, 1931.)

IV.—A Note on the Secular Changes of Rock Temperature on the Calton Hill. By F. J. W. Whipple, Sc.D. *Communicated by* A. H. R. Goldie, M.A.

(MS. received January 8, 1931. Read February 2, 1931.)

1. The observations of rock temperature on the Calton Hill have recently been analysed * by Mr R. W. Wrigley, with a view to the discovery of changes going on gradually and independent of weather conditions. Mr Wrigley took groups of years, such that the mean air temperature was the same for each group, and found the mean rock temperature for like periods. He trusted that in this way he would be able "to get rid of the surface temperature variations." The result of the calculations was that for different periods in which the mean air temperature was the same the temperature of the rock might vary by half a degree Fahrenheit. The rock temperature had a maximum about 1856, fell until 1892, and rose after that date.

Wrigley concluded that the fluctuations which he had discovered had their origin in the interior of the earth, and sought to correlate them with irregularities in the earth's rotation. There is, however, reason to doubt whether the premise is correct; in fact the statistics on which Mr Wrigley relies indicate that the variations in rock temperature were propagated downwards rather than upwards. The explanation of the paradox is that in grouping the observations only one meteorological factor was taken into account. The influence of sunshine, of rainfall, and of evaporation had to be ignored. Of these factors the first is quite competent to account for the slow variation in rock temperature. In recent years there has been a tendency for the duration of bright sunshine in cities to increase. The additional sunshine, due perhaps to the reduction of smoke in "Auld Reekie," would suffice to warm up the Calton Rock to the extent indicated by Mr Wrigley's figures.

2. The observations of rock temperature fall in two periods, 1837 to 1876 and 1880 to 1929, the original thermometers having been broken in September 1876. Mr Wrigley subdivides the periods into groups of years, each group with nearly the same mean air temperature. The corresponding mean values of the rock temperatures are reproduced in

* *Proc. Roy. Soc. Edin.*, vol. l (1930), p. 153.

the following tables. The dates for which the means are taken are not the same at the different levels.*

TABLE A.—FIRST PERIOD 1837–1876.

No. of Years.	Air.		6·4 feet.		25·6 feet.		Gradient °F/100 ft.	Computed Temp. at Surface.
	Mean Date.	Mean Temp.	Mean Date.	Mean Temp.	Mean Date.	Mean Temp.		
11	1843·5	47·00	1843·9	46·45	1845·0	47·12	3·5	46·23
14	1856·0	47·04	1856·4	46·76	1857·7	47·38	3·2	46·55
12	1869·0	47·06	1869·4	46·65	1870·6	47·30	3·4	46·43

TABLE B.—SECOND PERIOD 1880–1929.

No. of Years.	Air.		11·0 feet.		21·0 feet.		Gradient °F/100 ft.	Computed Temp. at Surface.
	Mean Date.	Mean Temp.	Mean Date.	Mean Temp.	Mean Date.	Mean Temp.		
5	1882·5	47·00	1883·0	46·02	1883·3	46·37	3·5	45·64
11	1890·5	46·94	1891·1	45·82	1891·7	46·22	4·0	45·38
10	1901·0	46·78	1901·6	46·14	1902·2	46·47	3·3	45·78
12	1912·0	46·78	1912·7	46·26	1913·3	46·60	3·4	45·89
10	1923·0	46·85	1923·6	46·40	1924·2	46·71	3·1	46·06

In addition to Wrigley's data the tables contain estimates of the temperate gradient in the rock and of the surface temperature. These are computed on the assumption of a uniform gradient up to the surface. The lapse of time is perforce ignored.

It will be seen that in each series of observations the fluctuations in rock temperature as recorded by thermometers at different depths are in sympathy. The point to which attention must be called is that the fluctuations at the greater depth have the smaller range. This suggests that we are dealing with oscillations which originate on the surface rather than in the interior of the earth. Again it will be noticed that the highest gradient occurred when the rock was cold, the lowest when the rock was

* Mr Wrigley does not state explicitly his reasons for adjusting the times in this way. Some curves prepared by Heath (*Trans. Roy. Soc. Edin.*, vol. xl (1901), Pl. I) indicate that such waves of temperature as it is desired to eliminate take only six months to reach the depth of 21 feet.

For the earlier years the annual means of air temperature were taken from Mossman's account of the climate of Edinburgh. For the years from 1896 onwards the means refer to Blackford Hill.

warm. It follows that there was the greatest upward flow of heat when the rock was cold, the least when it was warm, and it is reasonable to suppose that the flow was greater or less *because* the rock was cold or warm at the top.

Thus Wrigley's view that his statistics reveal a deep-seated fluctuation in the temperature of the earth's crust appears to be untenable. The changes from decade to decade in the temperature of the rock are probably appreciable only at moderate depths.

3. It may be suggested, however, that whilst the rock thermometers are capable of showing the slow fluctuations of temperature, the differences between the ranges at depths of 11 feet and 21 feet are not to be trusted. Even so we cannot take these fluctuations as evidence for corresponding changes of temperature at great depths. If the heat were brought to the surface by conductivity a wave of temperature with a period of 60 years would pass through the rock with a wave-length * of about 500 feet. The amplitude of the wave would be reduced in the ratio, $\exp(2\pi):1$ or $535:1$ in a wave-length. The time required for the wave to travel upwards through 20 miles of rock would be of the order 10,000 years, and the amplitude would be reduced in a ratio such as $10^{600}:1$.

Thus conduction of heat from far below can be ignored. If the fluctuations of temperature at 11 feet and 21 feet are not due to conduction of heat from above they must be generated in the rock at modest depths. No mechanism for this purpose is known; we can hardly postulate variable radio-activity in the rock. It seems better to remain satisfied with the hypothesis that the fluctuations do originate on the ground.

4. Accepting this hypothesis (that the fluctuations in temperature originate at the surface), let us ascertain the order of magnitude of the corresponding current of heat through the rock.

The specific heat c and the conductivity k are both known. The value of c was determined † by Regnault for Professor Forbes, viz. $c=0{\cdot}5283$. Heath found for k "the conductivity expressed in terms of the thermal capacity of a cubic foot of water" with the foot and year as units of length and time, the value 126.

For studying simple harmonic variations of temperature originating at the surface of the rock we may use the formula

$$\theta = \mathrm{A}\sin\left(\frac{2\pi t}{\mathrm{T}} - z\sqrt{\frac{c\pi}{k\mathrm{T}}}\right)\exp\left(-z\sqrt{\frac{c\pi}{k\mathrm{T}}}\right).$$

* In the notation of the next paragraph wave-length in the uppermost rock $=2(k\mathrm{T}\pi/c)^{\frac{1}{2}}=425$ ft.

† Heath, *loc. cit.*, p. 185.

Here θ is the departure of temperature from the mean for the depth z and T is the period of the oscillations. The ratio of the amplitudes at depths z_1, and z_2 is

$$\exp\left[(z_2 - z_1)\sqrt{\frac{c\pi}{k\mathrm{T}}}\right]$$

The diagram in Wrigley's paper suggests that the oscillations he has discovered have a period of 60 years. The ranges of the observed oscillations at 11 feet and 21 feet are ·58° F. and ·49° F. respectively. The ratio between these ranges is 1·18, whereas the theoretical expression reduces to 1·16. The agreement is satisfactory.

Taking ·29° F. as the amplitude of the oscillation of temperature at 11 feet, we find for A, the amplitude at the surface, the value ·34° F. This is consistent with the figures in the last column of Table B.

The variable part of the flow of heat through the surface of the rock is given by the equation

$$k\frac{\partial\theta}{\partial z} = -\mathrm{B}\sin\left(\frac{2\pi t}{\mathrm{T}} + \frac{\pi}{4}\right)$$

in which

$$\mathrm{B} = \mathrm{A}\sqrt{\frac{2ck\pi}{\mathrm{T}}}.$$

With $\mathrm{A} = {\cdot}34°$ F. and $\mathrm{T} = 60$ years, we find that

$$\mathrm{B} = 0{\cdot}90\ \frac{\text{ft. water °F}}{\text{year}}.$$

The mean gradient of temperature for the 60 years being ·0345° F. per foot, the normal flow of heat is $4{\cdot}35\ \frac{\text{ft. water °F.}}{\text{year}}$ and the fluctuations with which we are concerned are between 3·45 and $5{\cdot}25\ \frac{\text{ft. water °F.}}{\text{year}}$.

It is to be noted that the phase of the flow of heat differs by the eighth of a period from that of the temperature at the surface. The period being 60 years, the maximum upflow of heat precedes by 7½ years the minimum of surface temperature.

For comparison I have used the data in Heath's paper to compute the quantity of heat which flows in and out of the Calton Hill in alternate half-years. This amounts to 65 ft. water ° F.

Allowing for the steady upward flow, we may say that the heat absorbed by the rock during the summer half-year would suffice to raise the temperature of a foot of water by 63°, whilst the heat given out in the

winter is that lost by a foot of water in falling 67°. It will be seen that the variation discussed by Wrigley affects these estimates by about $\pm\frac{1}{2}$°.

It occurred to me that the fluctuations under discussion might be due to variations in the amount of sunshine falling on the Calton Hill. In the British Isles there is no close connection between the variations from year to year in air temperature and in the duration of bright sunshine, so Wrigley's grouping of the years according to temperature would not ensure steadiness in the amounts of sunshine. A decrease in the amount of sunshine from 1856 to 1890 and an increase thereafter would provide a satisfactory explanation of his results. Unfortunately there is no possibility of verifying this explanation.

There was no sunshine recorder at Edinburgh until 1890. In that year a recorder was set up at Newington. About the end of 1900 this instrument was moved to the Botanic Garden where it was in use for ten years. A second recorder has been in use at Blackford Hill since 1901. The older recorder has recently been examined at the Meteorological Office, London; the bowl is not of the standard dimensions, and although the focal length of the spherical lens is apparently that appropriate for the bowl, an extended test will be needed before it can be said whether sunshine records comparable with those of an instrument of the standard pattern can be obtained. Accordingly the fact that the average annual duration of sunshine recorded at Newington (1891–1900) 1186 hours, and at the Botanic Garden (1901–1911) 1193 hours, was much less than that recorded in recent years at Blackford Hill (1901–1929) 1372 hours, cannot be accepted as demonstrating more than the importance of avoiding discontinuities in meteorological records.

For the last two of the periods of Table B we can quote the mean annual duration of sunshine at Blackford Hill, viz. (1906–1917) 1356 hours, and (1918–1927) 1384 hours. A slight increase in the sunshine coincided with a slight rise in the rock temperature.

It is probable that in Edinburgh as in other cities less smoke has been produced in recent years than in the latter half of the nineteenth century, and that there has been a corresponding increase in the amount of sunshine. This is a possible explanation of the slightly higher temperature of the Calton Rock.

5. The question which Mr Wrigley has raised, whether it is possible to detect gradual changes in the flow of heat from the interior of the earth, is of great interest. It is not to be expected that the Calton Hill thermometers, which are all affected by the annual variation of temperature as well as by the changes in meteorological conditions from year to year,

will serve the purpose. The thermometers ought to be at much greater depths. The annual range of temperature, 1·9° F. at 21 feet, would be reduced to ·01° F. at 70 feet. Annual means differing by 0·6° F. were found at 21 feet for the years 1888 and 1890; such fluctuations may be regarded as due to an oscillation with period four years and range 0·6° F. The range of such an oscillation would be reduced to ·01° F. at 90 feet.

The oscillation investigated by Wrigley with a range of ·49° F. and a period of 60 years would penetrate farther into the rock. The range would be ·01° F. at 300 feet.

Thus it appears that, if it were decided to undertake at Edinburgh observations which would give definite information as to the variability of the flow of heat from the interior of the earth, it would be necessary to provide a borehole of considerable depth. Certainly the depth should not be less than 300 feet. Such a borehole might serve for other purposes —for example, for investigating how much of the heat which reaches the surface is generated by radio-activity in the upper layers of the rock. The enterprise would be appropriate tribute to the memory of James Forbes, Lord Kelvin, J. D. Everett, Piazzi Smyth, and Thomas Heath, whose researches have made the Calton Hill famous.

(*Issued separately April* 6, 1931.)

V.—Secular Changes of Rock Temperature.*

NOTE ON DR WHIPPLE'S PAPER.

By R. W. WRIGLEY, M.A.

(Read February 2, 1931. MS. received March 6, 1931.)

THE data which are available do not seem very suitable for Dr Whipple's method of analysis. As explained in my original paper, the groups into which the rock-temperature observations were divided depended entirely upon the method adopted to get rid of the effect of variations of air temperature at the surface, and the resulting means were for dates which had no connection with the long-period temperature fluctuations which were later disclosed. A series of observations extending over nearly a hundred years was thus reduced down to eight mean temperatures for periods of various lengths. From these eight points it definitely appears that the curve representing the rock temperatures is not a straight line; the form of the curve is roughly indicated, but it is not possible to determine with accuracy the amplitude or the maximum or minimum points. A similar eight points have been obtained for the temperature curve of the shallower thermometer, but, owing to the difference in depth, the dates are different, and it is therefore very difficult to deduce an accurate mean-temperature gradient to the surface. Further, the thermometer zero points cannot be tested; they are suitable rather to show the existence of fluctuations than to define their exact limits, and a change of the order of $0^\circ\cdot 1$ would be sufficient to upset Dr Whipple's argument.

It must be agreed that sunshine, rainfall, and evaporation all play a part in influencing the rock temperature, and to these agents may be added the strength and direction of the wind. But it is a difficult matter to estimate the amount of such influence. For example, the clear skies necessary for an excess of sunshine to warm the rock by day would cool it by permitting greater radiation at night. All the meteorological factors mentioned are intimately related to the air temperatures, and there is no evidence for any change in the general climate of Edinburgh during the last hundred years. If this period is divided up into large groups of years, each with the same mean air temperature, it seems reasonable to assume that for these groups the other disturbing elements will tend to average out also.

* *Proc. Roy. Soc. Edin.*, vol. li (1931), pp. 19–24.

Dr Whipple specially suggests sunshine as a probable agent, and attempts to connect a rise in the rock temperature with a small increase in the annual sunshine recorded at the Royal Observatory. This increase is very small (about 2 per cent.), and really seems to be due to a change in the method of measurement of the sunshine cards. Before 1914 due allowance was made for the lateral extension of the burn due to the sun's appreciable diameter and to the smouldering of the card, and a close approximation to the actual duration of sunshine was obtained. The Meteorological Office practice, as explained in the *Observer's Handbook*, has always been to make a smaller allowance for such lateral extension, and although by this method the resulting totals are admittedly too large, yet for the sake of continuity it has been maintained. In order to bring the totals measured at the Royal Observatory into conformity with those of the other stations featured in the Weather Reports, the Meteorological Office method was, by request, adopted in 1914. This appears to be a satisfactory explanation of the slight increase in annual sunshine since recorded, and disposes of any connection with the rise in rock temperature.

The main interest in these temperatures lies in the close correlation which they show with the minor fluctuations in the moon's longitude, by which they are connected with possible movements in the earth's crust. Conduction is not the only method by which heat can be transferred from below; changes of pressure would do it much more rapidly, and avoid the long period of time calculated by Dr Whipple. While no definite proof as to the source of the rock-temperature changes is possible at present, it is significant that the only correlation yet established suggests a crustal origin.

(*Issued separately April* 6, 1931.)

VI.—The Genus *Lyginorachis* Kidston. By R. Crookall, Ph.D. *Communicated by* Murray Macgregor, M.A., B.Sc. (With Three Plates.)

(MS. received December 4, 1930. Read January 12, 1931.)

I. Introduction.

The late Dr R. Kidston proposed the name *Lyginorachis* to include isolated petrified petioles which exhibited a structure so similar to the petioles of the well-known *Lyginopteris* (*Lyginodendron*) *oldhamia* (Binney) as strongly to suggest affinities with that plant.

Two species of *Lyginorachis* are known, both from the Lower Carboniferous rocks of Britain. In 1923 Dr D. H. Scott * gave an account of *L. papilio* † from the Cementstone Group (Calciferous Sandstone Series) of Norham Bridge, Tweedside, and briefly referred to a second species, *L. taitiana*,‡ from the Carboniferous Limestone Series. Kidston had passed on these sections to Dr Scott for description. Although photographs had been prepared, neither was figured, and, with characteristic kindness, Dr Scott has now provided me with a resumé of his notes. The photographs are by Dr Kidston, and I have to thank the Director of the Geological Survey for permission to reproduce them.

II. *Lyginorachis papilio* Kidston and Scott.

Photographs of this species, which is represented by a single specimen, are reproduced on Plate I, fig. 2, Plate II, figs. 4 and 5, and Plate III, figs. 6–8. The following account was given by Dr Scott: § "The petiole measures about 8 by 6 mm. in diameter. In form it is somewhat flattened on one side (no doubt the upper surface) and convex on the other. It contains a single, very large vascular bundle U-shaped in transverse section, with the concavity directed towards the upper surface. The resemblance to the wings of a butterfly suggested the appropriate specific name. The

* D. H. Scott, *Studies in Fossil Botany*, 3rd ed., vol. ii, pp. 57–59. London, 1923.

† "*Lyginorachis papilio* Kidston MS." was mentioned by Kidston in "Fossil Plants of the Carboniferous Rocks of Great Britain," *Mem. Geol. Survey*, Palæontology, vol. ii (1923–25), p. 19.

‡ "*L. taitiana* Kidston MS." was mentioned by Kidston (*op. cit.* (1923–25), p. 18) as "A petiole with *Lyginopteris* affinities."

§ D. H. Scott, *loc. cit.*

inner cortex contains sclerotic nests, and the outer zone probably has a dictyoxylon structure.

"The bundle was no doubt concentric, for such remains of the phloem as are preserved occur on all sides. The convex side of the xylem is irregularly indented, but this appears to be due in part to decay, for the irregularity varies in degree in different sections. Groups of small elements, presumably the protoxylem, are embedded in the wood near its convex side, just as in the petiolar bundle of *Lyginopteris oldhamia*. In the present case, however, they are more numerous than in the Coal Measure species, the number in *L. papilio* being about ten. A small strand, with its protoxylem directed outwards, is being given off from each extremity of the main bundle, just as in *L. oldhamia*, where similar strands pass out from the rachis into the pinnæ.

"Longitudinal sections show that most of the tracheids bear multiseriate bordered pits; scalariform elements are few, and are probably limited to the neighbourhood of the protoxylem groups. In the petiole of *L. oldhamia* the proportion of scalariform to pitted tracheids is greater.

"The inner cortex consists of a short-celled parenchyma, in which the sclerotic nests are embedded. The latter form somewhat flattened groups or plates, transversely placed and of no great extent. In all respects the agreement with the corresponding zone of the petiole in *L. oldhamia* is complete.

"The outer cortex, so far as can be judged from the somewhat imperfect tangential sections, is of the dictyoxylon type, the fibrous strands forming a network, as in *Lyginopteris*, and not running parallel, as in *Heterangium*."

III. *Lyginorachis taitiana*

Lyginorachis taitiana came from the Auldton Limestone (the equivalent of the Fourth Limestone of Whitehaven) at Auldton Limestone Pit, 1¼ miles S.S.E. of Lesmahagow, Lanarkshire. It was obtained by Mr David Tait of the Scottish Geological Survey, Edinburgh, from the then manager of the limestone pit. The specimen had been found as an isolated nodule in the limestone, and unfortunately, before being placed in Mr Tait's hands, had been partially ground down. It was entirely used up in sectioning.

Three transverse sections of *L. taitiana* are preserved in the collections of the Scottish Geological Survey, Edinburgh.* They are numbered

* Professor John Walton informs me that these transverse sections form part of a series which included 16 sections (No. 2809, A-P) preserved in the Kidston Collection of Slides in the Department of Botany, University of Glasgow, and that there are, in the same Collection, two longitudinal sections (Nos. 2810, 2811).

T. 699 D (representing the number of the specimen from which they were cut). Of these, one is broken and poorly preserved, but two are excellent sections, and the better bears an additional number (K. 91 : A1), as well as the word "Type" in Kidston's handwriting. This section is figured on Plate I at fig. 1.

The petiole measures 9 by 6·5 mm., and is in an excellent state of preservation. It contains a single large U-shaped vascular bundle, measuring 4 by 2 mm., which follows the contour of the petiole itself. The two ends of the U are somewhat open and slightly recurved. In addition there are two small lateral, roughly oval strands with a convex margin towards the lower, and a slightly indented margin towards the upper, side of the petiole. These have a diameter of 0·8 and 0·6 mm. respectively. They occur a short distance from the two extremities of the main vascular bundle. Their symmetrical position suggests opposite pinnæ. The recurved ends of the main bundle are slightly constricted, and were no doubt in process of becoming detached to form further lateral strands.

The structure of the vascular bundles of *L. taitiana* is concentric, and, in the case of the main bundle, the large-celled phloem is well preserved both on the convex side and on the wings of the concave side. In the bay between the wings, however, it is partly broken down. That this disintegration was only local is shown by an examination of a second section, where the unpreserved area is smaller and differently situated. There is no marked irregular indentation of the convex margin of the xylem, such as occurs in *L. papilio* (though, as Dr Scott observes, this was in part due to decay).

A narrow zone (2 to 6 cells wide) of small tracheids fringes the convex side of the xylem and passes some distance round the curved ends of the strand. The protoxylem groups are not very clearly marked, but appear as scattered groups of smaller elements along the inner edge of the small-celled zone (*i.e.* the structure was mesarch). Groups of protoxylem are also found round the ends of the "U." They total about 14.

The symmetrical position of the two detached lateral strands suggests that they supplied opposite pinnæ: apart from this feature, the structure of the petiole shows great similarity to that of *Lyginopteris oldhamia*. In each of these strands there is a narrow zone of small tracheids forming a horseshoe on the convex side of the xylem (*i.e.* towards the lower side of the petiole). As in the case of the main bundle, these zones include the somewhat indefinite protoxylem groups, numbering about four in each case.

The petiole differs from that of *Lyginopteris oldhamia* (and from *Lyginorachis papilio*) in the absence of sclerotic nests in the inner cortex. Secretory sacs occur very occasionally.

The hypodermal bands of stereome are well developed, and attain a thickness of about twenty cells in the radial direction. The constituent cells are very thick-walled and the state of preservation is excellent. The frequent lateral fusion of the bands, seen in transverse section, suggests a dictyoxylon structure, and this is confirmed by a comparison of the three sections.

The epidermis is also thick-walled and well-preserved. The presence of stomata is suggested at various points, but no indubitable instance could be found. On the type slide a gland-like hair occurs along the upper (flattened) margin of the petiole: there are dark contents in the base, but the apex is missing, so that it is impossible to say whether or not the structure represented a capitate gland.

In the table the petioles described above are compared with those of *Lyginopteris oldhamia*.

It may here be said that Dr Kidston's photographs of his slides include one of a small petiole (reproduced on Plate II at fig. 3).* He had labelled this, prior to instituting the genus *Lyginorachis*, "A small *Rachiopteris papilio*," but later was evidently in doubt as to the correctness of this determination, for he altered it to "*Rachiopteris sp.*"

I have not had an opportunity of examining the corresponding section (No. 695 B, Kidston Collection of Slides, Department of Botany, University of Glasgow), but it will be seen from the figure that the extremities of the vascular bundle are curved inwards (and not outwards, as in *L. papilio*). The irregular indentations in the convex (lower) side of the xylem, noted in *L. papilio*, are here absent. The phloem completely surrounds the xylem, and is particularly well preserved in the bay between the two curved arms.

The recent work of Dr R. G. Absalom † on petrifactions from Haltwhistle, Northumberland, is of particular interest in that he records a mixed, though mainly Lower Carboniferous coal-ball flora. Among the Upper Carboniferous forms cited is *Lyginopteris oldhamia*. Through the kindness of Dr Absalom I have been able to borrow the relevant sections and compare them with *Lyginorachis*. The identification was based on

* Dr S. Williams of the Department of Botany, University of Glasgow, kindly informs me that, according to Kidston's register, this specimen was from the Cementstone Group at Longton Burn, near Duns, Berwickshire.

† R. G. Absalom, "The Lower Carboniferous Coal-ball Flora of Haltwhistle, Northumberland," *Proc. Univ. Durham Phil. Soc.*, vol. viii (1929), pp. 73–87.

PETIOLES OF *Lyginopteris oldhamia* AND OF *Lyginorachis*.

	Lyginopteris oldhamia.	*Lyginorachis papilio.*	*Lyginorachis taitiana.*
Size . . .	up to 10 mm.	8 by 6 mm.	9 by 6·5 mm.
Upper margin .	concave.	slightly convex.	concave.
Lower margin .	convex.	convex.	convex.
Vascular bundle .	single.	single.	a main bundle, with two symmetrically placed lateral strands; each with a lower zone of small tracheids.
Structure of bundle	concentric.	concentric.	concentric.
Shape of bundle .	U-shaped; indented towards lower surface.	U-shaped, winged; irregularly indented on lower surface.	U-shaped; lower surface rounded.
Protoxylem . .	about 5 groups; near convex side.	about 10 groups; near convex side.	(main bundle) about 14 groups; near convex side.
Pitting on tracheids	a greater proportion of scalariform tracheids than in *L. papilio*.	mainly multiseriate: scalariform near protoxylem.	mainly multiseriate: scalariform near protoxylem.*
Inner cortex .	contains sclerotic nests and secretory sacs.	contains sclerotic nests.	without sclerotic nests; a few secretory sacs.
Hypodermal stereome.	forms a network.	probably forms a network.	forms a network.
Horizon . .	Lower Coal Measures.	Calciferous Sandstone Series.	Carboniferous Limestone Series.

petioles and leaflets only, and the author † speaks of "characteristic glands." We have seen that gland-like hairs occur on *Lyginorachis*, and that petioles showing very small differences from those of *Lyginopteris oldhamia* are known from the Lower Carboniferous. Under these circumstances, the presence of *Lyginopteris oldhamia* in this material (unfortunately entirely used up in sectioning) has still to be demonstrated,

* I am indebted to Dr S. Williams for this observation. The preservation is too poor to allow the preparation of satisfactory photographs.

† R. G. Absalom, *op. cit.* (1929), p. 78.

and Dr Absalom's "*L. oldhamia*" must be referred to Kidston's *Lyginorachis*. From the slides I have examined I am unable to suggest the species represented.

IV. The Affinities of *Lyginorachis*.

As will be seen from the table comparing the species, *L. papilio* and *L. taitiana*, though of much earlier geological age than *Lyginopteris oldhamia*, are very similar to the petioles of that plant. We cannot, however, with any degree of certainty refer them to the genus *Lyginopteris*, and Kidston's provisional name is fully justified.

Heterangium grievii Williamson is a well-known Lower Carboniferous plant. Originally described from the Calciferous Sandstone Series of Burntisland, Firth of Forth, it was recently recorded by Dr Absalom* from the Little Limestone Coal (Upper Bernician) at South Tyne Colliery, Haltwhistle, Northumberland. While the petioles of this species, as found in Britain, show constant differences from those of *Lyginopteris oldhamia* (chiefly consisting in the simpler shape of the bundle in transverse section and the Sparganum structure of the hypodermal stereome), Dr B. Kubart† has shown that the stems of these two types are connected by intermediate forms in the Millstone Grit and Coal Measures of Silesia.

As Dr Scott‡ points out, the best evidence for the occurrence of the genus *Lyginopteris* in rocks of Lower Carboniferous age is *Calymmatotheca stangeri* Stur§ (known in Britain from the Upper Limestone Group of the Carboniferous Limestone Series of Scotland and from the Yoredale Group of Wharfedale, England). That *C. stangeri* stands near to *Lyginopteris oldhamia* will be seen from the following table, in which the two are compared¶:—

* R. G. Absalom, *loc. cit.*

† B. Kubart, "Pflanzenversteinerungen enthaltende Knollen, etc.," *Sitzungsber. d. K. Akad. d. Wiss. in Wien*, Bd. cxvii, 1908; "Corda's Sphaerosiderite aus dem Steinkohlenbecken Radnitz-Braz in Böhmen, etc.," *ibid.* Band cxx, 1911; "Über die Cycadofilicineen *Heterangium* und *Lyginodendron* aus dem Ostrauer Kohlenbecken," *Österreichische bot. Zeitschrift*, 1914.

‡ D. H. Scott, *op. cit.* (1923), p. 57.

§ D. Stur, "Die Culmflora der Ostrauer und Waldenburger Schichten," *Abhand. der K. K. Reichsanstalt zu Wien*, Band viii, Heft 2, p. (151) 257, pls. viii, ix.

¶ Since the above was sent to press, Dr W. J. Jongmans (*Jaarverslag over* 1929, Geologisch Bureau voor het Nederlandsche Mijngebied te Heerlen (1930), p. 80) has described cupules attached to impressions of the fronds of *Sphenopteris hoeninghausi*. They are identical with those described by Oliver and Scott on the (petrified) *Lyginopteris oldhamia* and, Dr Jongmans observes, "are of the same type described by Stur for *Calymmatotheca stangeri*. Drawings shown to me by Prof. Oliver, and made after Stur's original specimen, so highly agree with the cupules of *S. hoeninghausi* that there is no doubt whatever as to the generical identity, even so that they strongly point into a specifical identity between *S. stangeri* and *S. hoeninghausi*," and concludes "identity, in my opinion, is not impossible."

Comparison of *Calymmatotheca stangeri* and *Lyginopteris oldhamia*.

	C. stangeri.	*L. oldhamia.**
"Female" fructification	6-rayed stellate linear valves (about 1 cm. long) borne on the ultimate divisions of the naked rachis, thought by Stur and confirmed by Oliver † to be cupules from which seeds had been shed (formerly thought to be sporangia).	similar cupules invested the seed of *L. oldhamia* (i.e. *Lagenostoma lomaxi*).
"Male" fructification.	unknown.	probably *Telangium scotti* Benson.‡§
Sterile leaves .	so similar to those of *L. oldhamia* (*i.e.* to *Sphenopteris hoeninghausi* Brongniart) as to be united to that species by Potonié,‖ though differences exist.¶	known as *Sphenopteris hoeninghausi* Brongniart.
Cortical sclerenchyma.	reticulate.	reticulate.
Spines . .	on fertile rachises and cupules.	on petioles and cupules.
Habit . .	probably formed a dense thicket ** of scrambling plants.	probably formed a dense thicket †† of scrambling plants.

* *Lyginopteris oldhamia* was first described in 1866 by E. W. Binney (*Proc. Lit. and Phil. Soc. Manchester*, vol. v (1866), p. 133) as *Dadoxylon*. W. C. Williamson (*Monthly Micro. Journ.*, vol. ii (1869), p. 66) transferred it to his new genus *Dictyoxylon* and later (*Phil. Trans. Roy. Soc.*, B, vol. clxiii (1873), p. 377) re-described it. Subsequent work on the species was done by Williamson and Scott, Lomax, Brenchley, and others. Williamson (*ibid.*, p. 393) regarded *L. oldhamia* as being related to *Lyginodendron landsburgi*. The genus *Lyginodendron* was instituted by W. Gourlie in 1843 (*Proc. Phil. Soc. Glasgow*, vol. i (1842–3), p. 108, pl. ii) for a stem bearing rounded longitudinal ridges but no leaf-scars. The type specimen, named after its discoverer, is in the Kidston Collection (No. 3220) and was refigured by Professor Seward (*Fossil Plants*, vol. iii (1917), fig. 401, p. 37). A similar fossil (No. 60) was collected by Dr H. M. Cadell from the Carboniferous Limestone Series at Grange Colliery, Bo'ness, Linlithgowshire. This form was described (but not figured) by Kidston in 1885 (*Ann. and Mag. Nat. Hist.*, 5th ser., vol. xvi (1885), p. 167) as "specimen No. 10". It is no doubt an old *Lepidodendron* stem with crinkled bark and obliterated leaf-scars, and Kidston compared it with *L. veltheimianum* Sternb., and pointed out that J. W. Dawson (*Acadian Geology* (1878), 3rd ed., p. 455, text-fig. 170 C) figured a similar specimen as "*Lepidodendron*—old bark." The Oldham plant was, therefore, for some time known as *Lyginodendron oldhamium*. In 1899, H. Potonié (*Lehrbuch der Pflanzenpalaeontologie* (1899), p. 170) suggested the name at present in general use, *Lyginopteris*. Recently, however, Dr Jongmans (*op. cit.* (1930), p. 80) has pointed out that Stur's name *Calymmatotheca* (Culm Flora, Heft 2, *Abhandl. k.k. Geol. Reichsanst.*, Band viii (1877), p. 149) has priority over Potonié's *Lyginopteris*, and that the plant should be named *Calymmatotheca hoeninghausi*

(the sterile leaves, occurring as impressions, being first described and figured by A. Brongniart (Hist. des végét. foss., vol. i (1829), p. 199, pl. lii) as *Sphenopteris hoeninghausi*).

† F. W. Oliver, "Über die neuentdeckten Samen der Steinkohlenfarne, *Biolog. Centralblatt*, Band xxv, No. 12, p. 401, fig. 6, 1905.

‡ M. Benson, "*Telangium scotti*," *Ann. Bot.*, vol. xviii (1904), p. 161.

§ R. Crookall, "*Crossotheca* and *Lyginopteris oldhamia*," *Ann. Bot.*, vol. xliv (1930), p. 1.

‖ H. Potonié, "Über einige Carbonfarne," pt. ii, *Jahrb. k. preuss. geol. Landes. für 1890*, p. 16, pls. vii–ix, 1891.

¶ See R. Kidston, "Fossil Plants of the Carboniferous Rocks of Great Britain," *Mem. Geol. Survey*, Palæontology, vol. ii (1923–25), pp. 333, 470. In both species there is considerable variation in the form of the pinnules according to their position on the frond. In *Lyginopteris oldhamia*, however, according to Kidston, the pinnules are smaller, more solid, and more closely placed than in *C. stangeri*.

** *Ibid.*, p. 334. †† *Ibid.*, p. 467.

EXPLANATION OF PLATES.

Plate I.

Fig. 1. *Lyginorachis taitiana*, T.S. petiole. × 12. Photograph by R. Kidston (Neg. No. 3012) of type section. Slide No. T. 699 D, Collection of the Geological Survey of Scotland, Edinburgh.

Fig. 2. *Lyginorachis papilio*, T.S. petiole. × 12. Photograph by R. Kidston (Neg. No. 268). Slide No. 696 B, Kidston Collection of Slides, Department of Botany, University of Glasgow.

Plate II.

Fig. 3. *Lyginorachis sp.* T.S. petiole. × 12. Photograph by R. Kidston (Neg. No. 274). Slide No. 695 B, Kidston Collection of Slides, Department of Botany, University of Glasgow.

Fig. 4. *Lyginorachis papilio*, T.S. portion of bundle. × 70. Photograph by R. Kidston (Neg. No. 269). Slide No. 696 D, Kidston Collection of Slides, Department of Botany, University of Glasgow.

Fig. 5. *Lyginorachis papilio*, T.S. portion of bundle. × 70. Photograph by R. Kidston (Neg. No. 270). Slide No. 696 D, Kidston Collection of Slides, Department of Botany, University of Glasgow.

Plate III.

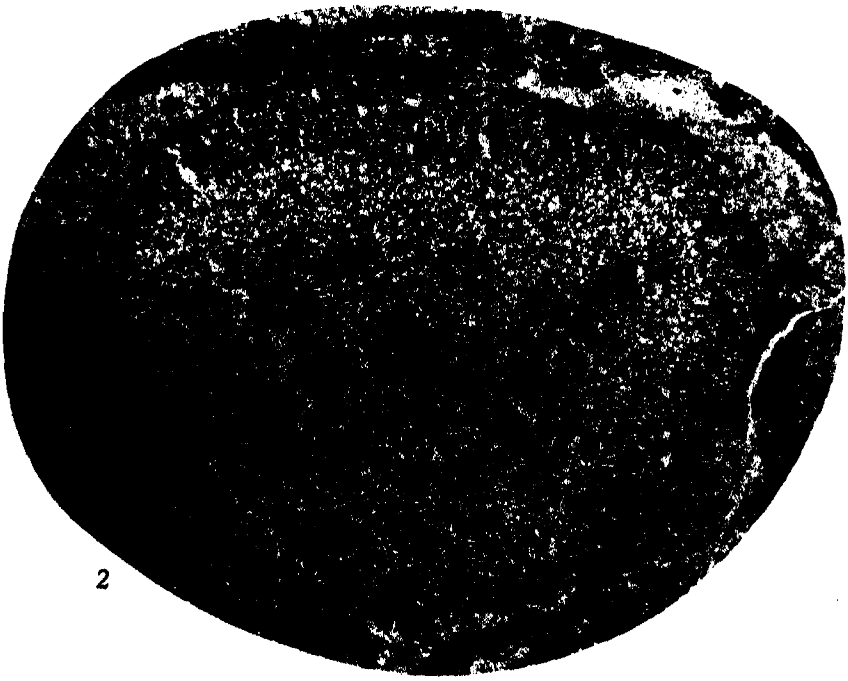
Fig. 6. *Lyginorachis papilio*, L.S. cortex. × 12. Photograph by R. Kidston (Neg. No. 273). Slide No. 696 F, Kidston Collection of Slides, Department of Botany, University of Glasgow.

Fig. 7. *Lyginorachis papilio*, L.S. bundle. × 70. Photograph by R. Kidston (Neg. No. 271). Slide No. 696 E, Kidston Collection of Slides, Department of Botany, University of Glasgow.

Fig. 8. *Lyginorachis papilio*, L.S. tracheids. × 130. Photograph by R. Kidston (Neg. No. 272). Slide No. 696 E, Kidston Collection of Slides, Department of Botany, University of Glasgow.

(*Issued separately May 16, 1931.*)

1



2

Lyginorachis taitiana & L. papilio

LYGINORACHIS PAPILIO, Etc.

6

7

8

LYGINORACHIS PAPILIO

VII.—On Charlier's New Form of the Frequency Function.

By **A. C. Aitken** and **A. Oppenheim**.

(MS. received February 5, 1931. Read March 2, 1931.)

§ 1. Introductory.

It is known (*cf.* Charlier's statistical memoirs, *passim*) that the frequency function of Type A can be written compactly in symbolic form,

$$f(z) = \exp\left(\sum_{r=3}^{\infty} a_r D^r\right)\phi(z),$$

where $r!\, a_r$ is equal to k_r, the r^{th} seminvariant of the distribution, $D = d/dz$ and

$$\phi(z) = (2\pi k_2)^{-\frac{1}{2}} \exp\left\{-\tfrac{1}{2}(z-k_1)^2/k_2\right\}.$$

For most purposes it is convenient to take the mean, k_1, as origin and the standard deviation, $k_2^{\frac{1}{2}}$, as unit. The function of Type A then becomes

$$f(z) = \phi(x) + A_3\phi'''(x) + A_4\phi^{\text{IV}}(x) + \ldots,$$

where $\phi(x) = (2\pi)^{-\frac{1}{2}}e^{-\frac{1}{2}x^2}$, and A_r is the coefficient of $(-)^r t^r$ in the expansion of $\exp\left(\sum_{r=3}^{\infty} a_r t^r\right)$.

There are, however, some inherent practical defects in the series of Type A. The chief of these is that as r increases, the order of magnitude of A_r does not decrease *regularly*; in fact, if N is the number of sources of deviation, whereas

$$a_r = O(N^{1-\frac{1}{2}r}),$$

it is found that

$$A_3 = O(N^{-\frac{1}{2}}),\ A_4 \text{ and } A_6 = O(N^{-1}),\ A_5,\ A_7, \text{ and } A_9 = O(N^{-\frac{3}{2}}), \text{ etc.}$$

(This follows readily from the fact that the A_r's are isobaric in the a's from a_3 onwards.)

Charlier has recently suggested (the reference is given at the end of this paper) that these inconveniences would be avoided if $\log f(x)$ were expanded instead of $f(x)$, and in a series of Hermite polynomials, thus,

$$\log f(x) = -\tfrac{1}{2}\log(2\pi) - \tfrac{1}{2}x^2 + c_3H_3 + c_4H_4 + \ldots,$$

where $H_r \equiv H_r(x) = (-)^r e^{\frac{1}{2}x^2}.D^r e^{-\frac{1}{2}x^2}$. He was able, by actual expansion of $\log(1 + A_3H_3 + A_4H_4 + \ldots)$ and subsequent reduction of the earlier coefficients, to prove that $c_r = O(a_r)$ as far as c_9, and was led to surmise that

$c_r = O(a_r)$ generally. We demonstrate below the truth of this, the proof resting on certain peculiar combinatory properties of Hermite polynomials.

§ 2. Formulation of the Problem.

The restriction $r \geqslant 3$ is special to the statistical problem, but not essential to the mathematical one. We examine then the order of c_r in the expansion

$$\log(1 + A_1H_1 + A_2H_2 + \ldots) = c_1H_1 + c_2H_2 + \ldots,$$

where

$$\exp(a_1t + a_2t^2 + \ldots) = 1 - A_1t + A_2t^2 - \ldots$$

Consider the coefficient of $a_\alpha a_\beta a_\gamma$ in the logarithmic expansion. (For the moment suppose α, β, γ *different* integers.) The exponential expansion has yielded the following terms in a_α, a_β, a_γ, namely,

$$a_\alpha H_\alpha,\ a_\beta H_\beta,\ a_\gamma H_\gamma,\ a_\alpha a_\beta H_{\alpha+\beta},\ \ldots,\ a_\alpha a_\beta a_\gamma H_{\alpha+\beta+\gamma}.$$

In the logarithm these terms combine in such a way as to give the following complete term in $a_\alpha a_\beta a_\gamma$, namely,

$$a_\alpha a_\beta a_\gamma\{H_{\alpha+\beta+\gamma} - 1!\,(H_\alpha H_{\beta+\gamma} + H_\beta H_{\alpha+\gamma} + H_\gamma H_{\alpha+\beta}) + 2!\,H_\alpha H_\beta H_\gamma\}.$$

Now

$$\begin{aligned} a_\alpha a_\beta a_\gamma &= O\{N^{3-\frac{1}{2}(\alpha+\beta+\gamma)}\} \\ &= O\{N^{1-\frac{1}{2}(\alpha+\beta+\gamma-4)}\} \\ &= O(a_{\alpha+\beta+\gamma-4}). \end{aligned}$$

We desire to show that the coefficient of $a_\alpha a_\beta a_\gamma$, the bracketed function of H's in the above, reduces to a *linear* function of H's involving $H_{\alpha+\beta+\gamma-4}$ at highest. More generally we shall prove that the symmetric function in the a's defined by

$$\begin{aligned} U_n(a_1, a_2, \ldots, a_n; x) \equiv H_{a_1+a_2+\ldots+a_n} &- 1!\,(H_{a_1}H_{a_2+a_3+\ldots+a_n} + \ldots) \\ &+ 2!\,(H_{a_1}H_{a_2}H_{a_3+\ldots+a_n} + \ldots) - \ldots, \end{aligned}$$

where the suffixes in the bracket preceded by $(-)^s s!$ range over all partitions into $s+1$ parts of the partible integer $a_1 + a_2 + \ldots + a_n$, reduces to a *linear* function of H's of highest order $a_1 + a_2 + \ldots + a_n - 2n + 2$. If this is achieved, then on rearrangement of terms each H_r will have a coefficient c_r of the order of a_r.

The case where equalities exist among $a_1, a_2, \ldots, a_n$ presents no difficulty. It is found that if a_r appears λ_r times, for $r = 1, 2, \ldots$, then the coefficient of $a_{a_1}a_{a_2}\ldots a_{a_n}$ is U_n divided by $\lambda_1!\,\lambda_2!\ldots$

§ 3. The Recurrence Relation of U_n.

H_r is of degree r, and is an even or odd function according as r is even or odd. U_n, being isobaric in the H's, is therefore an even or an odd

polynomial, and so expressible as

$$U_n = b_0 H_{a_1+a_2+\ldots+a_n} + b_1 H_{a_1+a_2+\ldots+a_n-2} + b_2 H_{a_1+a_2+\ldots+a_n-4} + \ldots,$$

the last term having either H_1 or a constant.

It is first noted that U_n *is zero if any a is zero.* For if a_1 is zero, the terms in U_n are each of the form $\lambda H_{\beta_1} H_{\beta_2} \ldots H_{\beta_s}$, where $(\beta_1, \beta_2, \ldots, \beta_s)$ is a partition of $a_2+a_3+\ldots+a_n$ into s parts. Now $(\beta_1, \beta_2, \ldots, \beta_s)$ may arise from putting a_1 equal to zero either in the partition $(a_1, \beta_1, \beta_2, \ldots, \beta_s)$ of $a_1+a_2+\ldots+a_n$ into $s+1$ parts, or in a partition $(\beta_1, \beta_2, \ldots, \beta_i+a_1, \ldots, \beta_s)$ of $a_1+a_2+\ldots+a_n$ into s parts. The latter event occurs in s ways, and so the coefficient of $H_{\beta_1} H_{\beta_2} \ldots H_{\beta_s}$ must be

$$(-)^s s! + (-)^{s+1}(s-1)!\, s = 0.$$

Again, since negative parts in partitions are excluded, U_n is taken to be zero in any formula where an a assumes a negative value.

We next establish a recurrence formula for U_n which generalises the recurrence formula for the Hermite polynomials.

$$\text{(A)} \qquad U_n(a_1+1, a_2, \ldots, a_n) - H_1 U_n(a_1, a_2, \ldots, a_n) - a_1 U_n(a_1-1, a_2, \ldots, a_n)$$
$$= a_2 U_{n-1}(a_1+a_2-1, a_3, \ldots, a_n) + \ldots + a_n U_{n-1}(a_2, a_3, \ldots, a_1+a_n-1).$$

PROOF.—In the three U's on the left of (A), group together such terms as have the respective a_1+1, a_1, or a_1-1 in the *same part of similar partitions*; for example, such a group might be

$$H_{a_1+1+\beta_1} H_{\beta_2} H_{\beta_3} \ldots H_{\beta_s} - H_1 H_{a_1+\beta_1} H_{\beta_2} \ldots H_{\beta_s} - a_1 H_{a_1-1+\beta_1} H_{\beta_2} \ldots H_{\beta_s},$$

where $(\beta_1, \beta_2, \ldots, \beta_s)$ is a partition of $a_2+a_3+\ldots+a_n$. [Any of the places β_i may, of course, be *unoccupied.*]

We thus obtain $H_{\beta_2} H_{\beta_3} \ldots H_{\beta_s}$ multiplied by

$$H_{a_1+1+\beta_1} - H_1 H_{a_1+\beta_1} - a_1 H_{a_1-1+\beta_1}, \text{ that is, by } \beta_1 H_{a_1-1+\beta_1}, \qquad (1)$$

in view of the familiar relation

$$H_{r+1} - H_1 H_r - r H_{r-1} = 0.$$

Thus the terms in which the place β_1 is unoccupied vanish. In the remaining terms β_1 contains parts belonging to partitions of $a_2+a_3+\ldots+a_n$. To pick out the coefficient of a_2 in these terms we note that, by (1) above, a_1+a_2-1 must always occur in the same part of a partition. Hence the terms of the coefficient of a_2 range over all partitions of the $n-1$ elements

$$a_1+a_2-1, a_3, \ldots, a_n,$$

and each will be preceded by the proper sign and factorial multiplier. But the aggregate of such terms is precisely $U_{n-1}(a_1+a_2-1, a_3, \ldots, a_n)$; and by symmetry the relation (A) follows.

§ 4. Induction from the Recurrence Relation.

It will be proved that in the linear expansion of U_n in Hermite polynomials of § 3,

$$(\mathrm{I}_n) \qquad b_i = 0 \text{ for } i = 0, 1, 2, \ldots, n-2\,;\; b_{n-1} = \frac{(a_1 + a_2 + \ldots + a_n - n)!}{(a_1 + a_2 + \ldots + a_n - 2n + 2)!}.$$

We shall first prove that if I_{n-1} holds for all $U_{n-1}(\beta_2, \beta_3, \ldots, \beta_n)$, then I_n holds for $U_n(1, a_2, a_3, \ldots, a_n)$.

For on this assumption, if $a_1 = 0$ in (A), the degree of terms on the right is $a_2 + a_3 + \ldots + a_n - 2n + 3$; and on the left only $U_n(1, a_2, \ldots, a_n)$ survives. This is the correct degree. Also the coefficient of $H_{a_2+a_3+\ldots+a_n-2n+3}$ is given as

$$a_2 a_3 \ldots a_n \frac{(a_2 + a_3 + \ldots + a_n - n)!}{(a_2 + a_3 + \ldots + a_n - 2n + 3)!}(a_2 + a_3 + \ldots + a_n - n + 1)$$

$$= 1 \,.\, a_2 a_3 \ldots a_n \frac{(1 + a_2 + a_3 + \ldots + a_n - n)!}{(1 + a_2 + a_3 + \ldots + a_n - 2n + 2)!},$$

again as it should be.

Next, we shall prove that if I_{n-1} holds for U_{n-1}, and I_n for every U_n up to $U_n(a_1, a_2, \ldots, a_n)$, then it also holds for $U_n(a_1 + 1, a_2, \ldots, a_n)$.

For, again referring to (A), we easily see that the degree is correct, while the coefficient of $H_{a_1+a_2+\ldots+a_n-2n+3}$ comes out as

$$a_1 a_2 \ldots a_n \frac{(a_1 + \ldots + a_n - n)!}{(a_1 + \ldots + a_n - 2n + 2)!} + a_2 a_3 \ldots a_n \frac{(a_1 + \ldots + a_n - n)!}{(a_1 + \ldots + a_n - 2n + 3)!}$$
$$\times \{(n-1)a_1 + a_2 + \ldots + a_n - n + 1\}$$

$$= a_2 a_3 \ldots a_n \frac{(a_1 + \ldots + a_n - n)!}{(a_1 + \ldots + a_n - 2n + 3)!}\{a_1(a_1 + \ldots + a_n - 2n + 3)$$
$$+ (n-1)(a_1 - 1) + a_2 + a_3 + \ldots + a_n\}$$

$$= (1 + a_1) a_2 a_3 \ldots a_n \frac{(a_1 + \ldots + a_n - n)!}{(a_1 + \ldots + a_n - 2n + 3)!}(1 + a_1 + a_2 + \ldots + a_n - n)$$

$$= (1 + a_1) a_2 \ldots a_n \frac{(1 + a_1 + \ldots + a_n - n)!}{(1 + a_1 + \ldots + a_n - 2n + 2)!},$$

again as it should be.

The induction is now complete, for $\mathrm{I}_1(a)$ is merely the trivial identity $U_1(a) = H_a$, and $\mathrm{I}_2(1, a)$ is the recurrence relation of Hermite polynomials,

$$H_{a+1} - H_1 H_a = a H_{a-1}.$$

The later coefficients $b_n, b_{n+1}, \ldots$ do not seem to follow any simple law, though it readily follows from the preceding work that $b_i(a_1, a_2, \ldots a_n)$ must be divisible by $a_1 a_2 \ldots a_n$.

The rearrangement of terms employed in applying the result I_n to Charlier's problem is valid, provided that the expansion of log $f(x)$ in Hermite polynomials is absolutely convergent. The conditions for convergence of such series have been recently discussed in two papers, by T. Kameda and E. Hille, cited at the end of this paper.

§ 5. Remarks on the Practical Use of Charlier's Series.

In spite of the theoretical advantages of the new "frequency function of Type C," there would seem to be still some practical disadvantages. The kind of curve fitted would usually be one in which the relative frequency was a maximum near the mean, and decreased to very small fractional values at the boundaries of the distribution. Now these small frequencies are the most unreliable in practice, yet in the present method they would receive undue importance, their logarithms being numerically large and negative. A system of weighting would thus seem to be called for, and this would introduce complications.

Again, the values of the coefficients in the Type C series would be found from the orthogonal properties of the Hermite polynomials, and would thus involve integrations between infinite limits. In practice these integrations would be approximated to by sums of discrete ordinates taken between finite limits; but the degree of accuracy would call for investigation.

§ 6. Combinatory Analysis involved in the Problem.

The properties of the function U_n lead to some peculiar results concerning symmetric functions of parts in a partition which seem worthy of notice and which, if they could be proved independently, would give a direct proof of the result I_n. The symmetric functions concerned are of the second order.

Consider a partible integer $a_1+a_2+ \, . \, . +a_n$, and suppose that $(\beta_1, \beta_2, \, . \, . \, , \beta_s)$ is a certain s-part partition of it. The second elementary symmetric functions $\sum\beta_i\beta_j$, $i \neq j$, may be summed for all s-part partitions; we shall call the resulting symmetric function $\sum\limits_s (a)_2$ for brevity. Again, if each of these second symmetric functions were raised to the r^{th} power, the resulting sum will be denoted by $\sum\limits_s (a)_2{}^r$. Finally, if in each term involving a β to a highest power m we replace β^m by $\beta(\beta-1) \, . \, . \, (\beta-m+1)$, or say $\beta^{[m]}$, we have symmetric functions

$$\sum_s (a_2)^{[r]}.$$

The identities in question refer to the last kind, and are these: for a partible integer of n parts a_i,

$$(n-1)!\,(a)_2^{[r]}-(n-2)!\sum_{n-1}(a)_2^{[r]}+(n-3)!\sum_{n-2}(a)_2^{[r]}-\,.\,.\,+(-)^n\sum_{2}(a)_2^{[r]}=0,$$

for $r=0, 1, 2, \ldots, n-2$.

These follow at once by substituting in the result I_n the expressions for products of Hermite polynomials as linear functions of Hermite polynomials, namely:

$$H_{a_1}H_{a_2}\,.\,.\,H_{a_n}=H_{a_1+a_2+\,.\,.\,+a_n}+(a)_2/1!\,H_{a_1+a_2+\,.\,.\,+a_n-2}+(a)_2^{[2]}/2!\,H_{a_1+a_2+\,.\,.\,+a_n-4}+\,.\,.\,.$$

[These expressions are readily established by observing that, as H_r is the coefficient of $t^r/r!$ in $\exp(-\frac{1}{2}t^2+tx)$, so $H_{a_1}H_{a_2}\,.\,.\,H_{a_n}$ is the coefficient of $t_1^{a_1}t_2^{a_2}\,.\,.\,t_n^{a_n}/(a_1!\,a_2!\,.\,.\,a_n!)$ in

$$\exp\left(-\tfrac{1}{2}\sum t_i^2+\sum t_i x\right)$$
$$=\exp\left\{-\tfrac{1}{2}\left(\sum t_i\right)^2+x\sum t_i\right\}.\exp\left(\sum t_i t_j\right),\ (i\neq j).$$

An application of the multinomial theorem then gives the result.]

These identities in the peculiar type of symmetric functions of partitions would be little more than curiosities if they did not also hold for more familiar symmetric functions. But the terms in an identity fall into groups of degree $2r$, $2r-1$, . ., in the a's, and each of these groups must also vanish identically. The terms of degree $2r$ give

$$(n-1)!\,(a)_2^r-(n-2)!\sum_{n-1}(a)_2^r+\ .\ .\ .\ +(-)^n1!\sum_{2}(a)_2^r=0,$$

for $r=0, 1, 2, \ldots, n-2$.

This can be further extended. Just as Vandermonde's Identity in Combinations is extended from positive integers to general variables by the consideration that it holds for a greater number of different integer values of a particular variable than its degree in that variable, so here we can extend the result to the case where the a's are no longer necessarily positive integers, but may be general variables. And lastly, by theorems on the mutual expressibility of symmetric functions, we can replace the elementary symmetric functions of the second order by any other type of symmetric function of the second order, such as the sum of squares, or the complete homogeneous product sum, and the identities still hold, provided they are written, *e.g.* for the sum of squares, S_2, as

$$(n-1)!\,S_2^r-(n-2)!\sum_{n-1}S_2^r+\ .\ .\ .\ +(-)^{n-1}\sum_{1}S_2^r=0.$$

To make this clear by an example, suppose that for the a's we have 5 units, and that we form partitions from these. Then $(1, 1, 1, 1, 1)$ arises in 1 way; $(1, 1, 1, 2)$ in 10 ways; $(1, 2, 2)$ in 15; $(1, 1, 3)$ in 10; $(2, 3)$ in 10; $(1, 4)$ in 5; and (5) in 1 way. The sum of squares, S_2, of parts in the respective partitions has values 5, 7, 9, 11, 13, 17, 25; and it is asserted that

$$4!\ 5^3 - 3!\ 10 \times 7^3 + 2!\ (15 \times 9^3 + 10 \times 11^3) - 1!\ (5 \times 17^3 + 10 \times 13^3) + 25^3 = 0,$$

which is true, and remains true when squares, first powers, or units are substituted for the cubes, or when any other type of symmetric function of the second order, for example, the complete homogeneous symmetric function, replaces S_2.

REFERENCES.

CHARLIER, C. V. L., *Meddelande Lunds Astr. Observ.*, 2, 51 (1928), "A New Form of the Frequency Function."

KAMEDA, T., *Journ. Fac. Sci. Imp. Univ. Tokyo*, 1_1 (1), (1925), "Theory of Generating Functions and its Application to the Theory of Probability," pp. 1–62, and specially 31–36.

HILLE, E., *Annals of Math.*, 2, 27 (1926), "A Class of Reciprocal Functions," pp. 427–464, and specially 457–464.

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VIII.—Some Noteworthy Examples of Parallel Evolution in the Molluscan Faunas of South-eastern Asia and South America.

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One of the main objects of the study of animal distribution is to determine the probable areas of origin and dispersal—Osborn's Centres of Radiation—of the different families and genera of the various classes of animals. This is not an easy matter, and, as reference to any standard work on molluscs would show, the questions of the origin, in time and space, of the various families of molluscs, as also their relationships, are often severely left alone. The classification is, as a result, often quite arbitrary and does not show the genetic relationships of the different members. In dealing with families with a world-wide and, more particularly, a discontinuous distribution, it is a matter of grave doubt whether several of the families, as defined, are homogeneous units which originated in any one area of the globe and spread to other regions. A serious difficulty in this connection is that the ordinary methods of dispersal of molluscs, so far as we are acquainted with them, do not help to explain the present-day distribution, while the scanty geological record in most cases makes it almost impossible to come to any satisfactory solution of the intricate problems.

The freshwater molluscan fauna of Peninsular India offers some very interesting forms for the study of the relationships and geographical distribution of a number of families of Gastropods and Pelecypods, not only for the area itself but also in connection with the origin of those families which have a wide range of distribution in Asia, Africa, and America.

In the present communication I do not propose to deal with the general questions of the origin and distribution of the various families as a whole, but will restrict my remarks to some noteworthy examples of the occurrence of very similar forms, which by several authors have been considered closely allied, in such widely separated areas as Peninsular India, Assam, Burma, and Southern Asia on the one hand and South America on the other. The three principal types which will be considered here belong to the

Gastropod family Pilidæ* and the Pelecypod families Unionidæ and Etheriidæ.

PILIDÆ.

The Pilidæ, or apple-snails as they are popularly termed, are amongst the largest freshwater molluscs known. They frequent ponds, pools, tanks, lakes, marshes, paddy-fields, and in some instances streams and rivers. They occur only in areas where succulent aquatic vegetation, on which they normally feed, is abundant. This last is a very important factor in the life of these snails, as they require a large amount of nourishment, which they obtain, to some extent, by cutting thin strips with their chitinous jaws, but mainly by scraping the soft epidermal surfaces of leaves or stems and algal outgrowths with the help of their radular teeth. These snails are in some respects the most highly specialised forms amongst freshwater molluscs as they have, in addition to a fully developed and functional ctenidium, a pulmonary sac. Both these structures are functional, and are used for respiration alternately as the animal is taking oxygen from water or air; at times, however, it is difficult to be certain which of the two structures is functioning and whether both of them are not equally helping in respiration. Most of the members of the family are dextral, but the African genus *Lanistes* Denys de Montfort is sinistral. The family inhabits fresh waters of the tropics in both the Old and the New World, and is not found anywhere in the Palæarctic Region except in Lower Egypt. Without going into details regarding the exact distribution and nomenclature of the various genera and subgenera of the family, it may be noted that the forms which mainly concern us here are the genera *Pila* Röding and *Turbinicola* Annandale and Prashad, in Asia, and *Pomacea* Perry, *Limnopomus* Dall, *Asolene* d'Orbigny, and *Pomella* Gray, in South America.

The genus *Pila*† is confined to the Oriental and Ethiopian Regions. In the Oriental Region it is found in India, except for the provinces of Punjab and Sindh, in Burma, Ceylon, Siam, Malay Peninsula, French Indo-China, Malay Archipelago as far as the Celebes and the Philippines. Species of the genus are distinguished from those of other genera of the family by their thick operculum, which is calcareous internally, and by

* In view of the unfortunate replacement of the generic name *Ampullaria* Lam. by *Pila* Röding, the family name for these snails also has to be changed from Ampullariidæ to Pilidæ.

† For details regarding the distribution of this genus in India, see Prashad, B., *Mem. Ind. Mus.*, viii, pp. 69–89, pls. xiii–xv (1925); and distribution in Asia and Africa generally, Kobelt, W., *Martini-Chem. Conch.-Cab.* (*N.F.*), xx (i), *Ampullaria* (1911–1915); and Pilsbry, H. A., and Bequaert, J., *Bull. Amer. Mus. Nat. Hist.*, liii, pp. 166–202 (1927).

having a well-developed left epipodial lobe; this channel-shaped lobe during aerial respiration forms an elongated, tubular siphon by the rolling of its sides one over the other. Members of this genus are generally confined to stagnant waters, or found in slow-running streams near the banks.

The genus *Turbinicola* Annandale and Prashad * is an inhabitant of hill streams. I have had occasion to study the genotype *T. saxea* (Reeve) both in its natural surroundings and in the laboratory, but the second species—*T. aperta* (Philippi) from Burma—is known only from dry shells. *T. saxea* is found on the west coast of Peninsular India, in the Bombay Presidency, in the hill streams. It is active only during and for some time after the rains, when there is water in the seasonal streams in which it lives; during the remainder of the year the animals of this species, which, like all other members of the family, can withstand a great deal of desiccation, remain embedded in the muddy beds of the streams. Animals of *T. saxea* during the active season are found living along the banks of the streams or on rocks which project well above the water level. They do not frequent the deeper waters, and even when forced into the deeper parts of the streams, they crawl back to the banks or projecting rocks. This is to be explained by the fact that they are better suited for aerial than aquatic respiration, and, as was found by several experiments, snails kept under water in an aquarium in the laboratory died in a couple of days, through asphyxiation. The structural modifications in the animals of this genus, which has certainly evolved from the common Pilid genus of the region—*Pila* Röding—are (i) the two nuchal lobes or pseudepipodia are equally developed, and the left lobe cannot, therefore, form an elongated respiratory siphon. During aerial respiration the left lobe forms a short funnel-shaped gutter, and as the opening of the pulmonary sac in *Turbinicola* is situated much more anteriorly, atmospheric air can be taken into the pulmonary chamber as easily by animals of this genus as by those of the genus *Pila* through their elongated siphon; (ii) the pulmonary sac is more capacious, its walls are more vascular, and its opening into the pallial cavity is comparatively larger and situated further forward towards the mantle margin; (iii) the epitænia is better developed, and serves as a more efficient agent in the separation of the branchial from the pulmonary part of the pallial cavity; and (iv) the ctenidium appears to have become degenerate, and as a result the ctenidial leaflets are comparatively smaller. All these structural modifications render the animals of *Turbinicola* much more suited to aerial

* For a detailed account of the habits and distribution of the two species, see Prashad, B., *Mem. Ind. Mus.*, viii, pp. 86–89 (1925), and *Proc. Twelfth Ind. Sci. Cong.*, pp. 134–136 (1925).

respiration, and appear to have been evolved in response to the environment in which these snails live. In addition to these changes in the soft parts of *Turbinicola*, the shells of the two species of the genus have, apparently in response to their peculiar habitat, become very much smaller and instead of the normal globose form of the members of the genus *Pila* they have assumed a more globose-ovate to almost ovate contour. The evolution of the genus *Turbinicola* in India and Burma appears to me to have taken place in response to the peculiar *milieu* independently in two distinct centres:—(i) in Peninsular India in the hill streams of the Western Ghauts, and (ii) in Burma in the hill streams of the Arakan Yomas.

The genus *Pomacea* Perry is the dominant type of the family Pilidæ in the New World. Its range of distribution extends from Central Mexico in North America to the La Plata River system in South America; a few species occur in Florida and Georgia, and some are also known from the West Indies. The systematics of the American species of the family are in a state of great confusion, and very little is known of the anatomy of the various forms. It is, therefore, not possible to determine clearly their relationships, but the anatomical characters, so far as they are known, seem to indicate a relationship of the genus *Pomacea* Perry with the Ethiopian genera *Saulea* Gray and *Afropomus* Pilsbry and Bequaert,* and possibly through these genera with *Pila* Röding, which is, as noted above, the dominant genus of the Oriental and the Ethiopian Regions.

Pomacea is provided with a very long respiratory siphon, which, as in the case of *Pila*, is formed by the left pseudepipodium, but is relatively longer. The shells of *Pomacea* and *Pila* differ in several minor characters, but the main distinguishing feature of *Pomacea* is its wholly corneous operculum. Most of the species of the genus *Pomacea* are found in marshy low banks of rivers or in swamps, ponds, and lakes, but a number of species which occur in distinct river systems in the highlands of Venezuela, Colombia, Ecuador, Peru, and which Dall † proposed to separate as a section or subgenus, *Limnopomus*, are of special interest. The ecological conditions under which this heterogeneous group of species live have never been studied, and most of the inferences as to the effects of their peculiar habitat on the snails must be purely hypothetical. The structure of the animals of this section is also unknown, but I agree with Pilsbry and Bequaert's remark (*loc. cit.*, p. 167, footnote 3) "that it will prove to be

* For further details in reference to *Saulea* and *Afropomus*, see Pilsbry and Bequaert's work (pp. 170, 171) cited already. Both these Old World genera are peculiar in possessing a corneous operculum, but unfortunately we know very little about their anatomy.

† Dall, W. H., *Journ. Conch.*, xi, p. 52 (1904).

a brevisiphonate form". The shells of this group of apple-snails are very thick and solid, depressed globose to ovoid or oblong-conoidal in outline, the spire is comparatively short, the whorls are not greatly swollen, the shell only narrowly umbilicate but more often quite imperforate and with a thick columellar callus forming a pad-like structure along the inner lip, reminding one, as Dall noted, of the structure in *Natica clausa.* As examples of this section may be noted, from the Orinoco River system, *Pomacea* (*Limnopomus*) *interrupta* (Sowerby) and *P.* (*L.*) *pertusa* (Sowerby) from Venezuela, and *P.* (*L.*) *castelloi* (Sowerby) from Colombia; from the headwaters of the Amazon system, *P.* (*L.*) *expansa* (Miller) and *P.* (*L.*) *solida* (v. d. Bupch) in Ecuador, and *P.* (*L.*) *sprucei* (Reeve) and *P.* (*L.*) *columellaris* (Gould) in Peru. Probably the species described under the names *Ampullaria impervia* Philippi and *A. producta* Reeve from the Amazon should also be referred to *Limnopomus*, but the shells of these species are less highly modified and there can be little doubt that they are found in slow-running waters of the lower reaches of the Amazon. The species mentioned above have, in my opinion, all evolved as a result of life in torrential streams from the common American genus *Pomacea*, but in view of the widely separated areas in which they are found and the important specific differences between the various species, there is a clear indication that they are not all the descendants of the same ancestral form. They probably have evolved independently from similar types, in response to identical environmental factors operating on them.

The genera *Asolene* d'Orbigny * and *Pomella* Gray,† both of which are represented by single species only, are two very interesting types. Both of them are found in running waters, but the exact ecological conditions under which they live are not known. Both of them are found in the Uruguay River, and though the modifications in the structure of their shells are along diametrically opposite lines, there can be no question that these must have been induced by life in running waters. In the case of *Asolene*, owing to the very great similarity in form of the shell of *A. platæ* (Maton)‡ to that of *Turbinicola saxea* (Reeve), it can safely be inferred that it is an inhabitant of rapid running waters in hill streams, and this

* d'Orbigny, A. D., *Voy. Amér. Mér. Moll.*, p. 364 (1837). Later, in 1840, on p. 379 the author changed the generic name to *Ampulloidea.*

† Gray, J. E., *Proc. Zool. Soc. London*, p. 148 (1847).

‡ The shell of this species is small, globose-ovoid, with a very short spire, normal ovoidal mouth, imperforate, and has a moderately developed columellar callus. For good figures of the species, see Kobelt, *loc. cit.*, pl. lxxii, figs. 4, 5 (1914). The second species *Asolene commissionis* (Ihering) Kobelt (*loc. cit.*, p. 202, pl. lxxii, figs. 6–8) is *Pomacea* (*Marisa*) *roissyi* (d'Orbigny), and not a member of the genus *Asolene.*

appears to be the case from von Ihering's description of one of the localities * whence he obtained specimens of this unique species. The only known species of the genus *Pomella* Gray—*P. megastoma* (Sowerby)—is found near the mouth of the Uruguay River. It is a large-sized species with a solid hemispherical shell, consisting of only a few very rapidly increasing whorls, the shell is imperforate, and the columellar callus, which is very broad and well developed, appears to form with the somewhat retroverted outer lip a very efficient structure for adhering to stones or rocks. The anatomy of *Pomella* is unknown, but Doello-Jurado † found that it is a brevisiphonate genus like *Asolene*, and I have no doubt that *Pomella*, like *Asolene*, has evolved from *Pomacea*.

The earliest representatives of the family Pilidæ, according to Zittel,‡ are found in the uppermost Cretaceous and the Tertiary deposits at Rognac near Marseilles, but Fischer § rightly questioned whether the forms which had been assigned to this family were not really Naticidæ. Arldt ‖ tries to get over the difficulty by suggesting that the four alleged European species of the family "sind dies jedenfalls noch marine Formen". Little is known as to the distribution of the family in the past in Asia, Africa, or South America, but the finding of a species ¶ in the Lower Siwalik Series in Poonch, Kashmir, seems to indicate that the range of distribution of the family in India in Cretaceous times was much more extensive in the north-west than it is to-day. The relationships of the Pilidæ are very confused, but the family seems to be allied to the Viviparidæ,** and may have originated, like the latter, from the ancestral type of the Trochidæ and Turbonidæ. Bouvier,†† on the other hand, though admitting their relationships with the Viviparidæ is inclined to connect them with the Naticidæ. Whatever may be its exact ancestry, there can be little doubt that the family arose, like the other families of freshwater Gastropods, from marine ancestors which had become established in estuarine waters. The less modified genera, the Ethiopian-Oriental *Pila* and *Pomacea*, were the first to take to living in fresh waters, and from these subsequently, as the species began to migrate into the upper reaches of the rivers in the rapid

* von Ihering, H., *Nachrbl. Deutsch. Malakozool. Ges.*, xxiii, p. 99 (1891).

† Doello-Jurado, M., *Physis*, ii, pp. 39, 40 (1915).

‡ Zittel, K. A. von, *Grundzüge der Palaeontologie*, i, "Invertebrata," p. 465 (1924).

§ Fischer, P., *Man. Conchyliol.*, p. 737 (1885).

‖ Arldt, T., *Handbuch Palaeogeogr.*, ii, p. 1165 (1922).

¶ Prashad, B., *Rec. Geol. Surv. Ind.*, liii, pp. 210-212, pl. xv (1925).

** See Prashad, B., *Mem. Ind. Mus.*, viii, p. 92 (1925); and Haller, B., *Morphol. Jahrb.*, xviii, p. 538 (1892).

†† Bouvier, M. E. L., *Ann. Sci. Nat. (Zool.)*, (7) iii, pp. 104, 105 (1887).

running waters, such genera as *Turbinicola* in India and Burma, and *Limnopomus* and *Asolene* in South America were evolved as a result of the effect of similar environment.

NAIADACEA.

In the superfamily Naiadacea Lam. em. Ortmann * is included an interesting assemblage of freshwater pearl-mussels. These mussels have a world-wide distribution not only on all the main continental areas, but also in such islands as Great Britain, Cuba, Madagascar, Ceylon, Sumatra, Java, Borneo, New Guinea, Australia, Tasmania, and New Zealand. With the exception of a species of *Nodularia* (*Cælatura*) † described in recent years from the island of Réunion, none are known from the Mascarenes, Celebes, Jamaica, or Haiti. The Naiades appear for certain to have been found as fossils up to the Triassic,‡ but did not become dominant till the early Cretaceous, and in some instances till the Tertiaries. The origin of the Naiades is still a matter for discussion. Neumayr,§ from his studies on the hinge of the family Unionidæ, concluded that they had originated from the marine family Trigonidæ, and, following Steinmann's suggestion, placed them in his new group, Schizodonts. A serious objection to the inclusion of the two families Unionidæ (or the superfamily of Naiadacea) and Trigonidæ in the same order is, that the two families differ very materially in the structure of their gills, the former being Eulamellibranchs, while the latter are Filibranchs.‖ Wöhrmann ¶ was definitely able to connect the Naiades with the Cardinidæ, and considered the marine genus *Trigonodus* Sandberger as the ancestral form of this family, but from his remarks "Auffallend ist, das man in den Raiblerschichten der Alpen *Trigonodus* nur in solchen Sedimenten findet, deren petrographische Beschaffenheit, sande, Gerölle, Mergel, für eine Ablagerung in nächster Nahe eine Küste spricht. Dasselbe ist auch in Nordamerika

* Ortmann, A. E., *Mem. Carnegie Mus.*, viii, p. 1 (1919).

† Germain, L., *Bull. Mus. d'Hist. Nat. Paris*, xxv, p. 122 (1919), and *Faun. Malacol. terr. fluv. Iles Mascareignes*, pp. 403, 404 (1921).

‡ The Devonian and Carboniferous forms described as Unionidæ by Sowerby, King, and MacCoy are now referred to the family Cardinidæ Zittel. For references to the works of the author cited and a discussion of the forms, see Pohlig, H., *Palæontographica (N.F.)*, vii, pp. 110–112 (1880).

§ Neumayr, M., *Sitzungsber. Kais. Akad. Wiss. Wien (Math.-Nat. Cl.)*, xcviii, pp. 5–27 (1889). Also see the same author's posthumous work in the *Denkschrift* of the same Academy, vol. lviii, pp. 701–801 (1891).

‖ See Pelseneer, P., In *Lankester's Treatise on Zoology*, pt. v, "Mollusca," pp. 258, 267 (1906).

¶ Wöhrmann, S. Frh. v., *Jahrbuch Kais.-Kon. Geol. Reichsan.*, xliii, pp. 1–28 (1893).

bei *Trigonodus cristonensis* der Fall. Es ist daher sehr wahrscheinlich, dass diese Nähe der Küste einzelne Examplare veranlasst haben, sich in Flussmündungen anzusiedeln und dort heimisch zu werden, wodann die Umwandlung in *Unio* sich vollzog," it is clear that he was inclined to consider the family as having originated from similar ancestors in a number of centres independently. Pohlig* a few years earlier had found certain Triassic brackish-water forms which he separated under the generic name *Uniona*, and came to the conclusion that the carboniferous genus *Anthrocosia* King, his new genus, and the Jurassic *Cardinia* Agassiz form a complete chain between the family Cyprinidæ d'Orbigny on the one hand, and Naiades on the other. Douvillé† derived the Taxodonts and the family Unionidæ from the Preheterodonts, through the Anthrocosidæ, the Arcidæ, and the Trigonidæ. Pelseneer,‡ basing his classification mainly on the anatomy, derived the family Trigonidæ from the Arcidæ, but made no reference to the origin of the Unionidæ. Odhner,§ from his studies on the excretory system of the different families, reconstructed the probable line of origin of the Unionidæ as being from the Protobranchial stock similar to the Solenomyidæ. From this ancestral stock arose the Arcidæ, with Trigonidæ as a side branch, and from this family originated the Unionidæ at a much earlier date. In the above review I have not considered the schemes of classification of the Pelecypods advanced by Lankester, Noetling, Dall, Fischer, Sharp, Borisjack, Grobben, Woodward, Bernard, Rice, and Ridewood,‖ as these schemes, though differing materially in certain respects, do not help us in understanding either the origin or the relationships of the Naiades. I hope at a later date to deal with the origin and relationships of this superfamily, but it may be noted here that, while mainly agreeing with Neumayr and Wöhrmann, I am of opinion that this superfamily, as now constituted, is not a natural one, but appears to have had a polyphyletic origin. Mutelid forms with a true taxodont dentition having been derived almost directly from the Arcidæ, while the Anodontidæ and Unionidæ originated from Uniona-like forms. This is indicated not only by the very different dentition, but also by the very marked dissimilarity in their gill-structure, the mantle connections, and, if von Ihering's

* Pohlig, H., *Palæontographica* (*N.F.*), vii, pp. 109–127, pls. xiii, xiv (1880).

† Douvillé, M. H., *Compt. Rend. Acad. Sci. Paris*, cliv, pp. 1677–1682 (1912).

‡ Pelseneer, P., *Bull. Sci. France Belgique*, xv, pp. 27–52 (1889).

§ Odhner, N. Hj., *Zeitschr. wiss. Zool.*, c, pp. 375–399 (1912).

‖ For an admirable review of the work of all these authors, reference may be made to Stenta, M., *Bull. Soc. Adr. Sci. Nat. Trieste*, xxv, pp. 1–150 (1908). Also see March, M. C., *Ann. Mag. Nat. Hist.*, (8) xii, pp. 91–116, pl. iii (1912).

observations * are correct, by the different types of larval forms of the families.

My reason for including the above brief summary of the literature is to show how confused is the information in reference to the origin and also the complicated geographical distribution of these mussels. Simpson considered the geographical distribution of the recent forms in detail and published first a preliminary account † of their classification and later a synopsis, ‡ but his generic and even family divisions in several cases are faulty. Attention will be directed here to two outstanding cases, which in my opinion are only examples of parallel evolution of similar types in response to the identical types of environments, but which Simpson was misled to including in families with which they are in no way connected. In the genus *Margaritana* Schumacher he § included with the widely distributed *M. margaritifera* (Linn.) and other species of the Holarctic Region, *Unio laosensis* of Lea.|| This species, which was described from the Laos Mountains, Cambodia, is now known to have a wide distribution in Cambodia, Siam, and Burma in South-eastern Asia, and Haas ¶ was right in separating it into a distinct genus *Margaritanopsis*. After a careful comparison of shells of the only known species of this genus, *M. laosensis* (Lea),** with those of the species of the genus *Margaritana*, I am of opinion that not only has it nothing in common with the genus *Margaritana*, but that it belongs to the family Unionidæ and not to the Margaritanidæ in which it was placed by Simpson. It appears to be closely allied to the genus *Trapezoideus* Simpson, and probably has evolved from a species like *T. prashadi* Haas.†† The superficial resemblance of *Margaritanopsis laosensis* to other species of the genus *Margaritana* is of the same nature as that of the Anodonta-like freshwater mussels of South America to those of the shells of the genus *Pilsbryoconcha* Simpson of Siam, Cambodia, Sumatra, and Java, as

* Ihering, H. von, *Archiv Naturgesch.*, i, pp. 45–140 (1893). In this connection also see Ortmann, A. E., *Mem. Carnegie Mus.*, viii, pp. 566–567 (1921).

† Simpson, C. T., *Proc. U.S. Nat. Mus.*, xviii, pp. 295-343, pl. ix (1896).

‡ Simpson, C. T., *Proc. U.S. Nat. Mus.*, xxii, pp. 501-1044, pl. xviii (1900).

§ Simpson, C. T., *loc. cit.*, p. 678 (1900).

|| Lea, I., *Proc. Acad. Nat. Sci. Philadelphia*, vii, p. 190 (1863).

¶ Haas, F., *Martini-Chemn. Conch.-Cab. (N.F.)*, ix (2), *Die Unioniden*, p. 121 (1912), and *Nachrbl. deutsch. Malakozool. Ges.*, xxv, p. 33 (1913).

** The second species described as *Margaritanopsis woodthorpi* from the Shan States, Burma, by Goodwin-Austen, *Rec. Ind. Mus.*, xvi, pp. 202-204, pl. xv (1909), is only *M. laosensis*.

†† Haas, F., *Senckenbergiana*, iv, p. 101 (1922), and *Abhandl. Senckenberg. Naturf. Ges.*, xxxviii, p. 199, pls. xv, xvi, fig. 3 (1924).

noted by Marshall.* The similarity in form of the remarkable Assamese form *Balwantia bensoni* (Lea)† and species of the closely allied genus *Solenaia* Conrad‡ from Siam and China of the family Unionidæ to species of various genera of Mutelidæ is still more pronounced. The shells of these elongated Solen-like forms of the family Unionidæ, which seem to have taken on the peculiar shape as an adaptation for burrowing in the hard rocky beds of streams, resemble so greatly those of the South American genera *Mycetopoda* d'Orbigny, *Mycetopodella* Marshall, *Diplondontites* Marshall, *Glabris* Gray, etc., and possibly the African genera *Mutela* Scopoli, *Pleiodon* Conrad, *Cameronia* Bourguiquat, etc., of the family Mutelidæ, that species have often been wrongly assigned to one or other of the families. The classical case is that of the species *Mycetopodella falcata* (Higgins),§ which, in spite of its having been definitely described by Higgins from "forest streams, near Chyavetas, Upper Amazons," was removed from its correct position in the family Mutelidæ to the genus *Solenaia* Conrad of the Unionidæ‖ by Simpson, and was given the supposed locality "South-eastern Asia". It was not till another shell of the same species from a tributary of the Amazon in Ecuador was described by Haas¶ as a new species under the name *Mycetopoda bolivari* that the correctness of the locality, as given by Higgins, was established. The above instances show that highly modified but almost similar types greatly resembling each other in form have been produced in the same or nearly allied families of freshwater mussels under the influence of similar environmental factors in such widely separated localities as South-eastern Asia on the one hand and South America and Africa on the other.

ETHERIIDÆ.

The Etheriidæ, or the freshwater oysters as these mussels are popularly known, constitute a small but highly interesting family. Two of its genera, *Bartlettia*** H. Adams and *Acostæa* d'Orbigny, are found in tropical South America, *Etheria* Lamarck is widely distributed in Africa

* Marshall, W. B., *Proc. U.S. Nat. Mus.*, lxxi, Art. 6, p. 3 (1927).

† Lea, I., *Lea Synonyms*, p. 57 (1870). See also Prashad, B., *Rec. Ind. Mus.*, xvi, pp. 290, 291 (1919); and Hora, S. L., *Journ. Asiat. Soc. Bengal*, (n.s.) xxii, pp. 71-76 (1927).

‡ Conrad, T. A., *Amer. Journ. Conch.*, iv, p. 249 (1868). For references to the Siamese and Chinese species, see Simpson, C. T., *Descr. Cat. Naiades*, pp. 457-463 (Detroit, 1914).

§ Higgins, T., *Proc. Zool. Soc. London*, p. 179, pl. xiv, fig. 6 (1868).

‖ For a complete history of the changes in the nomenclature, etc., of this species, see Marshall, W. B., *Proc. U.S. Nat. Mus.*, lxxi, Art. C, p. 2 (1927).

¶ Haas, F., *Trabajos Mus. Nac. Cienc. Nat. Madrid. (Zool. Ser.)*, No. 25, p. 57, fig. 2 (1916).

** For references to this family, see Anthony, R., *Ann. Soc. Zool. Malacol. Belgique*, xli, pp. 322-424 (1907); and Pilsbry, H. A., and Bequaert, J., *Bull. Amer. Mus. Nat. Hist.*, liii, p. 447 (1927).

and Madagascar, while *Pseudomulleria* Anthony is confined to Kadur District, Mysore, in Peninsular India. A map illustrating the distribution of the family as a whole was published by Anthony (*loc. cit.*, p. 418), and one showing the distribution of Etheria in Africa in greater detail is included in Pilsbry and Bequaert's invaluable paper (*loc. cit.*, p. 448) on the Aquatic Molluscs of the Belgian Congo. The family does not appear to be of any great age, as practically no fossil forms of earlier than the Pleistocene times have been found. The soft parts of the genera *Etheria** and *Pseudomulleria* have been investigated, but the South American genera *Bartlettia* and *Acostæa* are known from shells only. The anatomy of the African and Indian genera is remarkably similar, and from the great resemblance in the form of the shells of the Asiatic and the American species it may be inferred that the respective animals also will prove to be very similar. The relationships of the family are undoubtedly with the Unionidæ, and it has been suggested that the Etheriidæ are only modified Unionidæ which took to a sedentary mode of life, and later became fixed to the substratum by one or the other of the two valves. The modifications, both in the animal and the shell, though peculiar, are more or less uniform for all the genera of the family. In view of the very recent origin of the family, referred to above, and the very wide though localised distribution (except in the case of the African forms) of the different genera, there can hardly be any question of the Etheriidæ having originated or become modified from the Unionidæ of the different continents independently. This view gains support from almost similar forms having evolved in such widely separated areas as South America on the one hand, and Peninsular India on the other; the South American and Indian species were for a long time referred to the same genus, *Mulleria* Ferrusac, or, as it should more correctly be called, *Acostæa* d'Orbigny, but, as Anthony remarked (*loc. cit.*, p. 407), the two species differ markedly in shell characters, and even though nothing is known of the soft parts of the South American form, there can be no question about the Indian and the American species belonging to two distinct genera. The superficial resemblances in the shells of the two forms are purely fortuitous, and have been brought about by almost identical environmental factors working on similar types of animals. In fact, the Etheriidæ present the most noteworthy examples of the parallel evolution of similar forms from distinct ancestral types, living under identical conditions in widely separated countries.

* See Sassi, M., *Zool. Anz.*, xxxvi, pp. 25–31 (1910); and Woodward, M. F., *Proc. Malacol. Soc. London*, iii, pp. 87–91 (1898).

The presence of almost identical types belonging to the families Pilidæ, Unionidæ (Naiadacea), and Etheriidæ in quite distinct but similar habitats such as have been discussed above is to be explained as being due to the effects of the environment alone. It indicates neither generic relationships between the various forms, nor can there be any question of the types having originated and spread from any special centres of radiation. These cases cannot be explained as relict representatives of the fauna of one continental area * in another. The possibility whether the several families mentioned above, as indeed most other of the freshwater Gastropod and Pelecypod families, as understood at present, are not polyphyletic in their origin † also has to be considered, but the insufficient evidence which is available precludes a detailed consideration of this very important question.

* See, for example, Annandale, N., *IXe Cong. Internat. Zool. Monaco*, pp. 579–588 (Rennes, 1914).

† See Prashad, B., *Mem. Ind. Mus.*, viii, p. 245 (1928), for other references.

(*Issued separately May* 16, 1931.)

IX.—The Classification and Development of Carbonaceous Minerals. By Professor Henry Briggs, D.Sc., Ph.D.

(MS. received March 5, 1931. Read March 2, 1931.)

1. Method of Dealing with Analyses.

In this investigation a graphical method, based on the ultimate analysis, is applied to the classification of the carbonaceous minerals of organic origin. It is necessary in any such study to disregard those ingredients (*e.g.* ash-forming compounds, sulphur, nitrogen, and free moisture) whose presence, while greatly influencing the commercial value of a carbonaceous mineral, has no significance in regard to its classification. These extraneous or adventitious ingredients have therefore been eliminated by calculation, so as to reduce each analysis to terms of its essential elements —carbon, hydrogen, and oxygen. The fact that the relation

$$C + H + O = 100 \textit{ per cent. by weight}$$

holds true in all cases renders it theoretically immaterial which pair of the three elements are plotted against each other, since if two are given the proportion of the other can always be easily obtained. As a matter of convenience, however, I prefer the carbon-oxygen relation, and it has been adopted in the charts discussed below.

2. Previous Graphical Representation of Coal Species.

The well-known system of classification of coals due to Seyler, and dating from 1900, is based on a graphical presentation of the ultimate analyses.* Ralston† in 1915 and Hickling‡ in 1927 also graphed the percentages of carbon, hydrogen, and oxygen in coals, the former using trilinear and the latter rectangular co-ordinates. Both admirably succeeded in demonstrating that coals of the ordinary kind are members of one great species which embraces, in an unbroken series, every rank from lignite to anthracite. Their work enabled Hickling (*op. cit.*) and, more recently, the present writer § to suggest the process or processes by which each variety—lignite, sub-bituminous coal, bituminous coal, semi-bituminous coal, semi-anthracite, and anthracite—has been transformed, or is transforming itself, into fuel of successively increasing rank, thereby providing a scientific

* *Proc. S. Wales Inst. Engs.* xxi, 1900; *Fuel*, iii, 1924, pp. 15, 41, 53.
† U.S. Bureau of Mines Technical Paper, No. 93, Washington, 1915.
‡ *Trans. Inst. Mining Engs.*, lxxii, 1927, p. 261.
§ *Chemistry and Industry; Jour. Soc. Chem. Ind.*, l, 1931, p. 127.

background for the widely accepted peat-to-anthracite theory first propounded by von Beroldingen in 1778.

One of Hickling's charts (fig. 1) is reproduced here as the connecting

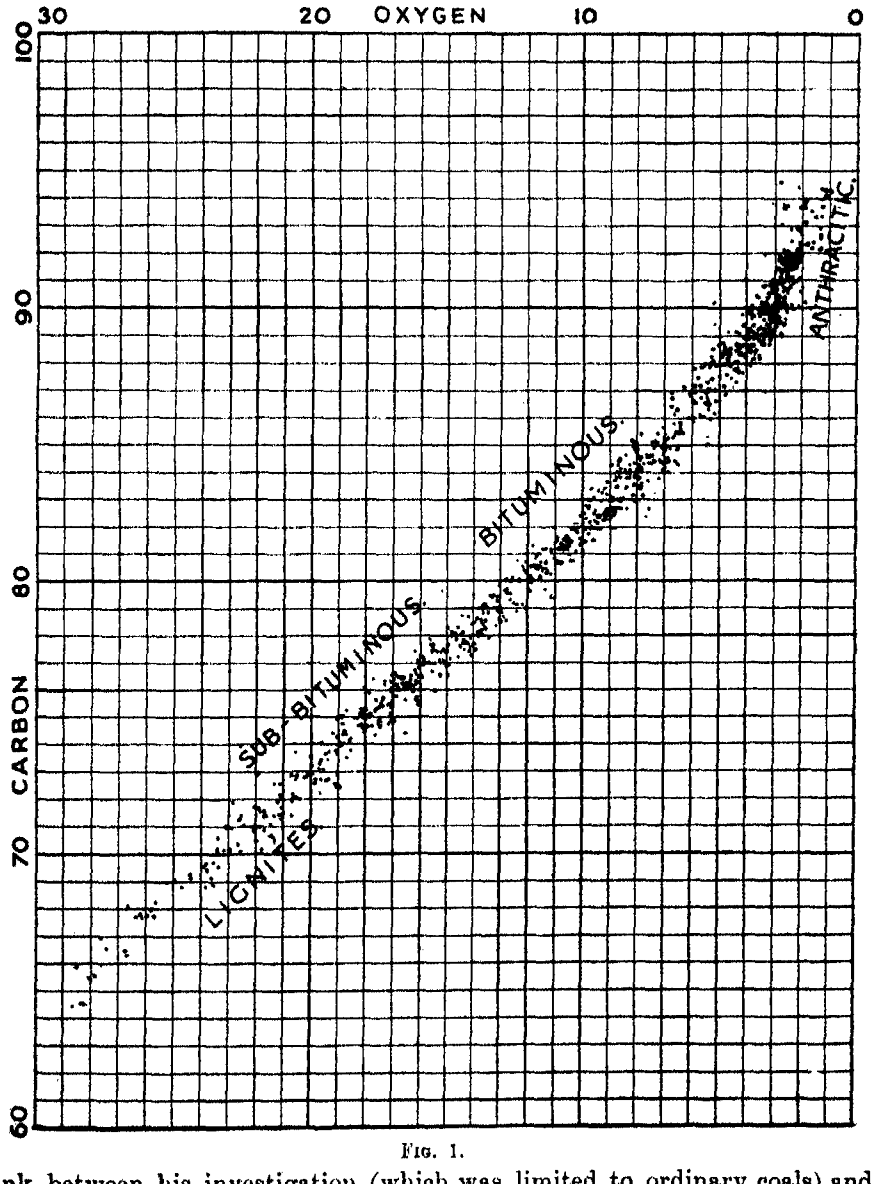

Fig. 1.

link between his investigation (which was limited to ordinary coals) and my own. It shows the carbon-oxygen relationship subsisting over the range lignite to anthracite, and, in constructing it, Hickling made use of over 1000 published analyses of American and British coals. The points

of the graph lie within a well-defined and continuous band, whose comparative narrowness enables a median to be drawn along it with considerable precision.

3. Development Lines.

Hickling's mean line, adjusted to the basis $C+H+O=100$ per cent., is given in *LABC* (fig. 2). A line of this kind denotes and defines the histolysis of the original vegetable matter and the histogeny of the mineral derived from it; it is the graphical expression of the development of the mineral species in rank, and I propose to call it the "development line" of the particular species.

Relatively few ultimate analyses of carbonaceous minerals other than ordinary coals are available; but sufficient of them have been collected from a variety of published records to enable the trend of their respective development lines to be traced out. In addition to that of ordinary coals, fig. 2 gives the development lines of the cannels (parrot coals), paraffin shales, torbanites, heavy (or asphaltic base) petroleums, and light (or paraffin base) petroleums in the order stated. Points on the chart which are of special interest have been numbered (solid minerals) or lettered (oils) to correspond with the table of the Appendix.

It now appears that there are two species of cannels having distinct development lines. The sub-cannels, which resemble ordinary coals in ultimate composition (though not in physical characteristics or in their proximate analyses), include among them the once famous Wigan cannel (Nos. 2, 3, and 4).

My data in regard to the crude petroleums do not justify more than a brief reference to them. Their analyses seem to adhere fairly well to one or other of two parallel lines whose slope is entirely different from the slope of the development lines of the solid minerals. It is not without interest to observe that asphalt (No. 47) takes a place low down in the torbanite series and not at the foot of the heavy oil group. The position of ozokerite (48), on the other hand, is where it might be expected, namely, near the bottom of the light oils line.

Of the others, the least regular series is that of the torbanites. Indeed, but for collateral evidence it would have been unwise to attempt to show even the tentative median which has been drawn.

4. Suitability for Oil Distillation.

Doubt exists as to the correct designation of one or two minerals whose analyses lie between the torbanite line and that of the paraffin shales. Dorset shale from the Main Bed, Corton (No. 19), is one of these. Whether

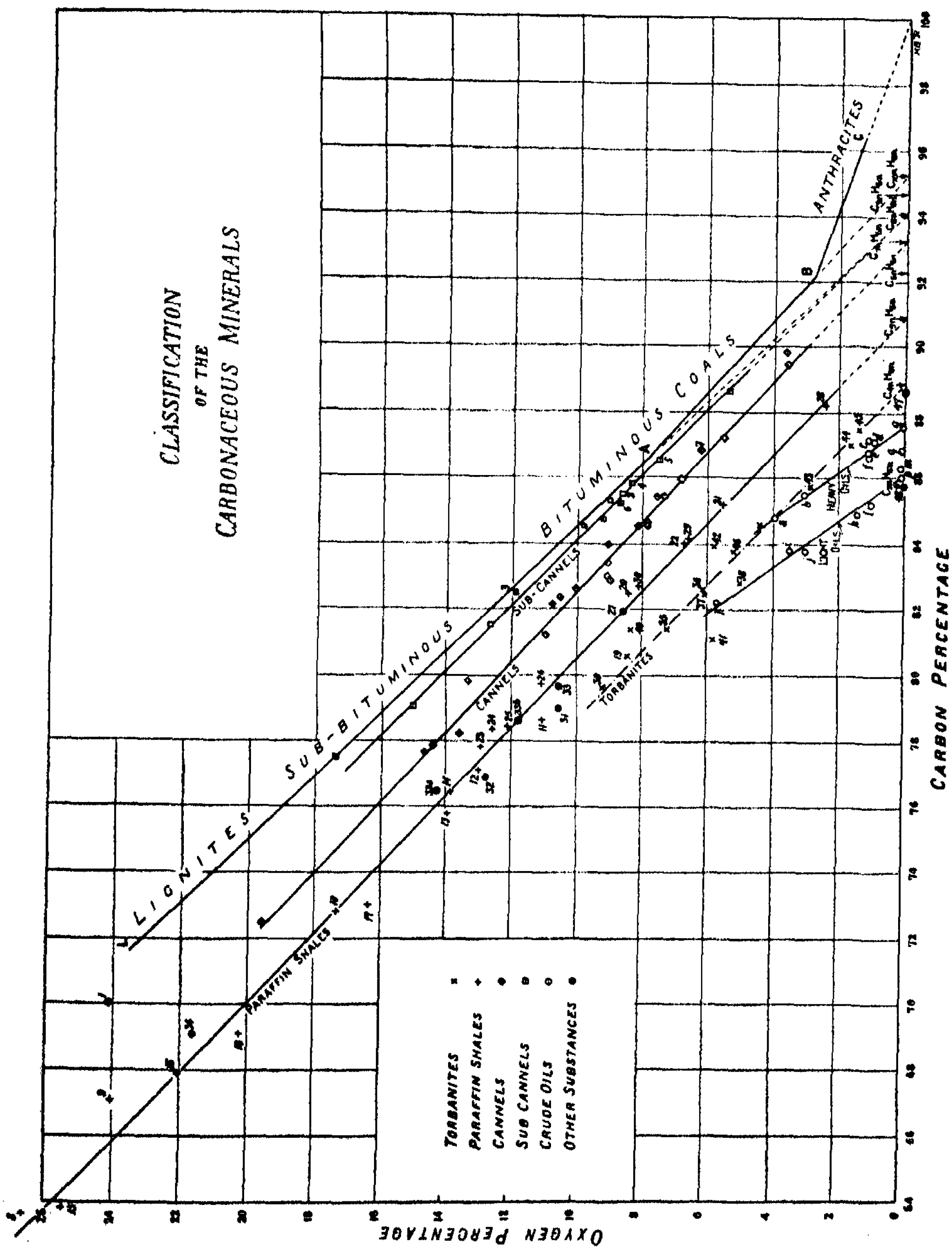
CLASSIFICATION
OF THE
CARBONACEOUS MINERALS
LIGNITES
SUB-BITUMINOUS
BITUMINOUS COALS
ANTHRACITES
SUB-CANNELS
CANNELS
PARAFFIN SHALES
TORBANITES
HEAVY OILS
LIGHT OILS
TORBANITES
PARAFFIN SHALES
CANNELS
SUB CANNELS
CRUDE OILS
OTHER SUBSTANCES
CARBON PERCENTAGE
OXYGEN PERCENTAGE

FIG. 2

regarded as a torbanite or a paraffin shale, its position on the chart is advantageous for a retorting mineral, and it is most unfortunate that its sulphur content is too high to allow of it being turned to account for oil production.

Without a known exception, the further from the coal line in the direction of the lower left-hand corner of the chart an analysis is situated (*i.e.* the higher the hydrogen proportion) the more suitable (apart from impurities) is the mineral as a source of oil. The substance taking pre-eminence in this regard is the remarkable Joadja Creek shale (No. 41); it is the richest of the "kerosene shales" of New South Wales, and is well known for its high yield and for the good quality of the oil obtained from it. A further illustration of the utility of the graphs as a means of distinguishing suitable subjects for distillation from those less suitable is afforded by the relative positions of the Leeswood (Flintshire) cannels and shale. The analyses of the Leeswood Smooth Cannel (Nos. 5 and 6), the Shale (No. 7), and the Curly Cannel (Nos. 21 and 22) place them respectively among the sub-cannels, the cannels, and the paraffin shales. The inference would be that (apart from impurities) the oil yielded by the last ought to be of good, that from the second of moderate, and that from the first of inferior quality. These indications are borne out by the records. The Leeswood cannels were extensively wrought and largely used for oil production in Mid-Victorian times. The Smooth Cannel, worth 9s. a ton in 1865, proved unserviceable as a source of oil, and was chiefly sold as a gas coal; the Shale, useless as a fuel by reason of a high ash-content, was worth 8s. 6d. a ton for retorting; while the Curly, which furnished a large yield of oil of good quality, fetched the high price of 28s. a ton.*

5. The Paraffin Shale Group.

Many of the points of the paraffin shale group were obtained from tables included in a paper with the title "Chemical Examination of the Organic Matter in Oil Shales" contributed to this Society in 1914 by Mr J. B. Robertson.† The species is remarkable in several respects, and not least in the great range in rank which the oil-shale kerogen evinces. If (as I do not doubt) the development line, like that of the coals, represents the progressive evolution from a less mature state at the upper end to a more mature state at the lower—in other words, if the series can be correlated, rank by rank, with the varieties of ordinary coal—Robertson's analyses of the Dunnet shale (Nos. 8 to 13 inclusive) show that the kerogen

* Special Reports on the Mineral Resources of Great Britain, vol. vii: *Mineral Oil, etc.* Memoir of the Geological Survey, 1920.

† *Proc. Roy. Soc. Edin.*, xxxiv, 1914, p. 190.

occurring in a single seam may vary widely, No. 8 being equivalent to a low-rank lignite and No. 11 to sub-bituminous coal. The origin of kerogen has often been discussed before this Society and others.* Some observers have affirmed its kinship with fossil resin, but the majority have disputed that relation on chemical grounds. That a similarity in composition exists seems apparent from the positions in fig. 2 of amber (No. 31), of amberite (No. 32), and of resin itself (No. 33). The proximity of lycopodium (No. 34) to the line in question is suggestive in this regard; it supports the view that the kerogen was originally resinous matter lying in the exines or walls of the spores or sporangia of lepidodendra and other club-mosses. It is also of interest to observe that a "resene" (No. 27)—incidentally, the only artificial substance included in the graphs—extracted by Bone from a Yorkshire bituminous coal by means of solvents† lies on the development line of the paraffin shales. The albertite of New Brunswick and New South Wales (Nos. 28, 29, 30), an oil-yielding mineral whose origin and character has been the subject of scientific and legal controversy, is seen to fall into place near the foot of that line.

6. Quantitative Definitions.

The equations of the lines set out below provide quantitative definitions for the cannels, sub-cannels, paraffin shales, and torbanites. Any undetermined mineral of these groups whose analysis is known may be tested against them and its proper designation obtained. The letters denote percentages by weight of the elements, the mineral being regarded as free from impurities and nitrogen.

Sub-cannels:

$$C + 81\,H = 572$$
$$C + 1{\cdot}012\,O = 94{\cdot}1$$

Observed range of hydrogen, 5·7 to 6·1 per cent.

Cannels:

$$C + 13{\cdot}6\,H = 814$$
$$C + 1{\cdot}08\,O = 93{\cdot}3$$

Observed range of hydrogen, 6·6 to 8 per cent.

Paraffin Shales:

$$C + 23\,H = 300$$
$$C + 1{\cdot}0455\,O = 90{\cdot}9$$

Observed range of hydrogen, 8·9 to 10·7 per cent.

* T. S. Traill, *Trans. Roy. Soc. Edin.*, xxi, 1857, p. 7; J. H. Bennett, *ibid.*, p. 173; E. H. Cunningham-Craig, *Proc. Roy. Soc. Edin.*, xxxvi, 1916, p. 44; H. R. J. Conacher, *Trans. Geol. Soc. Glasgow*, xvi, 1917, p. 164; E. M. Bailey in *Oil Shales of the Lothians* (3rd edition), Geol. Survey Memoir, 1927, p. 158.

† W. A. Bone, *Coal and its Scientific Uses*, 1918, p. 93.

Torbanites (tentative):

$$C + 23\ H = 344$$
$$C + 1{\cdot}0455\ O = 88{\cdot}9$$

Observed range of hydrogen, 10·2 to 13·1 per cent.

7. Observations on the Various Species and their End Products.

Reference has already been made to the occurrence of erratic points between the torbanite and the paraffin shale lines. In spite of these, the chart clearly indicates that the cannels, paraffin shales, and torbanites are distinct species following separate, though roughly parallel evolutionary courses, towards end products which are themselves distinct and characteristic. Considering the great geological and geographical range of the minerals concerned, and the probable variations in the analytical methods of the many chemists whose results I have taken, the aberrancies are fewer than might have been anticipated. Most of the minerals are true to type. The statement, common enough in the literature, to the effect that cannels pass by imperceptible degrees into shales and torbanites requires revision. Though similar in appearance, cannels are actually further removed from torbanites in composition than from common coals, and such a term as "torbanitic cannel" is inadmissible.

There is a close resemblance under the microscope between the kerogen of the bogheads and that of oil-shale, and the assumption has been that they were the same substance; but again similarity of aspect is misleading, for it now appears that torbanites and paraffin shales are different species.

When searching for a reasonable explanation of the relative regularity in form and arrangement of the development lines depicted in fig. 2, I met with an observation of Renault * to the effect that the oil-yielding matrix of boghead approximated to C_2H_3. As a broad statement this is clearly incorrect, for the composition is variable, depending, as it does, on the position on the development line of the mineral in question; moreover, all known kerogens contain oxygen. It seemed, however, possible that such might be the formula of the ultimate product towards which the boghead kerogen is slowly progressing, and when the compound was given its appropriate place on the base line of the chart it was seen to lie near the foot of the torbanite line as then drawn. The positions of several bodies of the series C_mH_{6n} were then set out (see fig. 2), when it was found that the medial line of the paraffin shales exactly ended at the point representing $C_{5n}H_{6n}$; that the portion LA of the coal curve, when continued,

* *Bull. Soc. Ind. Minerale,* xiii, ser. 3, 1899, p. 1056.

ran exactly through $C_{8n}H_{6n}$, and that, equally definitely, the part *AB* of the same curve intersected $C_{10n}H_{6n}$. The process of development of the sub-cannels, unless checked or deflected, will apparently continue with $C_{8n}H_{6n}$ as the goal. More clearly, the cannels have taken $C_{7n}H_{6n}$ as their base-line product. In view of this evidence the torbanite line was re-drawn through $C_{4n}H_{6n}$ as shown. The next body of the series, namely, $C_{3n}H_{6n}$ (or C_nH_{2n}), is ozokerite (No. 48).

As to the evolutionary process itself, it is perhaps worthy of record that a member of the paraffin shale group, situated at any position on its development line, would be able to progress along that line and finally to reach a composition connoted by the formula $C_{5n}H_{6n}$ by continuously expelling water and methane in the proportion of eight of the former to one of the latter by weight.

I have no intention of hazarding a guess as to the real meaning of such an expression as $C_{5n}H_{6n}$. It is sufficient for the present to show that the position and trend of the lines are not accidental; that each line is directed towards an end product whose composition, being definite, decides the spacing of the groups, and that the aim of the evolutionary process is apparently that of a progressive simplification of the carbon-hydrogen-oxygen complex of which these substances consist. As the coal curve indicates, the course of that process does not always run smooth. Conditions may arise, as at *A* and *B*, to deflect the aim. But though turned aside, the course is not haphazard; it is only re-oriented towards another member of the C_mH_{6n} series, until finally, in the region *BC*, it proceeds towards carbon itself.

In this aspect the inquiry opens questions offering to the chemist an untouched field for research.

[APPENDIX.

APPENDIX.

The minerals of the chart (fig. 2), to which numbers or letters have been attached, are briefly described below. Analysts' names are placed between brackets.

1.	Bovey Tracy Lignite	(Percy)
2.	Wigan Cannel	(Vaux)
3.	Do.	(Percy)
4.	Do.	(Do.)
5.	Leeswood Smooth Cannel	(Griffith)
6.	Do.	(Percy)
7.	Leeswood Shale	(Griffith)
8.	Dunnet Shale, top foot	(Robertson)
9.	Do. second foot	(Do.)
10.	Do. third foot	(Do.)
11.	Do. fourth foot	(Do.)
12.	Do. fifth foot	(Do.)
13.	Do. sixth foot	(Do.)
14.	Camps Shale, flat part	(Do.)
15.	Do. inclined part	(Do.)
16.	Broxburn Shale	(Do.)
17.	Good average oil shale	(Mills)
18.	Dorset Shale, Corton	(Berry)
19.	Do. Main Bed, Corton	(Do.)
20.	Blackstone, Kimmeridge, No. 1 Bed	(Do.)
21.	Leeswood Curly Cannel	(Griffith)
22.	Do.	(Percy)
23.	Methil Brown Shale	
24.	Do.	(Percy)
25.	Capeldrae Brown Shale	(Fyfe)
26.	Greta Shale, N.S.W.	
27.	"Resene" extracted from bituminous coal	(Bone)
28.	Albertite, New Brunswick	
29.	Do. Do.	(Percy)
30.	Do. N.S.W.	(Do.)
31.	Amber	(quoted by Dana)
32.	Amberite	(Do.)
33.	Resin	
33*a*.	Resinous Vessel Coats	(U.S. Geol. Surv.)
33*b*.	Fossil Resin	(Do.)
34.	Lycopodium	(Percy)
35.	Boghead Coal or Torbanehill Mineral	(Anderson)
36.	Do.	(Stenhouse)
37.	Do.	(Penny)
38.	Do.	(Fyfe)
39.	Do.	(Percy)

APPENDIX—*Contd.*

40.	Armadale Torbanite	(Robertson)
41.	Richest Joadja Creek Shale, N.S.W.	(Dixon)
42.	Australian Commonwealth Shale	(Robertson)
43.	Copaline, Highgate Hill	(Johnston)
44.	Autun Boghead	(Bertrand)
45.	Mt. Kembla Shale, N.S.W.	
46.	Tasmanite	(Percy)
47.	Trinidad Asphalt	(Richardson)
48.	Ozokerite, mean of seven analyses	(quoted by Dana)

CRUDE OILS.

a. Californian heavy.	*i.* Rangoon.
b. Mexican.	*j.* Roumanian.
c. Roumanian.	*k.* Pennsylvanian.
d. Java.	*l.* West Virginian.
e. Caucasian heavy.	*m.* Caucasian light. / Ohio.
f. Russian heavy.	*p.* Russian light.
g. Borneo.	*q.* Californian.
h. East Galician.	

(Issued separately May 16, 1931.)

X.—On the Identity of *Sacculina triangularis* and *Sacculina inflata*.

By **Dr H. Boschma**, Zoological Laboratory of the University of Leiden. *Communicated by* Professor J. H. Ashworth, F.R.S.

(MS. received February 25, 1931. Read May 4, 1931.)

In 1862 Anderson described a parasite of *Cancer pagurus* from the Firth of Forth, which he named *Sacculina triangularis*. The species differs from *Sacculina carcini* in its external form and the shape of the testes, particulars of which are described in Anderson's paper in sufficient detail to prove that the parasite is specifically distinct from *Sacculina carcini*. In a paper dealing with the European species of the Sacculinidæ (Boschma, 1927) I mentioned *Sacculina triangularis* as a distinct species, basing this statement on Anderson's description and figures. Afterwards I had the opportunity to study the anatomy of the species, owing to the kindness of Professor J. H. Ashworth, who sent me nine specimens of *Sacculina* on *Cancer pagurus* from the Firth of Forth. The following description of the characters of the species is based on this material.

The anatomy of the specimens on *Cancer pagurus* corresponds in every important detail with that of the specimen on *Hyas coarctatus* described previously (Boschma, 1927). Both forms consequently belong to the same species, and as the name *Sacculina inflata* was given by Leuckart (1859) to the parasite of *Hyas* three years before Anderson (1862) described his *Sacculina triangularis*, the former is the valid name of the species.

In the Sacculinidæ the region of the mantle opening is the anterior part of the body, the stalk occupies the median part of the posterior region. The animals are more or less flattened in a lateral direction. In *Sacculina inflata*, at least as far as concerns the specimens on *Cancer pagurus*, the long axis is the one passing through the stalk and the mantle opening. There are some specimens in which the distance between the dorsal and ventral region is approximately as long as that between the anterior and posterior part, or even the latter distance may be somewhat shorter than the former (fig. 1), but as a rule the parasites are elongated in the chief axis of the body. In the specimens figured by Anderson this peculiarity is clearly visible.

According to Anderson the parasites are usually gregarious: as many

as five individuals may be found on the abdomen of one crab. The specimens of the present material lived for the greater part as solitary parasites: in one case only, two specimens are found on the same host.*

The nine specimens are of different sizes, the smallest specimen has a greater diameter of approximately 2 mm., the largest specimen measures 11 mm. from the mantle opening to the stalk.

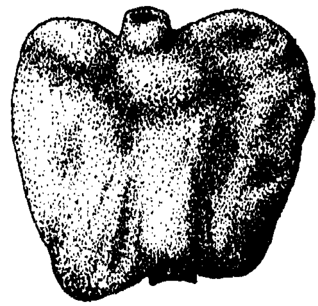

FIG. 1.—*Sacculina inflata*, specimen from *Cancer pagurus*, showing the surface, which was directed towards the thorax of the host. At the upper part of the figure the mantle opening; at the lower part the stalk. × 3.

In the smaller specimens the surface of the mantle is smooth to the naked eye, in the larger ones it is irregularly wrinkled and grooved, which may be caused partly by contraction after preservation. In the larger specimens the surface of the mantle is covered with dirt, so that the excrescences of the mantle are visible in some places only. A colony of Polyzoa has developed on a part of the mantle of one of these older specimens.

The internal anatomy has been studied in four specimens from which longitudinal sections have been made.

The visceral mass is covered with a well-developed muscular layer to which muscles are attached which pass through the visceral mass in different directions, most of them transversely. The bulk of the visceral mass is occupied by the large ovary. At the left and right sides of the ovary, situated somewhere nearer to the anterior part than to the posterior region, the colleteric glands are found. The posterior region of the visceral mass is attached to a muscular portion of the body, which terminates posteriorly in the stalk to which the mantle is attached. In this muscular region the testes are found; consequently these organs lie outside the true visceral mass. The mesentery is complete: it extends from the stalk to the mantle opening.

A fairly accurate description of the shape of the testes has already been given by Anderson (1862). They are more or less globular or oviform, and pass rather abruptly into the vasa deferentia, which are comparatively short, straight, or somewhat tortuous tubes. The latter terminate in the posterior part of the mantle cavity at the male genital openings (fig. 2).

The proximal part of each vas deferens is lined with a comparatively thick layer of chitin, as stated by Anderson.

* Professor Ashworth informs me that he has a specimen of *Cancer pagurus*, collected in the Firth of Forth in November 1927, which bears four examples of *Sacculina inflata*.

It is a peculiar fact that the two testes of *Sacculina inflata* are always of different sizes, the right one being more strongly developed than the left. In most specimens this difference in size is very striking, *e.g.*, in the specimen from which figs. 2 and 3 were drawn. Fig. 2 represents a section of the region in which the left testis attains its largest diameter, but the right testis is larger and, farther towards the dorsal part of the body, widens into a voluminous sac (fig. 3). One gets the impression that the

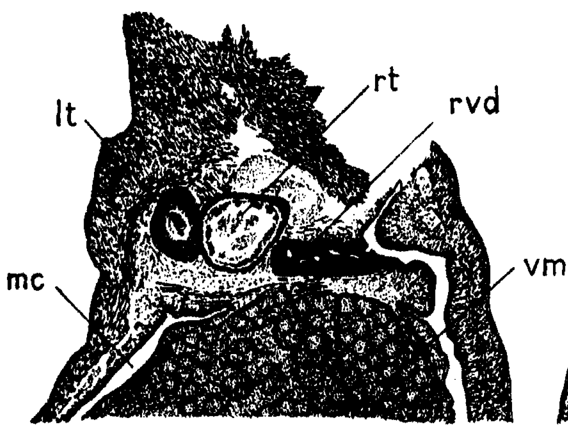

FIG. 2.—Posterior part of a longitudinal section of *Sacculina inflata* from *Cancer pagurus*. The section shows the left testis (*lt*) and the ventral part of the right testis (*rt*) with the right vas deferens (*rvd*). *mc*, mantle cavity; *vm*, visceral mass. ×18.

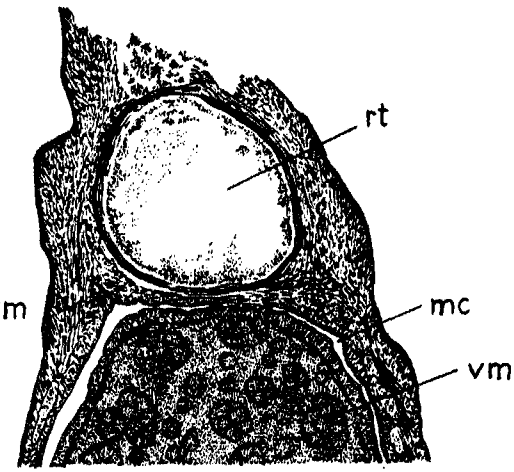

FIG. 3.—Posterior part of a longitudinal section of the same specimen. In this region the right testis (*rt*) attains its largest diameter. *mc*, mantle cavity; *vm*, visceral mass. ×18.

testis of the left side is more or less impeded in growth by the enormous development of that of the right side. In other specimens the left testis has a more or less flattened shape, whilst the right testis in section has a circular form.

In one specimen the difference in size of the two testes is less striking, though the right one is distinctly larger than the other. From this specimen sections of different regions of the male organs, taken at intervals of 200 μ, are represented in fig. 4. Fig. 4*a* is from the ventral section of the series, and fig. 4*k* from the dorsal section.

Fig. 4*a* shows the male genital opening of the right side and the ventral part of the left vas deferens. In fig. 4*c* the left vas deferens, lined with the thick layer of chitin mentioned above, is seen passing into the testis, and slightly projecting into the lumen of the testis. Sections of the largest part of the left testis are shown in fig. 4*d* and *e*; more dorsally (fig. 4*f*) the left testis gradually diminishes in size to its closed end (fig. 4*g*).

The right testis has a somewhat longer vas deferens than the left. In the region between the sections of fig. 4*d* and *e* the right vas deferens

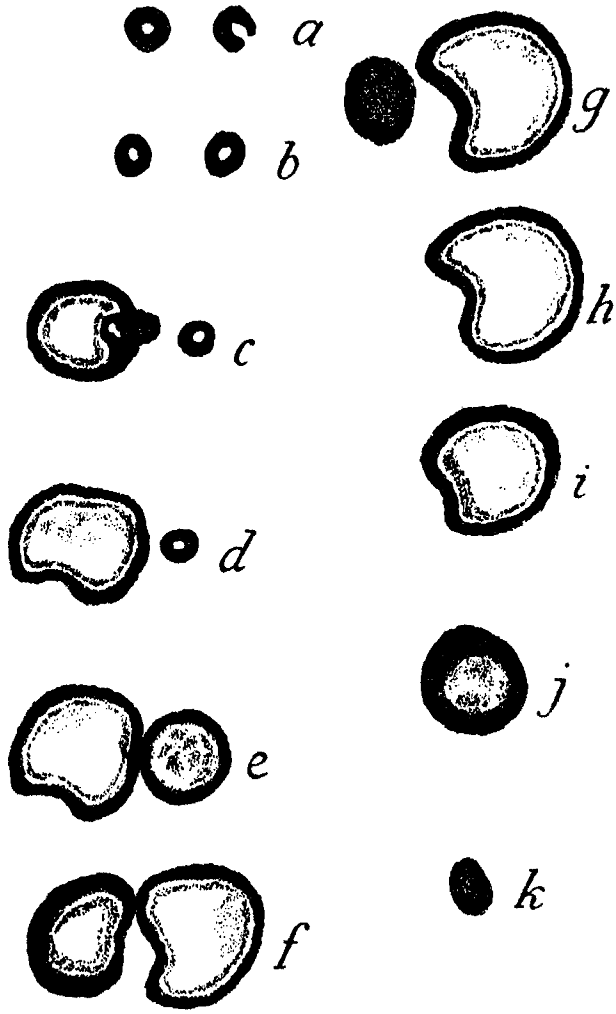

FIG. 4.—Sections taken at intervals of 200 μ of the male genital organs of another specimen of *Sacculina inflata* from *Cancer pagurus*. For further explanation see text. × 18.

passes into the testis of this side. As it is followed dorsally, the right testis gradually increases in size (fig. 4*g*, *h*) and then decreases (fig. 4*i*, *j*, *k*).

The colleteric glands of *Sacculina inflata* (fig. 5) are not so large as those of *Sacculina carcini*. They contain a number of comparatively wide tubes in many of which the products of the gland are visible. In the different specimens examined, the colleteric glands vary somewhat in size

and also in the number and width of the tubes. These glands are situated not far from the central parts of the lateral surfaces of the visceral mass, somewhat nearer to the anterior region than to the posterior.

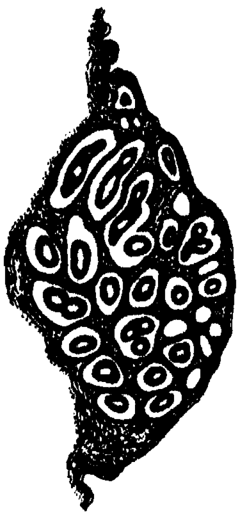

FIG. 5. — Longitudinal section of the right colleteric gland of a specimen of *Sacculina inflata* from *Cancer pagurus*. × 36.

Sections of the mantle show that it is muscular, especially around the mantle opening, where a strong sphincter is present.

The surface of the mantle possesses a fairly thick external cuticle, which, in the specimens examined, varies in thickness from 50 to 80 μ, and consists of layers of chitin which have been secreted parallel to the surface of the mantle (fig. 6*a*). The external surface of the cuticle is covered with small excrescences, which vary in length between 18 and 35 μ. In some specimens these excrescences are longer than in others, and, moreover, in different parts of the same specimen they are of different sizes. The excrescences consist of small hairs or papillæ (fig. 6*a*) covered with minute lateral hairs. These cuticular parts, therefore, do not differ noticeably from those found in the other European species of the genus *Sacculina* (*cf.* Boschma, 1927).

The internal cuticle of the mantle is a thin layer of chitin presenting

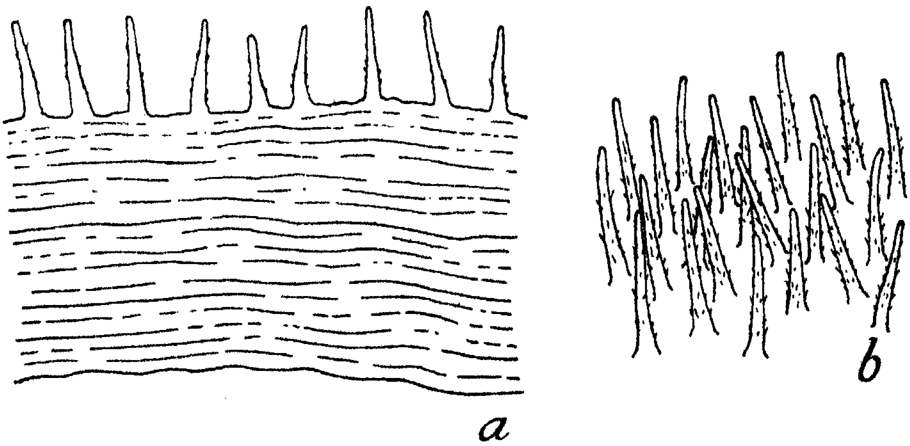

FIG. 6.—*Sacculina inflata* from *Cancer pagurus*. *a*, Section through the external cuticle of the mantle. *b*, Excrescences of the external cuticle as seen from above. × 660.

no striking peculiarities. Comparatively large parts of this cuticle from different specimens were examined, but in none of these was there a trace

of retinacula (the small cuticular excrescences which occur in many species of the genus), which probably, therefore, do not occur in this species.

The identity of the *Sacculina* living on *Cancer pagurus* with the parasite of *Hyas* is proved by the study of their internal anatomy. Unfortunately I have not been able to obtain more than one specimen of the *Sacculina* from *Hyas*, but this is so strongly similar to the parasites of *Cancer* that it is undoubtedly a representative of the same species. In the specimen from *Hyas* (*cf.* Boschma, 1927, fig. 14) one of the testes, here also that of the right side, is much larger than the other. Moreover, these organs have the same position as in the specimens from *Cancer*: they are embedded in the muscular mass at the posterior part of the body. The colleteric glands of the specimen from *Hyas* are somewhat smaller than those in the specimens from *Cancer*; but the tubes of which these organs are composed have approximately the same shape and size.

The excrescences of the external cuticle in the specimen from *Hyas* (up to 16 μ in length) are smaller than those of the corresponding parts in the specimens from *Cancer*. The shape of these hairs, however, corresponds in every detail, and the differences in length are too small to afford evidence for a specific distinction of the two forms. Moreover, in the specimen from *Hyas* no retinacula could be found on the internal cuticle of the mantle, just as in the specimens from *Cancer*.

Consequently, we may safely conclude that the parasite of *Cancer* is specifically identical with that of *Hyas*, and the valid name is *Sacculina inflata*, Leuck.

Leuckart's (1859) description indicates that the parasite has a different shape from that of *Sacculina carcini*: the larger dimension is that between the mantle opening and the stalk, or, at least, the breadth of the parasite does not exceed noticeably its height. Anderson (1862) also stated in his diagnosis of the species that the greater dimension is that from the mantle opening to the stalk. In both forms, however, among the parasites from *Cancer* as well as in those from *Hyas*, specimens may occur which are broader than usual, *e.g.*, the parasite dealt with in a previous paper (Boschma, 1927, fig. 1*m*, *n*).

In the parasite from *Hyas* the mantle opening is at the margin of one of the lateral surfaces (*cf.* Leuckart, 1859; Anderson, 1862), whilst in the parasites from *Cancer* it is directed forward. This difference may be caused by the fact that these parasites are living on different hosts.

Hitherto *Sacculina inflata* has been found as a parasite of *Hyas araneus* (Leuckart, 1859), of *Cancer pagurus* (Anderson, 1862), and of *Hyas coarctatus* (Smith, 1906). The name *Sacculina hyadis* (Malm, 1881)

for the parasite of *Hyas araneus* is a synonym of *Sacculina inflata.* Different authors (*e.g.*, Delage, 1884, and Smith, 1906) regarded *Sacculina inflata* as a synonym of *S. carcini*, but the study of its anatomy shows clearly that *S. inflata* differs from *S. carcini* by constant specific characters.

LITERATURE.

ANDERSON, J., 1862. "On the Anatomy of *Sacculina*, with a Description of the Species," *Ann. Mag. Nat. Hist.* (3), vol. ix.

BOSCHMA, H., 1927. "Über einige europäische Formen der Gattung *Sacculina*," *Zool. Jahrb.*, Abt. f. Syst., Bd. liv.

DELAGE, Y., 1884. "Évolution de la Sacculine," *Arch. Zool. Exp. Gén.* (2), tome 2.

LEUCKART, R., 1859. "Carcinologisches," *Arch. f. Naturgesch.*, Jahrg. 25, Bd. i.

MALM, A. W., 1881. "Om Cirripeder funna vid Bohusläns Kust. Göteborgs Naturhist. Mus.," *Zool. zoot. Afd.*, iii. Årsskr.

SMITH, G., 1906. "Rhizocephala, Fauna und Flora des Golfes von Neapel," xxix. Monographie.

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XI.—Electromagnetic Phenomena in a Uniform Gravitational Field.

By **D. Meksyn**, Ph.D., Mathematical Department, Edinburgh University. *Communicated by* Professor E. T. Whittaker, F.R.S.

(MS. received March 2, 1931. Read May 4, 1931.)

§ 1. Introduction and Summary.

In two recent papers Professor E. T. Whittaker* has solved the electromagnetic equations for the case of a uniform gravitational field. The fundamental tensor associated with such a field makes the Riemannian tensor vanish, since such a field can be transformed away by a suitable choice of co-ordinates. This property enables us to find the electromagnetic field in a uniform gravitational field without solving Maxwell's equations, but by a mere transformation of co-ordinates.

As is known from Differential Geometry, one surface is applicable to another by bending without stretching, if both have the same Gaussian curvature. Analytically this has the following interpretation: let the squares of the elements of length for the two surfaces be $g_{\mu\nu}dx_\mu dx_\nu$ and $g'_{\mu\nu}\,dx'_\mu\,dx'_\nu$, then a transformation of co-ordinates can be found

$$x = f(x') \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

which will satisfy the following equation:

$$g_{\mu\nu}\,dx_\mu\,dx_\nu = g'_{\mu\nu}\,dx'_\mu\,dx'_\nu \quad . \quad . \quad . \quad . \quad . \quad (2)$$

i.e. if the values of x given by (1) are inserted in the left side of (2) we obtain the right side of the same equation.

The condition that the Riemannian tensors of the two surfaces shall be equal is not only necessary, but also sufficient for the existence of such transformations.

Now the Riemannian tensor is equal to zero for Euclidean space, and, as we have pointed out, the same tensor also vanishes for a uniform gravitational field, hence it is possible to deduce a kind of Lorentz's transformations which connect these two spaces; and, if we have solved an electromagnetic problem for Euclidean space, we can obtain a solution of a corresponding problem for a uniform gravitational field by a mere transformation of co-ordinates.

We evaluate in the present paper the vector potential and the electromagnetic field of an electron moving freely in a uniform gravitational field.

* *Proc. Roy. Soc.*, A, 116, p. 720; A, 120, p. 1.

It appears that an electron radiates energy at the rate of $\frac{2}{3}\frac{e^2g^2}{c^3}$, which is Larmor's value.

It may appear at first that the solution of the electromagnetic equations for the uniform gravitational field is of little value since this is not the "natural" gravitational field. This is, however, not the case. The solution of this problem provides us with a good test of the principle of equivalence and the idea of curved space.

The idea of curved space is borne out only for mechanical phenomena and for the case of space slightly differing from Euclidean. The question, therefore, arises whether this principle is applicable only for the case of a "natural" field and for mechanical phenomena, or it is valid for electromagnetic phenomena and for every conceivable gravitational space.

The space of uniform gravitation, considered as a whole, is anything but the "natural space." It is bounded by a plane, say $x = -\alpha$; light emitted from any point at a finite distance from the boundary will never reach the boundary, which is also impenetrable for material bodies.

The electromagnetic equations can be solved rigorously, and in evaluating the radiation we have to carry out an integration with respect to the whole space.

The result of these calculations is Larmor's value for radiation; and, what is important, the rate of radiation is expressed, not through the acceleration of an electron, but through the metrical properties of space.

§ 2. The Transformation of Co-ordinates.

The fundamental form for a uniform gravitational field is *

$$\left.\begin{aligned} ds^2 &= (1+2\omega x)dt^2 - \frac{dx^2}{1+2\omega x} - dy^2 - dz^2 \\ \omega &= \frac{g}{c^2} \end{aligned}\right\} \qquad (3)$$

where g is the gravitational acceleration and $c=1$ is the velocity of light.

Let us find the transformation of this space into a Euclidean one.

We have to solve the equation

$$dt_0^2 - dx_0^2 - dy_0^2 - dz_0^2 = (1+2\omega x)dt^2 - \frac{dx^2}{1+2\omega x} - dy^2 - dz^2 \qquad (4)$$

We assume

$$y_0 = y;\ z_0 = z \qquad (5)$$

and from (4) and (5) we obtain

$$\left.\begin{aligned} d(t_0 - x_0) &= \lambda\left[\sqrt{1+2\omega x}\ dt - \frac{dx}{\sqrt{1+2\omega x}}\right] \\ d(t_0 + x_0) &= \frac{1}{\lambda}\left[\sqrt{1+2\omega x}\ dt + \frac{dx}{\sqrt{1+2\omega x}}\right] \end{aligned}\right\} \qquad (6)$$

* *Proc. Roy. Soc.*, **A**, 116, p. 722.

The condition that the right side of (6) is a total differential gives us two equations:

$$\left.\begin{aligned}\frac{\partial}{\partial x}\left[\lambda\sqrt{1+2\omega x}\right] &= -\frac{\partial}{\partial t}\left[\frac{\lambda}{\sqrt{1+2\omega x}}\right]\\ \frac{\partial}{\partial x}\left[\frac{\sqrt{1+2\omega x}}{\lambda}\right] &= \frac{\partial}{\partial t}\left[\frac{1}{\lambda\sqrt{1+2\omega x}}\right]\end{aligned}\right\} \quad . \quad . \quad . \quad (7)$$

which are to be satisfied by the same function λ.

We know *a priori* that these equations have a common solution.

Without going into further details we give here the required transformations. They are

$$\left.\begin{aligned}x_0 &= \frac{\sqrt{1+2\omega x}}{\omega}\cosh\omega t - \frac{1}{\omega}\\ t_0 &= \frac{\sqrt{1+2\omega x}}{\omega}\sinh\omega t\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

It is easily seen that, if ω tends to zero, $x_0 = x$ and $t_0 = t$; ω is a very small quantity, the second and higher powers of which can be neglected, and, again, to this order of approximation $x_0 = x$ and $t_0 = t$.

In (8) $\omega t = v$ is the velocity of the system (x_0, t_0); the latter started from rest and has been in motion relative to (x, t) during the time t.

Differentiating (8) we obtain

$$\left.\begin{aligned}dx_0 &= \frac{\cosh\omega t}{\sqrt{1+2wx}}\,dx + \sqrt{1+2\omega x}\sinh\omega t\,.\,dt\\ dt_0 &= \frac{\sinh\omega t}{\sqrt{1+2\omega x}}\,dx + \sqrt{1+2\omega x}\cosh\omega t\,.\,dt\end{aligned}\right\} \quad . \quad . \quad (9)$$

By a direct substitution of (9) in (4) we can confirm the validity of the transformations (8).

From (9) we find:

$$\left.\begin{aligned}\frac{\partial x^0}{\partial x} &= \frac{\cosh\omega t}{\sqrt{1+2\omega x}} & \frac{\partial x_0}{\partial t} &= \sqrt{1+2\omega x}\,.\sinh\omega t\\ \frac{\partial t_0}{\partial x} &= \frac{\sinh\omega t}{\sqrt{1+2\omega x}} & \frac{\partial t_0}{\partial t} &= \sqrt{1+2\omega x}\,.\cosh\omega t\end{aligned}\right\} \quad . \quad . \quad (10)$$

We can solve (8), express x, t as a function of x_0, t_0, and find the inverse differential coefficients:

$$\left.\begin{aligned}\frac{\partial x}{\partial x_0} &= \sqrt{1+2\omega x}\cosh\omega t & \frac{\partial x}{\partial t_0} &= -\sqrt{1+2\omega x}\sinh\omega t\\ \frac{\partial t}{\partial x_0} &= \frac{-\sinh\omega t}{\sqrt{1+2\omega x}} & \frac{\partial t}{\partial t_0} &= \frac{\cosh\omega t}{\sqrt{1+2\omega x}}\end{aligned}\right\} \quad . \quad . \quad (11)$$

§ 3. The Electromagnetic Field.

We can now easily find the vector potential and the electromagnetic field of an electron moving freely in a uniform gravitational field, and observed from a system at rest.

Let the electrostatic potential and the charge of an electron at rest in Euclidean space be

$$K_4{}^0(x^0, y^0, z^0); \; I_4{}^0 = \rho^0(x^0, y^0, z^0)$$

where

$$\nabla^2_{x_0, y_0, z_0} K_4{}^0 = -\rho^0 . \qquad (12)$$

Using the transformations for covariant tensors

$$\left.\begin{aligned} A_\mu &= \frac{\partial x_\alpha{}^0}{\partial x_\mu} A_\alpha{}^0 \\ A_{\mu\nu} &= \frac{\partial x_\alpha{}^0}{\partial x_\mu}\frac{\partial x_\beta{}^0}{\partial x_\nu} A_{\alpha\beta}{}^0 \end{aligned}\right\} \qquad (13)$$

we easily find from (10) and (13) the vector potenial, the stream vector, and the electromagnetic field, as measured by an observer at rest, for an electron moving freely from rest in a gravitational field.

They are as follows:

$$\left.\begin{aligned} K_1 &= \frac{\sinh \omega t}{\sqrt{1+2\omega x}} K_4{}^0(x_0, y_0, z_0) = \frac{\sinh \omega t}{\sqrt{1+2\omega x}} K_4{}^0\left(\frac{\sqrt{1+2\omega x}}{\omega}\cosh \omega t - \frac{1}{\omega}, y, z\right) \\ K_4 &= \sqrt{1+2\omega x}\cosh \omega t \,.\, K_4{}^0 = \sqrt{1+2\omega x}\cosh \omega t \,.\, K_4{}^0\left(\frac{\sqrt{1+2\omega x}}{\omega}\cosh \omega t - \frac{1}{\omega}, y, z\right) \end{aligned}\right\} (14)$$

and

$$I_1 = \frac{\sinh \omega t}{\sqrt{1+2\omega x}} \,.\, I_4{}^0 ; \quad I_4 = \sqrt{1+2\omega x} \,.\, \cosh \omega t \,.\, I_4{}^0 \qquad (15)$$

The electromagnetic tensor (covariant) is

$$\left.\begin{aligned} X = X^0 \qquad & Y = \sqrt{1+2\omega x}\,(Y^0 \cosh \omega t - \gamma^0 \sinh \omega t) \\ & Z = \sqrt{1+2\omega x}\,(Z^0 \cosh \omega t + \beta^0 \sinh \omega t) \\ \alpha = \alpha_0 \qquad & \beta = \frac{\beta^0 \cosh \omega t + Z^0 \sinh \omega t}{\sqrt{1+2\omega x}} \\ & \gamma = \frac{\gamma^0 \cosh \omega t - Y^0 \sinh \omega t}{\sqrt{1+2\omega x}} \end{aligned}\right\} \qquad (16)$$

For a contravariant electromagnetic tensor the expressions are as follows:

$$\left.\begin{array}{ll} X = X^0 & Y = \dfrac{Y^0 \cosh \omega t - \gamma^0 \sinh \omega t}{\sqrt{1+2\omega x}} \\ & Z = \dfrac{Z^0 \cosh \omega t + \beta^0 \sinh \omega t}{\sqrt{1+2\omega x}} \\ \alpha = \alpha_0 & \beta = \sqrt{1+2\omega x}\,(\beta^0 \cosh \omega t + Z^0 \sinh \omega t) \\ & \gamma = \sqrt{1+2\omega x}\,(\gamma^0 \cosh \omega t - Z^0 \sinh \omega t) \end{array}\right\} \quad . \quad (17)$$

where X^0, Y^0, Z^0, α^0, β^0, γ^0 is the electromagnetic tensor in Euclidean space.

For the case of an electron moving freely from rest in a gravitational field $\alpha^0 = \beta^0 = \gamma^0 = 0$.

Of course the expressions (14) and (15) satisfy the equations for the vector potential in the General Theory of Relativity (they are given below).

§ 4. Radiation.

In the classical electrodynamics the rate of radiation is given by the divergence of Poynting's vector:

$$\int \operatorname{div} P \,.\, dx\, dy\, dz \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Let us calculate this vector in our case.

We make use of the mixed tensor E^{ν}_{μ}. The equation of motion is

$$E^{\nu}_{\mu,\nu} = h_{\mu} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

where h_{μ} is the electromagnetic force. In our case $h_{\mu} = 0$.

Now

$$E^{\nu}_{\mu,\nu} = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x_{\nu}} (E^{\nu}_{\mu} \sqrt{-g}) - \frac{1}{2} \frac{\partial g_{\alpha\beta}}{\partial x_{\mu}} E^{\alpha\beta} \quad . \quad . \quad . \quad . \quad (20)$$

For the equation of energy $\mu = 4$, and as $g_{\alpha\beta}$ is independent at t, the last term vanishes; also $\sqrt{-g} = 1$, hence

$$E^{\nu}_{4,\nu} = \frac{\partial E^{\nu}_4}{\partial x_{\nu}} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

the first three terms in (21) represent the divergence of Poynting's vector.

The tensor E^{ν}_{μ} can be evaluated either from

$$E^{\nu}_{\mu} = -F^{\nu\alpha}F_{\mu\alpha} + \tfrac{1}{4} g^{\nu}_{\mu} F^{\alpha\beta} F_{\alpha\beta}$$

or by transformation of co-ordinates

$$E^{\nu}_{\mu} = \frac{\partial x_{\alpha}{}^0}{\partial x_{\nu}} \cdot \frac{\partial x_{\nu}}{\partial x_{\beta}{}^0} E_{\alpha}{}^{\beta 0}.$$

The expressions obtained are

$$\left.\begin{aligned}
E_4^1 &= -(1+2\omega x)\left[\frac{Y_0^2+Z_0^2+\beta_0^2+\gamma_0^2}{2}\sinh 2\omega t+(\beta_0 Z_0-\gamma_0 Y_0)\cosh 2\omega t\right]\\
E_4^2 &= \sqrt{1+2\omega x}[(X_0Y_0+\alpha_0\beta_0)\sinh\omega t-(\gamma_0X_0-\alpha_0Z_0)\cosh\omega t]\\
E_4^3 &= \sqrt{1+2\omega x}[(X_0Z_0+\alpha_0\gamma_0)\sinh\omega t-(\alpha_0Y_0-\beta_0X_0)\cosh\omega t]
\end{aligned}\right\} \quad (22)$$

In our case $\alpha_0=\beta_0=\gamma_0=0$ and X_0, Y_0, Z_0 are the expressions for the electrostatic force of an electron at rest.

The equation (18) becomes

$$\int(E_4^1\cos\xi+E_4^2\cos\eta+E_4^3\cos\zeta)dS \quad . \quad . \quad . \quad (23)$$

The evaluation of this integral (the details are given in the next paragraph) leads to the value

$$\frac{2}{3}\frac{e^2g^2}{c^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

for the rate of radiation due to motion of an electron.

§ 5. Evaluation of Radiation.

In order to evaluate the integral (23) we have to express the time as a function of the space co-ordinates of the field point.

The electromagnetic field emitted by the electron, which is at the origin of co-ordinates, propagates along the null geodesic of the space and reaches the field point.

The null geodesic for a uniform gravitational field is given in Whittaker's paper.* The equations are:

$$\left.\begin{aligned}
\frac{(1+2\omega x)^{\frac{1}{2}}\cosh(\omega t+\lambda)}{\omega}&=\alpha\\
y+\frac{(1+2\omega x)^{\frac{1}{2}}\sin\mu\sinh(\omega t+\lambda)}{\omega}&=\beta\\
z+\frac{(1+2\omega x)^{\frac{1}{2}}\cos\mu\sinh(\omega t+\lambda)}{\omega}&=\gamma
\end{aligned}\right\} \quad . \quad . \quad . \quad (25)$$

where α, β, γ, λ, μ are arbitrary constants.

Writing down the condition that the geodesic passes through the points $\bar{x}$, $\bar{y}$, $\bar{z}$, $\bar{t}$ and x, y, z, t we obtain the required equation for our geodesic

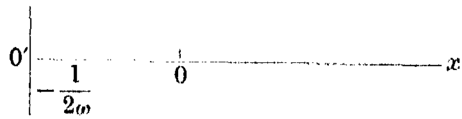
$$(1+2\omega x)^{\frac{1}{2}}(1+2\omega\bar{x})^{\frac{1}{2}}\cosh\omega(t-\bar{t})=1+\omega x+\omega\bar{x}+\frac{\omega^2}{2}\{(y-\bar{y})^2+(z-\bar{z})^2\} \quad (26)$$

* *Proc. Roy. Soc.*, A, 120, p. 6.

The electron is at the point (0, 0, 0, 0), hence (26) becomes

$$(1+2\omega x)^{\frac{1}{2}}\cosh\omega t = 1+\omega x+\frac{\omega^2}{2}(y^2+z^2) \qquad . \quad . \quad . \quad (27)$$

The space of gravitation is bounded by the plane $x=-\frac{1}{2\omega}$. We integrate (23) over the space



bounded by the plane $x=-\frac{1}{2\omega}$ and a hemisphere of infinite radius with a centre at the point $x=-\frac{1}{2\omega}$, 0, 0.

In the second integration the last two terms in (23) are of a lower order of magnitude than the first one, and in the first integration they do not appear at all; we therefore give here only the transformation of the first term.

We have from (22) and (8)

$$E_4^{\,1}=\frac{-(1+2\omega x)e^2(y^2+z^2)\sinh 2\omega t}{2\left\{\left[\frac{\sqrt{1+2\omega x}\cosh\omega t-1}{\omega}\right]^2+y^2+z^2\right\}^3} \quad . \quad . \quad . \quad (28)$$

We transfer the origin of co-ordinates to the point $x=-\frac{1}{2\omega}$, 0, 0, and denote

$$x'=x-\frac{1}{2\omega} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

also

$$\sinh 2\omega t=2\cosh\omega t\sinh\omega t=2\cosh\omega t\sqrt{\cosh^2\omega t-1} \quad . \quad . \quad (30)$$

We find now from (27), (28), (29), and (30)

$$E_4^{\,1}=-\frac{e^2(y^2+z^2)\left[\frac{1}{2}+\omega x'+\frac{\omega^2}{2}(y^2+z^2)\right]\sqrt{\left[\frac{1}{2}+\omega x'+\frac{\omega^2}{2}(y^2+z^2)\right]^2-2\omega x'}}{\left\{\left[x'-\frac{1}{2\omega}+\frac{\omega}{2}(y^2+z^2)\right]^2+y^2+z^2\right\}^3} \quad (31)$$

(a) We integrate over half a sphere of radius $r\to\infty$ with a centre at the point 0′. We transform (31) to spherical co-ordinates:

$$\left.\begin{aligned} x'&=r\cos\theta\\ y&=r\sin\theta\cos\phi\\ z&=r\sin\theta\sin\phi \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

the limits of integration are for θ from 0 to $\frac{\pi}{2}$, and for ϕ from 0 to 2π.

Now $\cos\xi=\frac{x'}{r}$ and $dS=r^2\sin\theta d\theta d\phi$, hence

$$\int E_4^1\cos\xi\, dS=$$

$$=-e^2\int\frac{\cos\theta\,.\,r^2\sin^2\theta\,.\left[\frac{1}{2}+\omega r\cos\theta+\frac{\omega^2}{2}r^2\sin^2\theta\right]\sqrt{\left[\frac{1}{2}+\omega r\cos\theta+\frac{\omega^2}{2}r^2\sin^2\theta\right]^2-2\omega r\cos\theta\,.\,r^2\sin\theta d\theta d\phi}}{\left\{\left[r\cos\theta-\frac{1}{2\omega}+\frac{\omega}{2}r^2\sin^2\theta\right]^2+r^2\sin^2\theta\right\}^3}.$$

As r tends to infinity we can omit in the above expression all terms of the lower order of magnitude, and obtain after easy simplifications:

$$\int E_4^1\cos\xi dS\simeq\frac{-e^2}{4\omega^2r^4}\int_{\theta=0}^{\theta=\frac{\pi}{2}}\int_{\phi=0}^{\phi=2\pi}\frac{\cos\theta\sin\theta d\theta d\phi}{\left[\sin^2\theta+\frac{4}{\omega r}\cos\theta\right]^3}\sim-\frac{\pi e^2}{2r^2}\quad . \quad . \quad (33)$$

or this expression is equal to zero for $r=\infty$.

(b) We integrate (28) along the plane $x'=0$; for this case $\cos\xi=-1$, $\cos\eta=\cos\zeta=0$. Putting in (31) $x'=0$ we obtain

$$\int E_4^1\cos\xi dS=\iint_{-\infty}^{+\infty}\frac{e^2(y^2+z^2)\left[\frac{1}{2}+\frac{\omega^2}{2}(y^2+z^2)\right]^2}{\left\{\left[-\frac{1}{2\omega}+\frac{\omega}{2}(y^2+z^2)\right]^2+y^2+z^2\right\}^3}dxdy\quad . \quad (34)$$

We transform (34) to polar co-ordinates:

$$y=r\cos\phi,\qquad z=r\sin\phi.$$

The denominator in (34) is equal to $\left[\frac{1}{2\omega}+\frac{\omega}{2}\ (y^2+z^2)\right]^6$, and we obtain from (34)

$$\int E_4^1\cos\xi dS=2^5e^2\omega^6\int_0^{2\pi}\int_0^{\infty}\frac{r^3drd\phi}{(1+\omega^2y^2)^5}=\frac{8\pi}{3}e^2\omega^2\ .\quad .\quad .\quad (35)$$

In order to express the radiation in the usual units we multiply (35) by $\frac{c}{4\pi}$ and obtain

$$\frac{2}{3}\frac{g^2e^2}{c^3}$$

where c is the velocity of light and g the acceleration of gravity.

§ 6. The Electromagnetic Equations.

We consider briefly the electromagnetic equations in the General Theory of Relativity. If K_μ I_μ are the vector potential and the stream

vector respectively, the equations for K_μ will be

$$\left.\begin{aligned} g^{\alpha\beta}K_{\mu,\,\alpha\beta} &= I_\mu - G^\epsilon_\mu K_\epsilon \\ K^\mu{}_{,\,\mu} &= 0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (36)$$

where G^ϵ_μ is the Riemannian contracted tensor. In our case $G^\epsilon_\mu = 0$.

The evaluation of (36) is simple; for the case of a uniform gravitational field it was given in Whittaker's paper.*

The obtained equations are

$$\left.\begin{aligned}
&\frac{1}{1+2\omega x}\frac{\partial^2 K_1}{\partial t^2} - \frac{2\omega}{(1+2\omega x)^2}\frac{\partial K_4}{\partial t} - 4\omega\frac{\partial K_1}{\partial x} - (1+2\omega x)\frac{\partial^2 K_1}{\partial x^2} - \frac{\partial^2 K_1}{\partial y^2} - \frac{\partial^2 K_1}{\partial z^2} = I_1 \\
&\frac{1}{1+2\omega x}\frac{\partial^2 K_2}{\partial t^2} - 2\omega\frac{\partial K_2}{\partial x} - (1+2\omega x)\frac{\partial^2 K_2}{\partial x^2} - \frac{\partial^2 K_2}{\partial y^2} - \frac{\partial^2 K_2}{\partial z^2} = I_2 \\
&\frac{1}{1+2\omega x}\frac{\partial^2 K_3}{\partial t^2} - 2\omega\frac{\partial K_3}{\partial x} - (1+2\omega x)\frac{\partial^2 K_3}{\partial x^2} - \frac{\partial^2 K_3}{\partial y^2} - \frac{\partial^2 K_3}{\partial z^2} = I_3 \\
&\frac{1}{1+2\omega x}\frac{\partial^2 K_4}{\partial t^2} - 2\omega\frac{\partial K_1}{\partial t} - (1+2\omega x)\frac{\partial^2 K_4}{\partial x^2} - \frac{\partial^2 K_4}{\partial y^2} - \frac{\partial^2 K_4}{\partial z^2} = I_4
\end{aligned}\right\} \quad (37)$$

Also the equation $K^\mu{}_{,\,\mu} = 0$ or $G^{\mu\nu}K_{\mu,\,\nu} = 0$ becomes

$$(1+2\omega x)\frac{\partial K_1}{\partial x} + 2\omega K_1 + \frac{\partial K_2}{\partial y} + \frac{\partial K_3}{\partial z} - \frac{1}{1+2\omega x}\frac{\partial K_4}{\partial t} = 0 \quad . \quad . \quad (38)$$

For an electrostatic field Euclidean space the vector potential and stream vector are equal to

$$\left.\begin{aligned} K_\mu &= (-F, -G, -H, \phi) \qquad & F = G = H = 0 \\ I_1 &= I_2 = I_3 = 0 & I_4 = \rho \end{aligned}\right\} \quad . \quad . \quad . \quad (39)$$

and

$$\nabla^2 K_4{}^0 = -\rho_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

hence for an electron moving freely from rest in a gravitational field the stream vector is given by (15).

It can be shown by direct substitution in (37) and (38) that the vector potential is given in the present case by (14), where $K_4{}^0$ satisfies (40) and the electron is at the origin of co-ordinates.

* *Proc. Roy. Soc.*, A, 116, p. 723.

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XII.—Further Numerical Studies in Algebraic Equations and Matrices. By A. C. Aitken.

(MS. received February 18, 1931. Read May 4, 1931.)

§ 1. Introductory.

In a former paper on the same subject* the writer pointed out that the sequence used by D. Bernoulli for approximating to the greatest root of an algebraic equation could be further utilised in such a way as to give all the roots. It is suggested in the present paper that there is really no need to compute a first Bernoullian sequence at all, but that by the theory of dual symmetric functions the coefficients in the given equation may be used with equal convenience. In a practical respect this simplifies the technique of root-evaluation.

The later sections of the paper will be concerned with the relation between the numerical processes of solving algebraic equations and the corresponding operation of matrix-powering for approximating to the latent roots of matrices. A theorem is disclosed and proved, giving a matrix which transforms the rational canonical form of a given matrix into the classical irrational canonical form.

In the earlier paper it was shown that if $f(t)$ satisfies the difference equation or recurrence relation

(I) $$f(t+n)-c_1f(t+n-1)+c_2f(t+n-2)- \ . \ . \ . \ +(-)^n c_n f(t)=0,$$

and if a persymmetric determinant be constructed, of order m and with diagonal elements $f(t)$, thus, in the usual notation for persymmetric determinants,

$$f_m(t)=\mathrm{P}\{f(t-m+1), f(t-m+2), \ . \ . \ ., f(t), \ . \ . \ ., f(t+m-1)\},$$

then $f_m(t+1)/f_m(t)$ tends in general, with increasing t, to $x_1x_2 \ . \ . \ . \ x_m$, the product of the m numerically greatest roots of the proposed equation

(II) $$x^n-c_1x^{n-1}+c_2x^{n-2}- \ . \ . \ . \ +(-)^n c_n=0.$$

The theorem of compound determinants,

(III) $$f_m(t)=\{[f_{m-1}(t)]^2-f_{m-1}(t+1)\,.\,f_{m-1}(t-1)\}/f_{m-2}(t),$$

* *Proc. Roy. Soc. Edin.*, **46** (1926), pp. 289–305.

was found to provide a ready means of constructing the sequences $f_m(t)$ for $m=1, 2, \ldots, n$, by a uniform iterative process developing by columns. The complications arising from complex, multiple, and numerically equal roots were discussed in detail, and sequences of more rapid approximation derived.

§ 2. Alternative Method of Root-Approximation.

The most convenient initial values to take for a first sequence $f(t)$ are $f(t)=0$ for $t=-n+1, -n+2, \ldots, -2, -1$, and $f(0)=1$. In this case $f(t)=h_t$, the complete homogeneous symmetric function of degree t in the roots of (II). But then the persymmetric determinant $f_m(t)$ is that special type of determinantal symmetric function called by Muir a "bi-alternant," with m elements h_t in the diagonal, say $h_{(tt\ldots t)_m}$. By the fundamental theorem of duality of such symmetric functions $h_{(tt\ldots t)_m}$ is identically equal to the conjugate bi-alternant in the elementary symmetric functions, $c_{(mm\ldots m)_t}$. Thus the rôles of h_r and c_r, m and t are simultaneously interchanged; and this does away with the need of calculating a first sequence, since the c_r's are already given as coefficients in the algebraic equation (II).

The arithmetical process is simple and uniform, the theorem (III) once again rendering service; we merely work downward by rows instead of across, as formerly, by columns. A first row of $n+1$ units is written down; next, in a second row, the coefficients $1, c_1, c_2, \ldots, c_n$. The rule for forming subsequent rows is then as follows: *each entry is the square of the one above minus the product of the two on either side of that one, divided by the one above that again.* The process is rapid and essentially suited to calculation by machine.

There is a twofold advantage in working by rows instead of by columns; in the first place the calculation of all the roots is proceeding *pari passu*, in the second place we are usually dealing with smaller divisors than in the columnar method. It is, of course, true that the arrays of numbers given by both methods are identical, for what emerges is actually a particular numerical table of dual symmetric functions of the roots of (II).

The discrimination and evaluation of complex, multiple, or numerically equal roots are carried out exactly as in the earlier paper.

Example. To solve the equation

$$x^4-6x^3-40x^2-19x+9=0.$$

[*Note.*—The characteristic in heavier type will indicate the number of significant digits preceding or of zeros following the decimal point of the number represented. Thus **0**·12345 will denote ·12345, **4**·12345 will denote 1234·5, $\bar{\mathbf{2}}$·123 will denote ·00123.]

t.	$f_1(t)$.	$f_2(t)$.	$f_3(t)$.	$f_4(t)$.
0	1·1	1·1	1·1	1·1
1	1·6	− 2·40	2·19	1·9
2	2·76	4·1486	3·721	
3	3·715	− 5·53835	5·21025	
4	4·7435	7·1940225	6·66754	
5	5·74600	− 8·69833935	8·20588815	
6	6·757901	10·25127202	9·64119443	
7	7·7666236	− 11·90404140	11·19903765	

Writing $f_m(t)/f_m(t-1)$ as $X_m(t)$, we find from the last six rows—

X_1.	X_2.	X_3.	X_4.
9·408	− 36·228	29·161	9
10·399	− 36·040	31·750	
10·034	− 35·993	30·843	
10·160	− 35·981	31·143	
10·115	− 35·979	31·042	

As a matter of fact, retaining as many places as the machine could accommodate and using the more powerful sequences described in the earlier paper,

$$X_m^{(2)}(t) = P\{X_m(t-1),\ X_m(t),\ X_m(t+1)\}/\delta^2 X_m(t),$$

where P is a persymmetric determinant, simply $\{X_m(t)\}^2 - X_m(t+1)X_m(t-1)$, and $\delta^2 X_m(t) = X_m(t+1) - 2X_m(t) + X_m(t-1)$, we obtain from the above

$X_1^{(2)}$.	$X_2^{(2)}$.	$X_3^{(2)}$.	$X_4^{(2)}$.
10·13188	− 35·97662	31·07814	9
10·12724	− 35·97781	31·05999	
10·12668	− 35·97767	31·06720	

and from these three by the same kind of iteration—

$X_1^{(3)}$.	$X_2^{(3)}$.	$X_3^{(3)}$.	$X_4^{(3)}$.
10·12661	− 35·97768	31·06694	9

This gives $x_1 = 10·12661$, $x_2 = -3·55279$, $x_3 = -0·86351$, $x_4 = 0·28970$, the last digit of each of which is possibly unreliable.

In a later section we shall develop yet another and very useful way of accelerating the approximation.

§ 3. The Related Process of Matrix-Powering.

The numerically greatest latent root λ_1 of a matrix A of real elements can be computed by an iterative process of matrix-powering, depending on the fact that the quotient of corresponding elements in A^{t+1} and A^t tends with increasing t to λ_1. The process is closely related to that just outlined for algebraic equations, as may be seen from a simple example. Consider the cubic equation

$$x^3 - c_1x^2 - c_2x - c_3 = 0.$$

It is the characteristic equation of the matrix

$$C \equiv \begin{bmatrix} c_1 & c_2 & c_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

and by induction or otherwise the matrix C^t can be readily obtained as, *e.g.*,

$$C^4 = \begin{bmatrix} (1\,1\,1\,1) & (1\,1\,1\,2) & (1\,1\,1\,3) \\ (1\,1\,1) & (1\,1\,2) & (1\,1\,3) \\ (1\,1) & (1\,2) & (1\,3) \end{bmatrix},$$

where in the general case $t > n$ and the elements are bi-alternants, as, for example,

$$(113) \equiv \begin{vmatrix} c_1 & c_2 & c_5 \\ c_0 & c_1 & c_4 \\ 0 & c_0 & c_3 \end{vmatrix}.$$

Now the elements in the first column of C^t are seen by the theorem of dual symmetric functions to be $h_t, h_{t-1}, h_{t-2}, \ldots$, which, it will be recalled, appeared in the first column of the scheme of computation for the roots of (II). Certain minors of C^t, it will also be found, reproduce later columns of that scheme. Evidently the processes are closely related.

We proceed to investigate the evaluation of all the latent roots of a matrix A from the sequence $A^t, A^{t+1}, A^{t+2}, \ldots$. It is known from theory that A can be reduced to canonical form HLH^{-1}, where H is non-singular and L is a matrix with zeros below the principal diagonal, the latent roots λ_r of A in the diagonal, and zeros above the diagonal, except possibly in

the super-diagonal, where there may be essential non-zero elements, indicating that the matrix has multiple latent roots with non-linear elementary divisors of the characteristic determinant. We shall distinguish between (*a*) the regular matrix with regular canonical form, and (*b*) the irregular matrix with irregular canonical form, and shall consider the cases separately, *e.g.*,

$$(a)\ \mathrm{L}=\begin{bmatrix}\lambda_1 & \cdot & \cdot & \cdot\\ \cdot & \lambda_2 & \cdot & \cdot\\ \cdot & \cdot & \lambda_3 & \cdot\\ \cdot & \cdot & \cdot & \lambda_4\end{bmatrix},\qquad (b)\ \mathrm{L}=\begin{bmatrix}\lambda_1 & 1 & \cdot & \cdot\\ \cdot & \lambda_1 & 1 & \cdot\\ \cdot & \cdot & \lambda_1 & \cdot\\ \cdot & \cdot & \cdot & \lambda_4\end{bmatrix},$$

where the diagonal λ's are not necessarily unequal in value.

§ 4. Latent Roots of Regular Matrices.

In the regular case we have $\mathrm{A}^t=\mathrm{HL}^t\mathrm{H}^{-1}$, where L^t is a purely diagonal matrix with $\lambda_1{}^t, \lambda_2{}^t, \ldots, \lambda_n{}^t$ in the diagonal. Hence the elements of A^t are polynomial functions trilinear in the elements of H, the elements of H^{-1}, and the $\lambda_r{}^t$'s. Hence the correspondingly placed elements in A^{t+1} are the same trilinear functions, but with λ^{t+1} instead of λ^t. Hence if $\lambda_1, \lambda_2, \ldots, \lambda_n$ are in descending order of absolute value, high powers of λ_1 dominate as t increases, and we have at once the known result that the quotient of any non-zero element in A^{t+1} by the corresponding non-zero element in A^t tends to λ_1.

It will next be proved that the ratio of corresponding minors * of order m in A^{t+1} and in A^t tends to $\lambda_1\lambda_2 \ldots \lambda_m$; for such minors are elements of the mth compound of A^t—that is, of the tth power of $\mathrm{A}^{(m)}$, the mth compound of A, in virtue of the theorem that the mth compound of a product of matrices is identical with the product of the mth compounds of the separate matrices. For the same reason

$$\mathrm{A}^{(m)}=\mathrm{H}^{(m)}\mathrm{L}^{(m)}\{\mathrm{H}^{(m)}\}^{-1}.$$

Hence the latent roots of $\mathrm{A}^{(m)}$ are the m-ary products † of the latent roots of A; the greatest of these is $\lambda_1\lambda_2 \ldots \lambda_m$, and so the result in question follows exactly as in the simple case.

The raising of A to consecutive high powers is most easily achieved by s repeated squarings, which give $\mathrm{A}^2, \mathrm{A}^4, \mathrm{A}^8, \ldots, \mathrm{A}^t$, where $t=2^s$, and a final multiplication by A itself, giving A^{t+1}. The row-by-column matrix-multiplications are rapidly performed by machine.

* We reserve the word "minor" for the determinant, "submatrix" for the array.

† Rados' Theorem (1891).

Example. Find the latent roots of the matrix (see the example of § 2)

$$A = \begin{bmatrix} 6 & 40 & 19 & -9 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

We find

$$A^8 = \begin{bmatrix} 77663941 & 320378159 & 138837375 & -68996124 \\ 7666236 & 31666525 & 13728719 & -6821109 \\ 757901 & 3118830 & 1350485 & -671400 \\ 74600 & 310301 & 134830 & -66915 \end{bmatrix}.$$

We shall not calculate A^9, for in this particular example A^8 contains elements of A^7, A^6, A^5, as was seen in § 3. The elements of the first column of A^8 have, in fact, already occurred in the illustrative example of § 2; while if we evaluate minors of A^8 of order 2 by Dodgson's process* of contraction we obtain numbers occurring in the second column of that example; and similarly for further contraction. It is clear that the method of evaluating roots of algebraic equations is a special case of the process of matrix-powering.

The sequences obtained are—

t.	Elements.	2-row Minors.	3-row Minors.	Det.
5	5·74600	− 8·69833935	8·20588815	
6	6·757901	10·25127202	9·64119443	
7	7·7666236	− 11·90404140	11·19903765	
8	8·77663941	13·32525531	12·61854181	9^8

and we derive

λ_1.	λ_2.	λ_3.	λ_4.
10·15953	− 3·54164	− ·865527	·288991
10·11509	− 3·55692	− ·862783	·289933
10·13065	− 3·55139	− ·863737	·289618

It is appropriate here to introduce a method, in general highly effective, of increasing the rapidity of root-approximation by these sequences. Suppose $Z_r(t)$, $Z_r(t+1)$ and $Z_{r+1}(t)$, $Z_{r+1}(t+1)$ are consecutive values

* Muir, *Theory of Determinants*, iii, p. 17.

tending to $\lambda_1\lambda_2 \ldots \lambda_r$ and $\lambda_1\lambda_2 \ldots \lambda_{r+1}$, while $\lambda_r(t)$ and $\lambda_{r+1}(t)$ are the computed quotients tending to λ_r and λ_{r+1}; then it is asserted that the weighted mean

$$\{\lambda_r(t)Z_r(t) - \lambda_{r+1}(t)Z_r(t-1)\}/\{\lambda_r(t) - \lambda_{r+1}(t)\}$$

gives a more rapid sequence for Z_r. This follows from a remark in the earlier paper (p. 301, line 5). So here we derive—

λ_1.	$\lambda_1\lambda_2$.	$\lambda_1\lambda_2\lambda_3$.	$\lambda_1\lambda_2\lambda_3\lambda_4$.
10·12661	−35·9777	−31·0670	9·0

whence

$$\lambda_1 = 10{\cdot}12661, \quad \lambda_2 = -3{\cdot}55279, \quad \lambda_3 = -0{\cdot}86351, \quad \lambda_4 = 0{\cdot}28970,$$

in agreement with the example of § 2.

§ 5. Latent Roots of Irregular Matrices.

We turn now to the irregular case (*b*) of matrices A with multiple roots associated with non-linear elementary divisors, a latent root λ_r, say, being of multiplicity m and maximum divisor-exponent p. The matrix of greatest order associated with λ_r in the canonical form L is then of order p and type

$$\Lambda_r = \begin{bmatrix} \lambda_r & 1 & \cdot & \cdot & \cdots \\ \cdot & \lambda_r & 1 & \cdot & \cdots \\ \cdot & \cdot & \lambda_r & 1 & \cdots \\ \cdot & \cdot & \cdot & \lambda_r & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdots \end{bmatrix}.$$

Then the corresponding submatrix in the canonical form of A^t is

$$\Lambda_r^t = \begin{bmatrix} \lambda_r^t & t\lambda_r^{t-1} & \binom{t}{2}\lambda_r^{t-2} & \binom{t}{3}\lambda_r^{t-3} & \cdots \\ \cdot & \lambda_r^t & t\lambda_r^{t-1} & \binom{t}{2}\lambda_r^{t-2} & \cdots \\ \cdot & \cdot & \lambda_r^t & t\lambda_r^{t-1} & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdots \end{bmatrix}.$$

The proof of this is simple. Let the matrix* with 1's in the super-diagonal, zeros everywhere else, be denoted by U, the unit-matrix being I. Then U^s is at once seen to have 1's in the sth super-diagonal, $s<n$, zeros elsewhere. Now

$$\Lambda_r = \lambda_r I + U,$$

* This matrix has been called the "auxiliary unit-matrix" by Prof. H. W. Turnbull, to whom I am indebted for bringing its properties to my notice.

and so

$$\Lambda_r^t = \lambda_r^t \mathrm{I} + t\lambda_r^{t-1}\mathrm{U} + \binom{t}{2}\lambda_r^{t-2}\mathrm{U}^2 + \ . \ . \ . \ + \binom{t}{p-1}\lambda_r^{t-p+1}\mathrm{U}^{p-1},$$

which gives the result.

Hence, in A^t, if λ_1 is the greatest latent root, of maximum divisor-exponent p, the powers of λ_1 will dominate as before, but whereas the part of an element in A^t involving λ_1 will be of the form

$$k_0\lambda_1^t + k_1 t\lambda_1^{t-1} + k_2\binom{t}{2}\lambda_1^{t-2} + \ . \ . \ . \ + k_{p-1}\binom{t}{p-1}\lambda_1^{t-p+1},$$

the corresponding element in A^{t+1} will have

$$k_0\lambda_1^{t+1} + k_1(t+1)\lambda_1^t + k_2\binom{t+1}{2}\lambda_1^{t-1} + \ . \ . \ . \ + k_{p-1}\binom{t+1}{p-1}\lambda_1^{t-p+2}.$$

Thus the quotient of corresponding elements in A^{t+1} and A^t will still tend to λ_1, but in a different manner, and, because of the terms in t, much less rapidly than in the regular case, and too slowly to be of practical use. However, if λ_1 be of multiplicity m, the quotient of corresponding minors of order m in A^{t+1} and A^t will give λ_1^m in a rapid regular sequence, so that λ_1 may still be found. As for the intermediate powers of λ_1 and minors of A^t of lower order than m, it will be shown that there is an operation which converts the slow quotient-sequences into rapid regular ones.

λ_1 being evaluated from λ_1^m by minors of order m, let us denote the operation $f(t+1) - \lambda f(t)$ by $\Delta_\lambda f(t)$. We shall call this the first λ-difference of $f(t)$. Then we have at once, by finite differences,

$$\Delta_{\lambda_1}^{p-1}\left\{k_0\lambda_1^t + k_1 t\lambda_1^{t-1} + k_2\binom{t}{2}\lambda_1^{t-2} + \ . \ . \ . \ + k_{p-1}\binom{t}{p-1}\lambda_1^{t-p+1}\right\} = k_{p-1}\lambda_1^{t-p+1}.$$

Hence, in the case of a root λ_r with highest exponent p in the elementary divisors, the $(p-1)$th λ_r-differences of terms in a sequence of quotients of corresponding minors of order r in $\mathrm{A}^t, \mathrm{A}^{t+1}, \ . \ . \ .$ tend to $\lambda_1\lambda_2 \ . \ . \ . \lambda_r$ as a regular rapid sequence.

Incidentally the precise order of the λ-differences which produced regular sequences of minor-quotients for λ_r^k, where $k = 2, 3, \ . \ . \ . \ , p-1$, was determined by the writer by calculation for various values of p and k, since it affords a heuristic method for finding the maximum exponent of elementary divisors in the kth compound of a matrix with canonical form Λ_r. The exponent was inferred to be $k(p-k)+1$, a fact which is capable of independent proof by matrix-algebra. The number $k(p-k)+1$ appears elsewhere in determinantal theory as the maximum number of linearly independent determinants existing in a general rectangular matrix of k rows and p columns.

§ 6. Relation between Two Forms of Canonical Matrix.

The "classical" canonical form, *e.g.*

$$L \equiv \begin{bmatrix} \lambda_1 & 1 & . \\ . & \lambda_1 & . \\ . & . & \lambda_2 \end{bmatrix},$$

of a matrix A is an irrational form; it involves the prior solution of the characteristic equation, *e.g.*

$$\lambda^3 - c_1\lambda^2 - c_2\lambda - c_3 = 0,$$

which is in general an irrational process. The other canonical form,

$$C \equiv \begin{bmatrix} c_1 & c_2 & c_3 \\ 1 & . & . \\ . & 1 & . \end{bmatrix},$$

is a rational canonical form. In the more general case a matrix A has two canonical forms in which the respective types L and C appear as submatrices diagonally juxtaposed.

Our preceding numerical work suggests a relation between these two forms. The transformation of "similarity," by which $HLH^{-1} = C$, turns out to be such that H is an "alternant" matrix.

In fact, if any matrix A has the canonical form HLH^{-1} where λ_1 is the leading element in L, it is seen without difficulty that the elements in any column of A^t tend to be in a certain proportionality, namely, that of the elements in the first column of H, while those in any row of A^t tend toward the ratios of the elements in the first row of H^{-1}. [Thus matrix-powering gives information of an approximating kind, not merely about the values of the latent roots of a matrix, but also about its classical canonical form. Indeed, facts similar to the above hold for minors of all orders in H, H^{-1}, and A^t, but they need not detain us.] Now it was noticed, in connection with algebraic equations, that the elements in any column of C^t tended to the ratio $\lambda_1^{n-1} : \lambda_1^{n-2} : \lambda_1^{n-3} : \ldots \lambda_1 : 1$. We are thus led to surmise a general theorem and to assert that, for example, if $\lambda_1, \lambda_2, \lambda_3$ are different,

$$\begin{bmatrix} \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 & . & . \\ . & \lambda_2 & . \\ . & . & \lambda_3 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 & c_3 \\ 1 & . & . \\ . & 1 & . \end{bmatrix} \begin{bmatrix} \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ 1 & 1 & 1 \end{bmatrix},$$

which is true, each side, the left directly, the right in view of

$$\lambda^3 = c_1\lambda^2 + c_2\lambda + c_3, \text{ for } \lambda = \lambda_1, \lambda_2, \lambda_3,$$

giving

$$\begin{bmatrix} \lambda_1^3 & \lambda_2^3 & \lambda_3^3 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \\ \lambda_1 & \lambda_2 & \lambda_3 \end{bmatrix},$$

another alternant matrix.

Hence $HL = CH$, or $C = HLH^{-1}$. Also, H is non-singular, since $|H|$ is the difference-product $(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)$, or $\zeta^{\frac{1}{2}}(\lambda_1, \lambda_2, \lambda_3)$.

As for the irregular case, the λ-differencing process suggests that where there are multiple roots and non-linear elementary divisors we should employ for H the *differentiated* alternant matrix,* or matrix of the "confluent difference-product," as follows:

Let

$$L = \begin{bmatrix} \lambda_1 & 1 & . & . \\ . & \lambda_1 & 1 & . \\ . & . & \lambda_1 & . \\ . & . & . & \lambda_4 \end{bmatrix}, \qquad (\lambda_1 \neq \lambda_4)$$

with characteristic equation

$$\lambda^4 - c_1\lambda^3 - c_2\lambda^2 - c_3\lambda - c_4 = 0.$$

Then it is asserted that

$$\text{(IV)} \quad \begin{bmatrix} \lambda_1^3 & 3\lambda_1^2 & 3\lambda_1 & \lambda_4^3 \\ \lambda_1^2 & 2\lambda_1 & 1 & \lambda_4^2 \\ \lambda_1 & 1 & . & \lambda_4 \\ 1 & . & . & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 & 1 & . & . \\ . & \lambda_1 & 1 & . \\ . & . & \lambda_1 & . \\ . & . & . & \lambda_4 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 \\ 1 & . & . & . \\ . & 1 & . & . \\ . & . & 1 & . \end{bmatrix} \begin{bmatrix} \lambda_1^3 & 3\lambda_1^2 & 3\lambda_1 & \lambda_4^3 \\ \lambda_1^2 & 2\lambda_1 & 1 & \lambda_4^2 \\ \lambda_1 & 1 & . & \lambda_4 \\ 1 & . & . & 1 \end{bmatrix}$$

where the alternant matrix has columns for the multiple root λ_1 differentiated by the operation $(d/d\lambda)^r/r!$. For we have, λ_1 being here a triple root,

$$\begin{aligned} &\lambda^4 - c_1\lambda^3 - c_2\lambda^2 - c_3\lambda - c_4 = 0, \quad \text{for } \lambda = \lambda_1, \lambda_4, \\ &\left.\begin{aligned} &4\lambda^3 - 3c_1\lambda^2 - 2c_2\lambda - c_3 = 0, \\ &6\lambda^2 - 3c_1\lambda - c_2 = 0, \end{aligned}\right\} \text{for } \lambda = \lambda_1, \end{aligned}$$

and the right side of (IV) in virtue of these equations, the left by simple properties of binomial coefficients, both give the alternant matrix

$$\begin{bmatrix} \lambda_1^4 & 4\lambda_1^3 & 6\lambda_1^2 & \lambda_4^4 \\ \lambda_1^3 & 3\lambda_1^2 & 3\lambda_1 & \lambda_4^3 \\ \lambda_1^2 & 2\lambda_1 & 1 & \lambda_4^2 \\ \lambda_1 & 1 & . & \lambda_4 \end{bmatrix}.$$

* The determinant of such matrices has been studied. See Muir's *History*, iv, p. 178.

Here again H is non-singular, for $|H|$ is the confluent difference-product $(\lambda_1-\lambda_4)^3$, or $\zeta^{\frac{1}{2}}\{(\lambda_1, \lambda_1, \lambda_1), \lambda_4\}$.

We infer similarly, in the general case, that the classical irrational canonical form of a matrix may be transformed into the rational form, or *vice versa*, by means of a transforming matrix H composed of diagonally juxtaposed alternant submatrices, having columns successively differentiated if necessary for orders of non-linear multiplicity in the latent roots.

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XIII.—On the Operational Solution of the Homogeneous Linear Equation of Finite Differences by Generalised Continued Fractions. By L. M. Milne-Thomson. *Communicated by* Professor Bevan B. Baker.

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SUMMARY.

THE homogeneous linear equation of the second order may be solved by means of a continued fraction whose elements are coefficients in the equation. The two dimensional continued fraction is generalised to m dimensions and yields the solution of the equation of the mth order.

The homogeneous linear difference equation of the second order may be exhibited in the form

$$u_x = a_x u_{x-1} + b_x u_{x-2} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

a_x, b_x being given functions of the variable x whose domain is the positive integers including zero. It is assumed that a_x, b_x do not become infinite for any value of x in this domain. The general solution of (1) is a function of x, containing two independent arbitrary constants, which when substituted for u_x in (1) renders it an identity. The general solution is a homogeneous linear function of the arbitrary constants which we shall take to be the initial values of u_x, in this case, u_0, u_1.

Denote by $\dfrac{p_x}{q_x}$ the xth convergent of the continued fraction $a_1 + \dfrac{b_2}{a_2 +}\;\dfrac{b_3}{a_3 +} \cdots$

Then

$$p_x = a_x p_{x-1} + b_x p_{x-2},$$

$$q_x = a_x q_{x-1} + b_x q_{x-2}.$$

It follows that p_x and q_x are particular solutions of (1).

Now

$$p_2 = a_2 p_1 + b_2 p_0.$$

If we regard p_1 and p_0 as arbitrary and denote them by u_1 and u_0, we have

$$p_2 = a_2 u_1 + b_2 u_0.$$

Assigning an arbitrary value u_0 to p_0 is actually equivalent to writing u_1 for a_1 and $b_2 u_0$ for b_2.

If we write $\frac{\beta_{x-1}}{\alpha_{x-1}}$ for the $(x-1)$th convergent of the continued fraction $\frac{b_2}{a_2+}\,\frac{b_3}{a_3+}\,\ldots$, we have

$$\frac{p_x}{q_x}=u_1+\frac{\beta_{x-1}}{\alpha_{x-1}}u_0,$$

and it is seen that p_x is derived from p_2 by writing α_{x-1} for a_2 and β_{x-1} for b_2, so that

$$p_x=\alpha_{x-1}u_1+\beta_{x-1}u_0,$$

and hence the general solution of (1) is

$$u_x=\alpha_{x-1}u_1+\beta_{x-1}u_0.$$

We have thus expressed the general solution of (1) in terms of the components of the $(x-1)$th convergent of the continued fraction $\frac{b_2}{a_2+}\,\frac{b_3}{a_3+}\,\ldots$, which contains no arbitrary elements and which is written down from the given equation. It will be observed that the values of a_x, b_x for $x=0$, 1 are irrelevant.

It is proposed to generalise the above result to the homogeneous equation of order m.

The continued fraction $a_1+\frac{1}{a_2+}\,\frac{1}{a_3+}\,\ldots$ regarded as an algorithm based on the ratio of numbers has recently been extended to several dimensions by Messrs Paley and Ursell.*

For the present purpose we require a generalisation of the continued fraction $a_1+\frac{b_2}{a_2+}\,\frac{b_3}{a_3+}\,\ldots$ where we regard the fraction as given. The notation about to be described, while differing in detail is fundamentally the same as that given in the paper just cited. On the other hand, we are not concerned with the meaning of the generalised continued fraction as such, nor do questions of convergence arise. Although the theory is in essence operational and algebraic, it is convenient, but not essential, to use a geometrical terminology.

The Operator Associated with a Point.

Consider the point $(a_{r,x}\,a_{r-1,x}\,\ldots\,a_{2,x}\,a_{1,x}\,a_{m,x}\,a_{m-1,x}\,\ldots\,a_{r+1,x})$ in space of m dimensions, the first suffix being arranged cyclically in descending order from m. For brevity we denote this point by (r, x).

* *Proc. Camb. Phil. Soc.*, **26**, p. 127, 1930.

The operator associated with this point is denoted by $(r{\cdot}x)$, where

$$(r{\cdot}x) = (a_{r,x} \; . \; . \; . \; a_{1,x} \; \dot{a}_{m,x} \; . \; . \; . \; a_{r+1,x}),$$

and is interpreted as follows:—

$(z_1 \, z_2 \; . \; . \; . \; z_m)$ being any point,

$$(r{\cdot}x)(z_1 \, z_2 \; . \; . \; . \; z_m) = (z'_1 \, z'_2 \; . \; . \; . \; z'_m),$$

where

$$\begin{aligned}
z'_1 &= a_{r,x} \, z_{r+1} + z_1 \\
z'_2 &= a_{r-1,x} \, z_{r+1} + z_2 \\
&\cdot \quad \cdot \quad \cdot \quad \cdot \\
z'_r &= a_{1,x} \, z_{r+1} + z_r \\
z'_{r+1} &= a_{m,x} \, z_{r+1} \\
z'_{r+2} &= a_{m-1,x} \, z_{r+1} + z_{r+2} \\
&\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
z'_m &= a_{r+1,x} \, z_{r+1} + z_m.
\end{aligned}$$

Thus the operator $(r{\cdot}x)$ converts any given point into a definite point.

If all the a are zero except that which has the dot, and if this is unity, the operator leaves unaltered the point on which it operates. We denote by I_r this identical operator, the non-zero term being in the $(r+1)$th place from the left.

Thus

$$\mathrm{I}_r(z_1 \, z_2 \; . \; . \; . \; z_m) = (0 \; 0 \; . \; . \; . \; 0 \; 1 \; 0 \; . \; . \; . \; 0)(z_1 \, z_2 \; . \; . \; . \; z_m) = (z_1 \, z_2 \; . \; . \; . \; z_m).$$

If $r \geqslant m$ we can write

$$r = km + s, \qquad k > 0, \; m > s \geqslant 0,$$

and we define

$$(r{\cdot}x) = (s{\cdot}x).$$

It will be observed that s is the least positive residue of r modulo m.

Note that in particular

$$(m{\cdot}x) = (0{\cdot}x) = (\dot{a}_{m,x} \; a_{m-1,x} \; . \; . \; . \; a_{1,x}).$$

We can pass from any operator to the point with which it is associated by writing (r, x) for $(r{\cdot}x)$.

The Operator V_k.

This is defined by the relation

$$\mathrm{V}_k(z_1 \, z_2 \; . \; . \; . \; z_m) = z_k.$$

Thus the operator V_k isolates for consideration the kth co-ordinate of the point on which it operates.

Generalised Continued Fractions.

We define the generalised cyclic continued fraction of m cycles as the sequence of operations

$$\mathrm{I}_{m-2}(m-1\cdot 1)(0\cdot 2)(1\cdot 3)(2\cdot 4) \ldots (x-2\cdot x) \ldots,$$

or as any sequence which can be obtained from the given sequence by suppressing a finite number of consecutive operations from the left. In this notation the continued fraction $a_1 + \frac{b_2}{a_2+} \frac{b_3}{a_3+} \ldots$ could be denoted by

$$(a_1\ 1)(b_2\ a_2)(a_3\ b_3)(b_4\ a_4) \ldots$$

If in the generalised continued fraction given above we suppress all the operations to the right of $(x-2\cdot x)$ and replace $(x-2\cdot x)$ by the associated point, we obtain the xth "convergent"

$$(\lambda_x \ldots \beta_x\ \alpha_x\ \mu_x) = \mathrm{I}_{m-2}(m-1\cdot 1)(0\cdot 2) \ldots (x-2,\ x),$$

and in particular

$$\alpha_x = \mathrm{V}_{m-1}\ \mathrm{I}_{m-2}(m-1\cdot 1)(0\cdot 2) \ldots (x-2,\ x),$$

so that

$$\alpha_0 = \mathrm{V}_{m-1}(0\ 0 \ldots 0\ 1\ 0) = 1,$$

$$\alpha_1 = \mathrm{V}_{m-1}(m-1,\ 1) = a_{11} = a_{11}\ \alpha_0,$$

$$\alpha_2 = a_{12}\ \alpha_1 + a_{22}\ \alpha_0.$$

If we define $\alpha_s = 0$ for s negative, we see that α_1 and α_2 satisfy the relation

$$\alpha_x = a_{1,\,x}\ \alpha_{x-1} + a_{2,\,x}\ \alpha_{x-2} + a_{3,\,x}\ \alpha_{x-3} + \ldots + a_{m,\,x}\ \alpha_{x-m}\,. \qquad (2)$$

That this relation is general we prove by induction. For

$$\alpha_{x+1} = \mathrm{V}_{m-1}(m-1\cdot 1) \ldots (x-3\cdot x-1)[(x-2\cdot x)(x-1,\ x+1)],$$

$$\alpha_x = \mathrm{V}_{m-1}(m-1\cdot 1) \ldots (x-3\cdot x-1)[(x-2,\ x)].$$

Thus α_{x+1} is obtained from α_x by replacing $(x-2,x)$ by $(x-2\cdot x)(x-1, x+1)$; that is, $a_{r,\,x}$ is replaced by $a_{r,\,x}\ a_{1,\,x+1} + a_{r+1,\,x+1}$ if $r \neq m$, while $a_{m,\,x}$ is replaced by $a_{m,\,x}\ a_{1,\,x+1}$. Hence assuming that (2) is true for x,

$$\alpha_{x+1} = a_{1,\,x+1}(a_{1,\,x}\ \alpha_{x-1} + \ldots + a_{m,\,x}\ \alpha_{x-m}) + a_{2,\,x+1}\ \alpha_{x-1} + \ldots + a_{m,\,x+1}\ \alpha_{x-m+1}$$

$$= a_{1,\,x+1}\ \alpha_x + a_{2,\,x+1}\ \alpha_{x-1} + \ldots + a_{m,\,x+1}\ \alpha_{x-m+1},$$

whence (2) follows by induction from α_1. That the other components of the xth convergent satisfy the same relation follows from the above proof by writing V_k for V_{m-1}. It follows at once from (2) that α_x is a particular solution of the homogeneous linear equation of order m:

$$u_x = a_{1,\,x}\ u_{x-1} + a_{2,\,x}\ u_{x-2} + \ldots + a_{m,\,x}\ u_{x-m}\,. \qquad (3)$$

The general solution of this equation involves m arbitrary values of u_x, say the first m,

$$u_0\, u_1\, u_2\, \ldots\, u_{m-1}.$$

Now

$$a_m = a_{1,m}\, a_{m-1} + a_{2,m}\, a_{m-2} + \ldots + a_{m,m}\, a_0 \quad . \quad . \quad . \qquad (4)$$

and by definition

$$a_m = \mathrm{V}_{m-1}(m-1\cdot 1)(0\cdot 2)\ \ldots\ (m-2,\ m).$$

But

$$(m-2,\ m) = (a_{m-2,m}\ \ldots\ a_{1,m}\ a_{m,m}\ a_{m-1,m})$$

is the first convergent of the continued fraction

$$(m-2\cdot m)(m-1\cdot m+1)\ \ldots\ (x-2\cdot x)\ \ldots$$

If, then, we write

$$(m-2\cdot m)\ \ldots\ (x-2,\ x) = (\kappa'_{x-m+1}\ \ldots\ \alpha'_{x-m+1}\ \mu'_{x-m+1}\ \lambda'_{x-m+1}),$$

we see that a_x is obtained from (4) by writing α'_{x-m+1} for $a_{1,m}$, β'_{x-m+1} for $a_{2,m}$, and so on. Consequently if we ascribe to $a_{m-1}, a_{m-2}, \ldots, a_0$ the values $u_{m-1}, u_{m-2}, \ldots, u_0$ and suppress the primes, we have as the general solution of (3)

$$u_x = \alpha_{x-m+1}\, u_{m-1} + \beta_{x-m+1}\, u_{m-2} + \ldots + \mu_{x-m+1}\, u_0 \quad . \quad . \qquad (5)$$

We have thus expressed the general solution of (3) in terms of the components of the $(x-m+1)$th convergent of the generalised continued fraction

$$(m-2\cdot m)(m-1\cdot m+1)\ \ldots\ (x-2\cdot x)\ \ldots$$

which contains no arbitrary elements and which is derived direct from the given equation. The values of the coefficients in (3) for $x=0,\ 1,\ 2,\ \ldots,\ m-1$ do not enter the solution.

We can now make the solution completely operational by introducing the operator

$$\mathrm{U}(u_2\, u_3\, \ldots\, u_{m-1}\, u_0\, u_1),$$

which is defined by the relation

$$\mathrm{U}(u_2\, u_3\, \ldots\, u_{m-1}\, u_0\, u_1)(\kappa\ \ldots\ \beta\ \alpha\ \mu\ \lambda)$$
$$= u_{m-1}\,\alpha + u_{m-2}\,\beta + \ldots + u_0\,\mu,$$

and we can now assert that the general solution of the homogeneous linear equation of order m is

$$u_x = \mathrm{U}(u_2\, u_3\, \ldots\, u_{m-1}\, u_0\, u_1)(m-2\cdot m)(m-1\cdot m+1)\ \ldots\ (x-2,\ x).$$

This is an explicit solution of the equation. For a given x the operations can of course always be performed.

As an application of the foregoing results consider the homogeneous equation of the third order,

$$u_x = s_1\, u_{x-1} + s_2\, u_{x-2} + s_3\, u_{x-3},$$

where $s_1 = a+b+c$, $-s_2 = bc+ca+ab$, $s_3 = abc$, a, b, c being unequal constants.* The solution is

$$u_x = a_{x-2}\, u_2 + \beta_{x-2}\, u_1 + \gamma_{x-2}\, u_0$$

where u_0, u_1, u_2 are initial values of u_x, and

$$(a_{x-2}\ \gamma_{x-2}\ \beta_{x-2}) = (s_1\ \dot{s}_3\ s_2)(s_2\ s_1\ \dot{s}_3)(\dot{s}_3\ s_2\ s_1)\ \ldots\ (x-2,\ x).$$

In particular

$$(a_1\ \gamma_1\ \beta_1) = (s_1,\ s_3,\ s_2),$$
$$(a_2\ \gamma_2\ \beta_2) = (s_1\ \dot{s}_3\ s_2)(s_2,\ s_1,\ s_3)$$
$$= (s_1^2 + s_2,\ s_1 s_3,\ s_3 + s_1 s_2),$$
$$(a_3\ \gamma_3\ \beta_3) = s_1^3 + 2s_1 s_2 + s_3,\ s_1^2 s_3 + s_2 s_3,\ s_1^2 s_2 + s_2^2 + s_1 s_3).$$

Thus

$$a_1 = a + b + c,$$
$$a_2 = a^2 + b^2 + c^2 + ab + bc + ca,$$
$$a_3 = a^3 + b^3 + c^3 + a^2 b + b^2 c + c^2 a + ab^2 + bc^2 + ca^2 + abc.$$

If P_x denotes the sum of the homogeneous products of degree x of a, b, c, we see that

$$a_1 = P_1, \quad a_2 = P_2, \quad a_3 = P_3.$$

Now

$$P_x = -\frac{a^{x+2}(b-c) + b^{x+2}(c-a) + c^{x+2}(a-b)}{(a-b)(b-c)(c-a)} \quad . \quad . \quad . \quad (6)$$

Using the recurrence relation

$$a_x = s_1 a_{x-1} + s_2 a_{x-2} + s_3 a_{x-3},$$

it follows at once by induction that $a_x = P_x$. Again,

$$\gamma_1 = s_3, \quad \gamma_2 = s_3 a_1, \quad \gamma_3 = s_3 a_2,$$

whence by the recurrence relation $\gamma_x = s_3 P_{x-1}$. Lastly,

$$\beta_1 = -s_1 P_1 + P_2, \quad \beta_2 = -s_1 P_2 + P_3, \quad \beta_3 = -s_1 P_3 + P_4,$$

so that

$$\beta_x = -s_1 P_x + P_{x+1}.$$

Hence we have the complete solution

$$u_x = u_2 P_{x-2} + u_1[P_{x-1} - (a+b+c)P_{x-2}] + u_0 P_{x-3}\, abc$$

where P_x is given by (6).

* See L. M. Milne-Thomson, *Proc. Camb. Phil. Soc.*, **27**, p. 26, 1931, where a general operational method, applicable to all linear equations with constant coefficients, is given. The above equation is here taken merely as the simplest illustration of the continued fraction method.

(*Issued separately May* 20, 1931.)

XIV.—Male Haploidy and Female Diploidy in *Sirex cyaneus* F. (Hymen.).

Professor **A. D. Peacock**, D.Sc., and **R. A. R. Gresson**, B.Sc., Ph.D., University College (University of St Andrews), Dundee. (With One Plate.)

(MS. received March 11, 1931. Read May 19, 1931.)

INTRODUCTION.

THE following short paper presents the first cytological study made of a member of the Siricidæ * (Tenthredinoidea),† and follows a similar study by Peacock and Sanderson (6) of *Pteronidea* (*Nematus*) *ribesii* Scop., from the adjacent family Tenthredinidæ. Both papers attempt to show chromosomal conditions in what are usually regarded as the most generalised of Hymenoptera; but, whereas the *ribesii* paper enters into some detail concerning somatic tissue and the female maturation stages, the present paper does not. This is because it is difficult to obtain the amount of material necessary, owing to the specialised wood-boring habit of the insect. The inquiry has proceeded far enough, however, to warrant a preliminary report.

A discussion on Hymenopterous cytology, and the list of references thereto, are given in the above-mentioned paper by Peacock and Sanderson.

We acknowledge with thanks the kindness of Dr R. Neil Chrystal of the Imperial Forestry Institute, Oxford, in providing, at considerable trouble to himself, the specimens examined. Thanks are also due to the Royal Society of London for a grant by means of which this and other Tenthredinid researches have been carried out.

MATERIAL AND METHODS.

Material was obtained from larval and pupal specimens of *Sirex cyaneus*, the ovaries being removed from larvæ in December 1928, and the testes from larvæ and pupæ in June 1929.

In dissections saline solution was used and the gonads immediately transferred to Bouin's picro-formal fixative; sections were cut at 5 μ

* Nomenclature according to Waterson in "The Sirex Wood-wasps and their Importance in Forestry," by R. N. Chrystal, *Bull. Ent. Res.*, xix, pt. 3, 1928.

† Nomenclature according to Enslin's *Die Tenthredinoidea Mitteleuropas*, 1912.

and subsequently stained in iron hæmatoxylin. The only available testis treated according to Feulgen's "Nuclealreaktion" (2) was fixed in corrosive-acetic fixative and sections cut at 5 μ.

Cytological Findings.

The main conclusions concerning chromosome constitution may be summarised as follows: the first spermatocyte division is abortive; the second spermatocytes show clearly 8 chromosomes, while the oogonia show 16. The significance of these findings will be discussed later.

The testes of advanced larvæ and pupæ contained only sperms, and, consequently, the following description is confined to the conditions observed in the testes of the younger larvæ.

Spermatogonial divisions were not observed, and the majority of spermatocytes were passing through the early stages of the abortive division. Some few follicles, however, contained cells at earlier and later stages, so that the behaviour of the first and second spermatocytes could be followed with ease. The following description applies to iron hæmatoxylin preparations.

The resting first spermatocytes were revealed as small cells with disproportionately large nuclei, the chromatin being clumped together in a deeply staining nucleolar-like body (fig. 1, Plate). This body for convenience is referred to as the "nucleolus." The next stage is marked by the appearance of granules and threads of chromatin which appear to have origin in the "nucleolus." A chromatoid body situated outside the nuclear membrane was frequently observed at this stage (fig. 2, Plate). The granules and threads increase in number, and the "nucleolus" disappears, so that the nucleus is now shown as a clear body, containing at its centre a loose skein of threads in which are situated darkly staining granules of varying size. Simultaneously one pole of the spermatocyte becomes elongated and pointed, the whole cell consequently becoming somewhat pear-shaped (fig. 3, Plate). In many cases a thick thread of chromatin is shown extending from the main chromatin skein or mass to the nuclear membrane (figs. 4 and 5, Plate). At this stage a centrosome appears at one pole, and spindle fibres become visible and pass from the centrosome to the vicinity of the nuclear membrane (fig. 5, Plate). That the other pole of the cell becomes slightly drawn out and pointed is not readily obvious except after examination of a large number of spermatocytes (figs. 6 and 7, Plate); spindle fibres and two centrosomes are thereby easily distinguished. It should be particularly

noted that during these phenomena the nuclear membrane does not disappear and, although the masses of chromatin assume more or less definite shapes, true chromosomes do not appear to be formed (figs. 6, 7, and 8, Plate).

In a certain few cells a small dark granule is present outside the nuclear membrane and situated in the vicinity of the thread of chromatin mentioned above; also, a small knob of chromatin occurs at the end of the chromatin thread, inside the nuclear membrane (figs. 4 and 5, Plate). The position of the granule in the cytoplasm, and the presence of chromatin thread and knob in contact with the nuclear membrane, suggest that chromatin may be extruded from the nucleus to the cytoplasm (see p. 100). In a few instances more than one exterior granule was observed; in fig. 8, Plate, small granules surrounded by faintly stained material are shown at one pole. The three larger granules are probably chromatoid bodies.

The chromatin now coalesces again and clumps (figs. 9 and 10, Plate), and the spindle fibres, although they may persist until a slightly later stage, tend to disappear. As the chromatin becomes clumped, numerous small dark granules appear in the cytoplasm, and the chromatoid body may increase greatly in size (figs. 11 and 12, Plate). The appearances of many cells suggest that a bud containing granules is constricted off the cell process at the more pointed pole (figs. 12 and 13, Plate). It is very difficult to determine with certainty whether the buds become absolutely free; indeed, it is possible that they are not freed. After the formation of the buds the cells become round in outline, the buds disappear, and the cells enter on the early stage of the normal division of the second spermatocyte.

Neither chromosome size and shape nor the phases of nuclear division were studied in detail, as the primary question posed was the problem of haploidy and diploidy. An examination of a large number of second spermatocyte plates and spindles proved that in the majority of cases the chromosomes were quite distinct and separate, so that counts could be made with ease and certainty. Consequently there is no doubt that the haploid number is 8 (figs. 14, 15, 16, and 17, Plate).

Many of the follicles contained spermatids which, owing to their characteristic appearance, could easily be distinguished from the other cells. It is of interest to note that a chromatoid body occurred in several spermatids, in most cases being situated in the vicinity of the nuclear membrane (fig. 18, Plate).

The only testis available for treatment by Feulgen's technique gave

the correct reaction, the chromatin granules and threads of the first spermatocytes and the chromosomes of the second spermatocytes staining a purplish colour, the other cell structures remaining unstained. Stages of the abortive division and of the division of the second spermatocyte, such as were observed in material stained in iron hæmatoxylin, could be recognised, but, owing to fixation distortion and shrinkage in using corrosive-acetic fixative of the strength recommended by Ludford (4) for Feulgen's method, this material was unsuitable for chromosome counts and the detailed study of the abortive division. Shortage of material has prevented a repetition of the process using weaker fixative. The chromatoid body and extra-nuclear "chromatin" granules were not visible although a large number of cells was examined. The non-visibility of the "chromatin" granules, however, may be due to their small size or the shrinkage effect of the fixative. In contrast to these effects on the testis, it is worthy of note that Tenthredinid ovaries treated by Feulgen's method in previous work (Gresson, 3), gave more satisfactory fixation results.

The investigation of the larval ovaries showed that differentiation of the oocytes had not yet taken place. The greater number of the cells was in the resting condition, but small groups of oogonia in certain parts of the ovaries proved to be in active division. Chromosome counts were made from the most favourable of these cells and showed 16 beyond doubt (fig. 19, Plate).

Attempts to obtain exact counts from the ovarian follicle-cells did not prove successful owing to the small size of the chromosomes and the manner in which they clumped during division. But it was clearly observed that the chromosome number could not be less than 16.

Thoracic sections of female pupæ were examined in the hope of obtaining somatic counts, but the only cells in division were giant cells and, as is well known for other animals, the number of chromosomes in these is greater than that found in the normal somatic cells.

Discussion.

The literature relating to the abortive division in the Hymenoptera has recently been discussed by Vandel (7), and will be further treated in some detail in a forthcoming paper on Tenthredinid spermatogenesis by our colleague Miss Sanderson; it is not proposed to deal with it here except in so far as it refers to Sirex.

No definite resting stage between the abortive division and second spermatocyte division, as recorded by Doncaster for Neuroterus (1), by

Wieman for Dryophanta (8), and by Patterson and Porter for Paracopidosomopsis (5), was observed. However, the manner in which the chromatin becomes clumped at the end of the abortive division suggests that a short resting stage intervenes before the cell enters on the second division.

It is worthy of note that what is generally termed the chromatoid body did not give the chromatin reaction by Feulgen's technique.

In the spermatocytes treated with iron hæmatoxylin this body was very conspicuous, but in spermatocytes treated by Feulgen's method there was not revealed any cytoplasmic body which gave the chromatin reaction. This observation suggests that caution must be observed in interpreting the "chromatoid" body as being chromatin.

Regarding the study of the nucleolus of the resting first spermatocyte, the available material treated by Feulgen's technique did not give unequivocal results, for the stages earlier than the abortive division (described above) were too indistinct to read. But whether this was due to imperfect technique or to the nature of the stages concerned cannot be decided. Further material is necessary.

The cytological features described in this Tenthredinid, namely, the abortive first spermatocyte division, and chromosome counts in spermatocytes and oogonia, are exactly comparable with the features recorded by Peacock and Sanderson (6) for the Tenthredinid *Pteronidea ribesii*, and also described for other Hymenoptera in their paper.

Summary.

1. From the above it is therefore justifiable to conclude that *Sirex cyaneus* is typically hymenopterous, in that:

   (*a*) the ripe-male reproductive tissue is haploid (8 chromosomes), while the female oogonia are diploid (16 chromosomes);

   (*b*) the abortive character of the first spermatocyte division is evidence that chromosome reduction does not occur during the maturation of the male gametes.

2. Support is thus afforded to the hypothesis commonly held, that the male hymenopteron is a haploid organism and is the product of an unfertilised (haploid) egg, while the female is a diploid organism arising from a fertilised egg.

3. Feulgen's "Nuclealreaktion" holds for the chromatin involved in the abortive division and for the chromosomes of the second spermatocyte; but the large chromatoid body and cytoplasmic granules seen in these same stages by the iron hæmatoxylin technique did not give the chromatin reaction. The nature of these chromatoid bodies remains to be discovered.

REFERENCES.

(1) Doncaster, L., "Gametogenesis of the Gall-fly, *Neuroterus lenticularis* (*Spathegaster baccarum*), Part 1," *Proc. Roy. Soc.*, B, vol. lxxxii, 1910.

(2) Feulgen, R., "Die Nuclealfarbung," Abderhalden's *Handbuch der biologischen Arbeitsmethoden*, Abt. V, Lief. 213, 1924.

(3) Gresson, R. A. R., "Certain Phenomena of Tenthredinid Oogenesis as revealed mainly by Feulgen's Nuclear-Reaction," *Quart. Journ. Micro. Sci.*, vol. lxxiii, 1930.

(4) Ludford, R. J., "Studies on the Microchemistry of the Cell. I. The chromatin content of normal and malignant cells, as demonstrated by Feulgen's Nuclealreaktion," *Proc. Roy. Soc.*, B, vol. cii, 1928.

(5) Patterson, J. T., and L. T. Porter, "Studies on the Biology of Paracopidosomopsis. II. Spermatogenesis of males reared from unfertilised eggs," *Biol. Bull.*, vol. xxxiii, 1917.

(6) Peacock, A. D., and A. R. Sanderson, "Cytological Evidence of Male Haploidy and Female Diploidy in a Saw-fly (Hymenoptera)," *Proc. Second Intern. Cong. for Sex Research*, 1931.

(7) Vandel, A., *La Parthénogénèse*, Paris, 1931.

(8) Wieman, H. L., "Observations on the Spermatogenesis of the Gall-fly, *Dryophanta erinacei* (Mayr.)," *Biol. Bull.*, vol. xxviii, 1915.

DESCRIPTION OF PLATE.

The drawings were made by means of a Zeiss camera lucida and a Watson "Service" Microscope, a Reichert $\frac{1}{12}$ oil immersion objective, and a Hawksley eye-piece No. 4 × 10. The figures were subsequently enlarged $2\frac{1}{2}$ times and the detail completed with the aid of the microscope.

Lettering.

ce, centrosome.
ch, chromatin.
ch.b, chromatoid body.
gr, dark granules in cytoplasm.
n, "nucleolus."

Fig. 1. Resting first spermatocyte.

Fig. 2. Later stage showing granules and threads of chromatin.

Fig. 3. Early stage of abortive division.

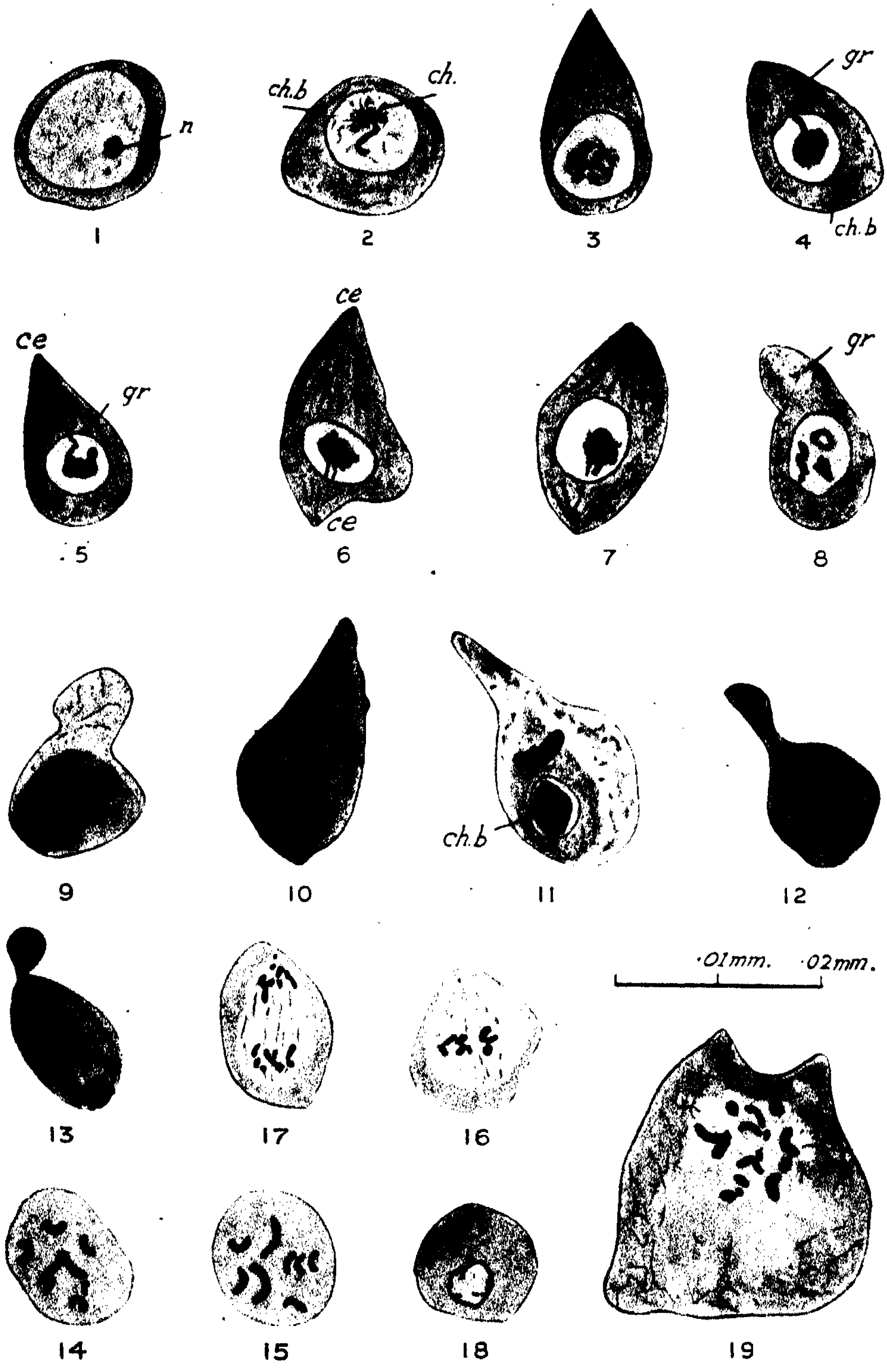
Fig. 4. Abortive division; thread of chromatin extending from main chromatin skein to nuclear membrane; dark granule outside nuclear membrane.

Fig. 5. Abortive division; chromatin thread, granule outside nuclear membrane, spindle fibres, and a single centrosome shown.

Figs. 6 and 7. Abortive division; spindle fibres and 2 centrosomes shown.

Fig. 8. Abortive division; small granules surrounded by faintly stained material shown at one pole; 3 dark bodies probably chromatoid.

Figs. 9 and 10. Abortive division; late stages showing clumped chromatin.



A. D. Peacock and R. A. R. Gresson.

Fig. 11. Abortive division; chromatin clumped; dark granules in cytoplasm; note extremely large chromatoid body.

Figs. 12 and 13. Abortive division; chromatin clumped; dark granules in cytoplasm; note bud formation.

Figs. 14 and 15. Second spermatocytes; plates showing 8 chromosomes.

Fig. 16. Second spermatocyte; metaphase showing 8 chromosomes before splitting.

Fig. 17. Second spermatocyte; anaphase; not more than 8 chromosomes at either pole.

Fig. 18. Spermatid; chromatoid body above nucleus.

Fig. 19. Oogonium showing 16 chromosomes.

Addendum, May 19, 1931.

Since this paper was completed an examination of more recently available material by our colleague, Miss Ann R. Sanderson, B.Sc., has shown that the spermatogonia also possess the haploid number of 8 chromosomes.

(*Issued separately July* 7, 1931.)

XV.—Some New Facts about the Structure of the Cuticles in the Russian Paper-Coal and their Bearing on the Systematic Position of Some Fossil Lycopodiales. By Jessie A. R. Wilson, B.Sc. (With One Plate and Seven Text-figures.)

With a note on

The Absence of Eligulate Heterosporous Lycopodiales in the Fossil-Record. By John Walton, M.A., D.Sc., Regius Professor of Botany in the University of Glasgow.

(MS. received April 2, 1931. Read May 19, 1931.)

THE fossil cuticles of the paper-coal from the Moscow Basin in Central Russia were first described and figured by Auerbach and Trautschold* in 1860. Since then these cuticles have been the subject of much discussion, especially with regard to the interpretation of their numerous regularly disposed perforations. According to the various views of previous investigators, several generic and specific names have been assigned to these cuticles. Auerbach and Trautschold, regarding the perforations as the result of the fall of the leaf-cushions as well as the leaves, considered the cuticles to belong to the genus *Lepidodendron*, and named the plant to which they belonged *Lepidodendron tenerrimum*. Previously Eichwald,† in *Lethœa Rossica*, had described impression material, also from the Moscow Basin, under the name *Lepidodendron Olivieri*, with which, according to Zalessky,‡ the cuticles described by Auerbach and Trautschold are specifically identical.

Göppert (*fide* Zalessky §) also regarded the cuticles as parts of a *Lepidodendron* but referred them to *Lepidodendron obovatum* Sternberg. He considered the perforations as marking the exit of the vascular tissue to the leaves. Until the investigations of Zeiller, the membranes found in the paper-coal were regarded as the epidermis of the stem, but he demonstrated clearly that they must be regarded as the cuticle alone.‖ Interpreting the gaps in the cuticles as due to the fall of the leaves,

* Auerbach and Trautschold (1860), p. 40, pl. iii, figs. 1–6.
† Eichwald (1854), p. 108, pl. v, figs. 10–13.
‡ Zalessky (1915), p. 42. § Zalessky, *ibid.*, p. 28.
‖ Zeiller (1882), p. 221, pl. x, figs. 13–14.

Zeiller refers these fossils to the genus *Bothrodendron*, and from a comparison with Lindley and Hutton's specimens to the species *Bothrodendron punctatum* L. and H.* Zeiller figures the cuticles upside down, showing the openings constricted towards the upper end. He attributes the shape of each opening to the fusion of two gaps in the cuticle, the lower and larger representing the leaf-scar, and the smaller, upper one the opening to the ligular pit.

The classification of these remains with *Bothrodendron* was also adhered to at first by Nathorst,† but he later‡ changed his mind, preferring the generic name *Porodendron* previously proposed by him for some Spitsbergen fossils which he considered were specifically identical with the Russian cuticles. Zalessky § at first adopted this nomenclature and referred some fossils from the lower Carboniferous of the Mugodschari Mts. to *Porodendron tenerrimum*. In 1915, however, Zalessky, as a result of a thorough investigation of Eichwald's specimens and also those of Auerbach and Trautschold, readopted the name Lepidodendron for the cuticles.‖ With regard to the peculiar strap of cuticle which extends over the perforations, Zeiller and Zalessky both considered that it represented a strip of cuticle from the surface of the leaf—according to Zeiller from the abaxial surface, which view was necessitated by the manner in which he orientated his specimens, but according to Zalessky from the adaxial surface.

Fresh light was thrown on the nature of the cuticles by Professor Walton's interpretation of the strap of cuticle as the cuticular lining of the ligular pit.¶ Viewed in this light, the perforations must needs be regarded as due to the loss of the leaves and not so much as due to the disappearance of the leaf-cushions, and on the strength of this interpretation the cuticles are referred to the genus *Bothrodendron*. Bode recently put forward another interpretation for these straps of cuticle. He regards them as due to the rolling up of a cuticle which he supposes formed over the abscission surface after the leaf fell off.** This implies that the perforations correspond to the leaf-scars, and as Bode could find no indication of the presence of a ligule, he classes these plant-remains amongst the eligulate Lycopods, and with Nathorst's genus *Porodendron*.†† This genus is characterised by the absence of a ligule, by the possession of small, compact leaves, not situated on leaf-cushions, and by the presence

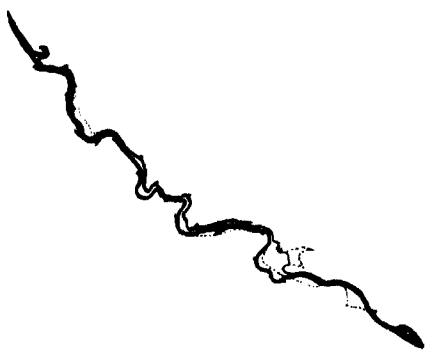
* Zeiller (1882), p. 224, pl. x, figs. 1–12.
† Nathorst (1894), p. 45.
‡ *Idem.* (1914), p. 68.
§ Zalessky (1909), p. 5, pl. i, figs. 1–4.
‖ Zalessky (1915), p. 42.
¶ Walton (1926), p. 121.
** Bode (1929), p. 131.
†† *Ibid.*, p. 133.

of only one print upon the leaf-scar. Among the cuticles Bode distinguishes two types, to which he gives separate specific names, viz. *Porodendron lepidodendroides* and *Porodendron pinakodendroides*, the characters of which are based upon the form and arrangement of the leaf-scars.*

Thus from the foregoing it is apparent that the systematic position of the plants which produced these cuticles may be closely correlated with the interpretation of this strap of cuticle, which extends into the openings from their upper edge, and on Professor Walton's suggestion a reinvestigation of the cuticles was undertaken, in view of the facts recently stated by Bode.

Description of Some New Specimens.

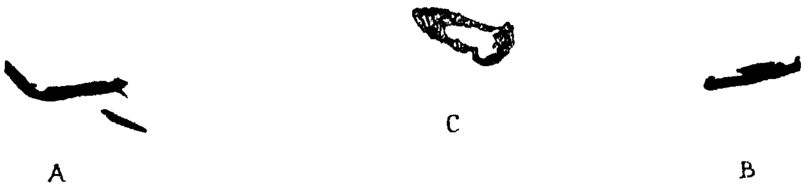
Fragments of cuticle for examination were prepared by Schultze's Maceration Method, described by Bode,† from a sample of paper-coal from Tovarkovo in Central Russia. In addition, microtome sections were cut of samples of the cuticle with perforations which showed a well-defined strap. This material was dehydrated, embedded in paraffin wax (M.P. 52° C.), and sections cut at 10 μ thickness. As the sections were liable to be washed off the slide, collodion dissolved in clove oil was employed to fix the paraffin ribbons to the slide instead of egg albumen. Sections cut transversely through the strap show in most cases that it consists of two membranes closely pressed together (text-fig. 1). In some sections these membranes have become more widely separated, and it is then evident that they are usually continuous with each other at the two ends (text-fig. 2, and Plate, fig. 5). This implies that the strap is really a tube which has become flattened. The main cuticle in transverse section appears smooth on one side and irregular on the other, the irregularities being due to the moulding of the cuticle to the contour of the epidermal cells.‡ Although its cuticle is much thinner, a similar



Text-fig. 1.—Transverse section of a strap of cuticle, which extends into a leaf-scar perforation. The small projections occurring at regular intervals on the outer side of the membranes are the portions of cuticle above the lateral walls of the cells which surrounded the ligular pit. × 285.

* Bode (1929), p. 135. † *Ibid.*, p. 129. ‡ Zeiller (1882), p. 221.

appearance is evident in the section of the strap, where the smooth sides of the membrane face each other, and small projections are seen at regular

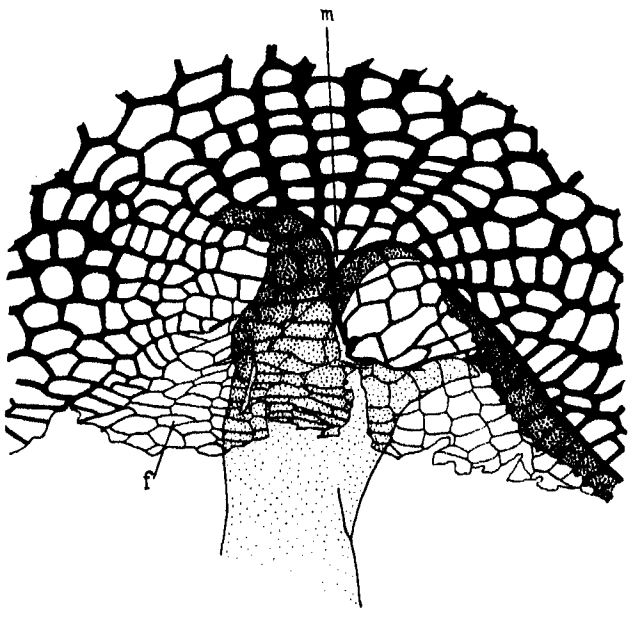


TEXT-FIG. 2.—Transverse section of cuticle through a perforation, bearing a cuticular strap such as is shown in Plate, fig. 4. The outlines of the cells which surround the ligular pit are evident at the left side of the section of the strap.

A and B, stem cuticle at sides of leaf-scar; C, cuticular strap. × 90.

intervals on their outer side (text-fig. 1). This would seem to indicate that the tube was surrounded on all sides by tissue.

Other fragments of cuticle were examined entire. This leads to the

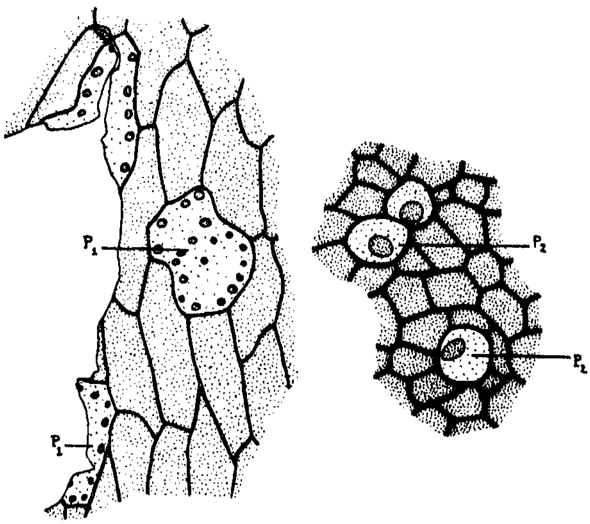


TEXT-FIG. 3.—Drawing of specimen shown in Plate, fig. 4. Portion of cuticle showing upper part of a leaf-scar perforation with the strap of cuticle attached.

m, mouth of ligular pit; *f*, flap of thinner cuticle representing the epidermis which extends between the mouth of the ligular pit and the upper edge of leaf-scar. The upper part of the cuticular strap is torn at the right-hand side. × 135.

same conclusion that the strap of cuticle is a tube, open at both ends, the lower opening of which is plainly shown in Professor Walton's figure (*loc. cit.*, fig. 6), while the upper opening is indicated here in text-fig. 3.

In this figure the portion of cuticle which intervenes between this opening and the perforation still persists as a flap, stretching across the upper end of the tube and continuous with the rest of the cuticle on either side. A striking feature of the cuticle at this region is the radiation of linear series of epidermal cells from this opening, as indicated by the cell outlines; this is seen on the flap as well as on the surrounding cuticle



TEXT-FIG. 4.

A. Portion of thinner cuticle at the margin of a perforation.
P_1, papillate cell of the first type, with very thin cuticle and numerous small cuticular papillæ. × 285.

B. Portion of cuticle at the margin of a perforation.
P_2, papillate cells of the second type, with thin cuticle and a single large papilla. × 285.

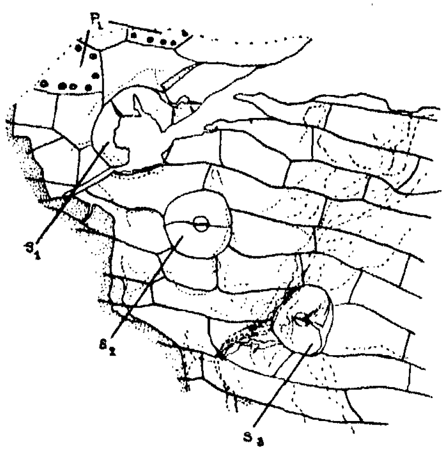
(text-fig. 3). The outlines on the tube show that the cells here were elongated in the longitudinal direction (fig. 4).

In some perforations where the outline is rather irregular, the margin consists of a much thinner cuticle as compared with the rest of the membrane (Plate, figs. 1 and 2). Some of the cells in these thin regions produced a still more delicate cuticle with small cuticular papillæ regularly arranged in their surface (text-fig. 4, A; Plate, fig. 2, P_1). Less frequently another type of papillate cell is met with in this region, where a single large papilla occupies a central position on the cell (text-fig. 4, B). Another remarkable feature of this thin area of cuticle is the presence of stomata (Plate, figs. 2 and 3; text-fig. 5). These have only been found toward the lower edge of the perforation. The stomata are

orientated in the direction of the long axis of the leaf-scar perforation, and as indicated by the cell outlines on the cuticle, have possessed two semicircular guard-cells with a rim of cuticular thickening bordering the pore on either side. It can further be deduced that the stomata were unaccompanied by special subsidiary cells, the surrounding cells being similar in shape and direction to the other epidermal cells, which were bounded by straight walls and were more or less rectangular, with the long axis of the cells running longitudinally.

CONCLUSIONS.

A consideration of these facts leads one to favour the interpretation of the strap as the cuticle which covered the surface of the ligular pit, in which case the perforations in the paper-coal cuticles are due mainly to the loss of the leaves. The small projections, seen in transverse section of the strap, present on the outer sides of its membrane are then the projections of the cuticle external to the anticlinal walls of the cells which line the pit (text-fig. 1). The openings of the tube now become intelligible; the lower indicates the point of attachment of the ligule, which would therefore seem to have been similarly placed to the ligule in *Lepidodendron* (see below, remarks on *Lepidodendron selaginoides*), while the upper opening is the actual mouth of the pit. In *Bothrodendron* stems linear series of epidermal cells radiate from the mouth of the ligular pit similarly to what has already been noted on the cuticles. (My attention was directed to this feature by Professor Walton in the case of *Bothrodendron minutifolium*.) The flap of thin cuticle which hangs down in front of the tube and is continuous with the edge of the tube facing the perforation evidently represents that part of the leaf-surface intervening between the mouth of the pit and the upper edge of the abscission scar. This flap is represented diagrammatically in text-fig. 6 at *b*.

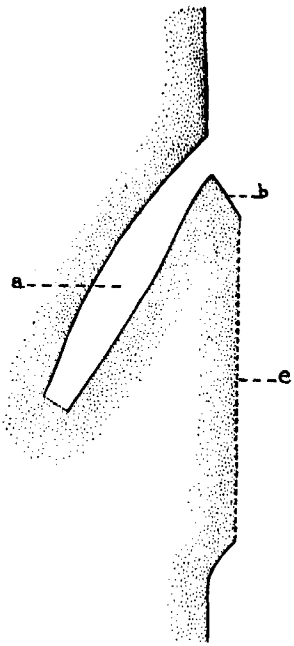


TEXT-FIG. 5.—Portion of the thinner cuticle at the lower edge of the perforation shown in Plate, fig. 1.

P_1, papillate cells; s_1, s_2, s_3, stomata. Broken lines represent fine wrinkles on the surface of the cuticular membrane. × 285.

Since the perforations correspond to the leaf-scars, the irregular thin

areas present round their margins must be cuticle from the leaf-base or cushion, and showing characters of the leaf-cuticle, which was probably in this instance thinner than that of the stem-cuticle. The remarkable structures found in these regions bear this out; papillate cells and stomata being foliar characters. These papillate cells of the first and most frequent type are reminiscent of cells in the leaf-epidermis of various species of *Selaginella*, such as have been described by Harvey Gibson,* and the stomata are also similar to those found in *Selaginella*, in which genus they are oval to circular in shape, as seen in surface view, have no subsidiary cells, and are usually disposed in rows parallel to the mid-rib of the leaf. Bode explained this delicate cuticle round the edge of the gaps as remnants of an absciss cuticle—the main area of which having rolled up to form the strap; but the occurrence of leaf-characters on these areas refutes this idea, and perhaps the round cell seen towards the lower edge of a perforation in one of his figures (*loc. cit.*, pl. xxi, fig. 16), and for which he could give no interpretation, may be something in the nature of our second type of papillate cell.



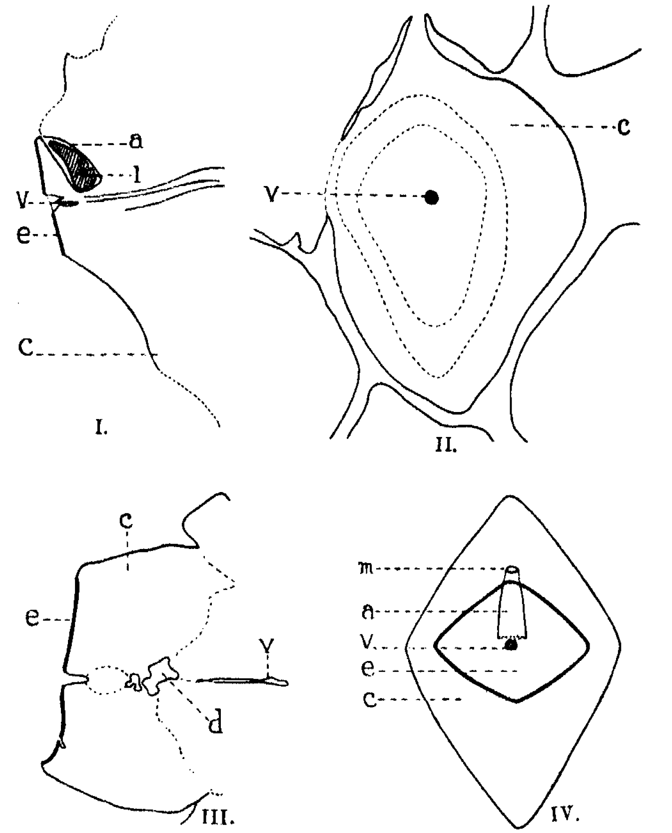
TEXT-FIG. 6.—Diagrammatic longitudinal radial section of stem at base of leaf of the plant, which produced the paper-coal cuticles.

a, ligular pit; *e*, leaf-scar; *b*, that part of leaf-surface which extends from the mouth of the ligular pit to the upper edge of the leaf-scar. × 85.

The interpretation of Bode concerning the strap of cuticle cannot be maintained in view of the above facts. If the strap were indeed the result of rolling up, this would be seen in the transverse section, but no evidences of such can be established. On the contrary, many of the sections appear as complete rings, flattened indeed but with no obvious slit at any point. Examination of numerous perforations reveals no correlation between the shape of the opening and of the tongue, as would be expected if Bode's hypothesis were correct, since the gap represents the abscission surface, but the tongue is at times even longer than the opening, depending upon the amount of leaf-base left attached at the fall of the leaves. The shape of the cell outlines on the tongue also belies the possibility of it being an absciss cuticle, the cell of which would most probably appear iso-diametric. Although isolated cutinised patches may be formed upon the

* Harvey Gibson (1897), p. 134, pl. ix, fig. 16.

cells of an absciss layer, it is questionable if complete cuticularisation of an abscission surface is known to occur in any plant. The presence of the



TEXT-FIG. 7.

I. Diagram of longitudinal section of leaf-cushion of *Lepidodendron selaginoides*. (Drawn from slide No. 922 of the Kidston Slide Collection, Dept. of Botany, University of Glasgow.)

II. Diagram of tangential section of a leaf-cushion of *Lepidodendron selaginoides*. (Drawn from slide No. 926 of the Kidston collection.)

III. Diagram of section transverse to the stem axis of a leaf-cushion of *Lepidodendron selaginoides*. (Drawn from slide No. 1355 of the Kidston collection.)

IV. Diagrammatic representation of leaf-cushion of *Lepidodendron selaginoides* constructed from I, II, and III, to show the relation between ligular pit and leaf-scar.

m, mouth of ligular pit; *a*, ligular pit; *d*, soft tissue at base of pit; *e*, leaf-scar; *c*, leaf-cushion; *v*, vascular strand; *l*, ligule. All × 10 approx.

flap of cuticle, occurring in front of the tongue (Plate, fig. 4; text-fig. 3) is also inconsistent with Bode's view, for if the tongue represented the absciss cuticle, then the upper end of the tongue would coincide and be

continuous with the upper edge of the leaf-scar perforation, but this is not so in the most completely preserved samples.

This collective evidence favours the interpretation of the tongue as the cuticular lining of the ligular pit and not as a rolled-up absciss cuticle.

Text-fig. 7, I, II, and III, are diagrammatic representations of various sections of leaf-cushions of *Lepidodendron selaginoides* from specimens in the Kidston slide collection. From these a diagram of the leaf-cushion in surface view has been proportionally constructed (text-fig. 7, IV). *Bothrodendron* and *Lepidodendron* are intimately related genera, the former only differing in the flattening out and disappearance of the leaf-cushions in the older stems,* and we see that if only the cuticular parts of this *Lepidodendron* leaf-cushion remain, then we get exactly what we have here before us in these paper-coal cuticles: the cuticle of the ligular pit would hang down as a tube behind the leaf-scar.

The plants from which these cuticles have been derived have obviously been in possession of a ligule, and must be classed amongst the Ligulate Lycopods. Since the perforations represent the leaf-scars and there is no evidence of any flattening of raised portions of the cuticle surrounding them, the membranes belong to a Bothrodendroid stem. As previously mentioned, Zalessky, who had access to the actual specimens of Eichwald and of Auerbach and Trautschold, and after a thorough investigation of these, considered *Lepidodendron Olivieri* and *Lepidodendron tenerrimum* to be identical. In view of this and with due regard to the fact that Eichwald's specimens were first described, these fossil-cuticles should be known as *Bothrodendron Olivieri* Eichw. sp.

It is worthy of note that Zalessky † in 1918 figured some fossil cuticles bearing striking resemblance to those of the Moscow paper-coal. The perforations are smaller and more circular than those of the Russian specimens, but a similar strap of cuticle is present attached to the upper edge of the perforation. Zalessky interprets these openings as due to the loss of the leaf-cushions, and the small strap as a portion of cuticle derived from the adaxial surface of the leaf. The cuticle at the lower edge of the perforation is remarkably thinner than the rest of the membrane (*loc. cit.* pl. lxiii, fig. 2). Zalessky regards these cuticles as belonging to a Bothrodendroid stem, but institutes for them a new genus *Angarodendron*. Figured along with these cuticles are impressions of stems under the generic name *Cœnodendron*. These show oval-shaped leaf-cushions, possessing circular leaf-scars, the upper edge of which is almost coincident with the upper edge of the cushion (*loc. cit.* pl. lxiii, fig. 2 *a*). This

* Seward (1910), p. 253.

† Zalessky (1918), p. 9, Atlas, pls. lx–lxiii.

relation of the leaf-scar to the leaf-cushion is suggestive in connection with the cuticles of *Angarodendron*. From comparison with the impressions the perforations of the cuticle may be regarded as corresponding to the leaf-scars and the small area of thinner cuticle found at their lower edge to the leaf-cushions. This comparison may be carried further and applied to the Moscow paper-coal cuticles. In these it is significant that the thinner cuticle occurs only on the lower edge of the perforations (Plate, fig. 2). If this cuticle represents that which covered the leaf-cushion, and the actual opening corresponds to the leaf-scar, then these plants which produced the cuticles may have possessed a very small triangular leaf-cushion on the lower side of the leaf-base. Even although the loss of this leaf-cushion were involved in the fall of the leaves, it would not prevent the preservation of the cuticle of the ligular pit, the position of which is above the leaf-scar.

I wish to acknowledge my indebtedness to Professor J. Walton for his constant help and advice, for his assistance in the production of the photographs, for liberty to use literature from his own collection, and for permission to consult the necessary slides and literature from the Kidston collection.

BIBLIOGRAPHY.

AUERBACH, J. and H. TRAUTSCHOLD, "Uber die Kohle von Zentral-Russland," *Nouv. Mem. Soc. Imp. d. Nat. d. Moskow*, Bd. 13, 1860.

BODE, H., "Zur Kenntnis der Gattung Porodendron Nathorst (*non* Zalessky)," *Palæontographica*, Bd. 72, 1929.

EICHWALD, E., *Lethæa Rossica, Période ancienne*, 1854.

HARVEY-GIBSON, R. J., "Contributions towards a Knowledge of the Genus Selaginella, Spr.," *Ann. of Bot.*, vol. xi, 1897.

NATHORST, A. G., "Zur paläozoischen Flora der Arktischen Zone," *K. Sveska. Vetensk. Ak. Handl.*, Bd. 26, No. 4, 1894; "Zur fossilen Flora der Polarländer," *Nachträge z. Paläozoischen Flora Spitzbergens*, 1914.

SEWARD, A. C., *Fossil Plants*, vol. ii, Cambridge, 1910.

WALTON, J., "Note on the Structure of the Plant Cuticles in the Paper-Coal from Toula in Central Russia," *Mem. and Proc. Manchester Lit. and Phil. Soc.*, vol. lxx, 1926.

ZALESSKY, M. D., "Note sur les débris végétaux du terrain carbonifère de la chaine de Mugodzary," *Bull. Com. Géol.*, T. 28, 1909.

ZALESSKY, M. D., "Observations sur le *Lepidodendron Olivieri* Eichw. et le *Lepidodendron tenerrimum* A. et T.," *Mem. du Comité Géol.*, Nouv. Sér., Livr. 125, 1915.

ZALESSKY, M. D. "Flore palæozoique de la série d'Angara," *Mem. du Comité Géol.*, Nouv. Ser., Livr. 174, 1918.

ZEILLER, R., "Observations sur quelques cuticles fossiles," *Ann. d. Sci. Nat.*, Ser. VI, T. 13, 1882.

DESCRIPTION OF PLATE.

(*All figures from untouched photographs*).

Fig. 1. Photograph of a portion of cuticle, showing a complete leaf-scar perforation, at the lower edge of which is an area of thinner cuticle (t). (× 50.)

Fig. 2. Photograph of lower edge of the perforation of fig. 1. Note the occurrence of papillate cells (P_1) of the first type and three stomata (s_1 s_2 s_3) on the thinner cuticle. (× 150.)

Fig. 3. Photograph of one of the stomata (s_2) seen in fig. 2. (× 570.)

Fig. 4. Photograph of a portion of the cuticle, showing attachment of the tongue to upper edge of leaf-scar perforation. Explanation of various parts as for text-fig. 3. (× 110.)

Fig. 5. Photograph of a transverse section cut by microtome through a tongue of cuticle seen also in text-fig. 2. (× 380.)

NOTE BY PROFESSOR J. WALTON ON

THE ABSENCE OF ELIGULATE HETEROSPOROUS LYCOPODIALES IN THE FOSSIL-RECORD.

There is no satisfactory evidence that eligulate heterosporous Lycopodiales ever existed. There is a possibility, however, that plants answering to that description may be found among the numerous Palæozoic Lycopodiales which are at present imperfectly known. *Cyclostigma kiltorkense* and *Pinakodendron* spp. are known to have been heterosporous, and so far no trace of a ligule or ligular pit has been detected on any parts of them. It is still, however, possible that they possessed a ligule. If the abscission of the leaf were effected in the region between the ligular pit and the stem no trace of a ligule would be shown on the leafless stem, and on a stem with the leaves still in attachment the overlapping leaves would possibly hide the ligule. The sporophylls were borne on the main branches in *Pinakodendron*, and as Kidston has remarked, it is only by the sporangia that it is possible to distinguish between fertile and sterile stems in this genus. In all known species of *Lepidodendron*, *Sigillaria*, and other ligulate Lycopodiales the ligule is situated between the sporangium and the distal extremity of the sporophyll, so that when the sporophyll fell off no indication of a ligule was left on

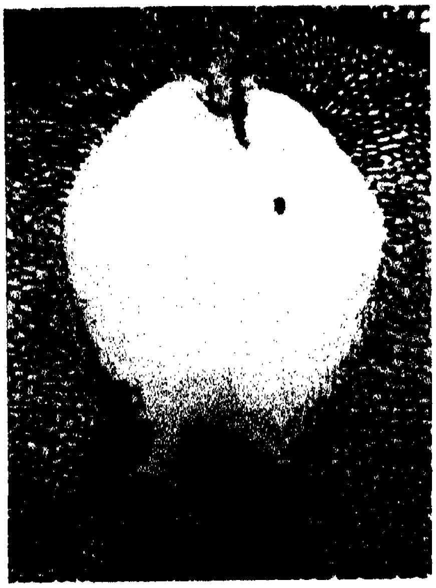


Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

Photo by J. W. & J. A. R. W.

BOTHRODENDRON OLIVIERI EICHW. SP.

Jessie A. R. Wilson.

the stem, and at the same time it is, as far as the writer knows, impossible to detect the ligule or its pit on the surface of incrustations of such detached sporophylls.

With such difficulties before us it is necessary to proceed with caution, and to establish the existence of eligulate heterosporous Lycopodiales among fossils will be a very formidable task. Bode * in a recent paper suggests that there may be some relationship between the plants which produced the cuticles which form such a large proportion of the paper-coal and some cones referred to the genus *Porostrobus* found associated with them. These cuticles and cones possibly belong to the same plant, but it is to be noted that Bode considers the cuticles to belong to an eligulate plant, while at the same time he shows that the cones contain megaspores. These megaspores are of two kinds, large ones 3–3·5 mm. in diameter, and smaller ones 0·8–1·0 mm. in diameter. Both kinds of spores occur, according to Bode, on the same sporophyll. In this connection it may be recalled that there is considerable range in size in the megaspores of *Mazocarpon shorense* Benson † (from 2 mm.–1·3 mm. in diameter), and the suggestion may be made that these smaller spores are not microspores, as Bode suggests, but small megaspores. In *Mazocarpon* the size of the spores is in some inverse relation to the number of spores in the sporangium.

If Bode's interpretation of the structure of the cuticles was correct he would then appear to have established the existence of eligulate heterosporous Lycopodiales, but from the foregoing account of the cuticles by Miss Wilson it is clear that the writer ‡ was justified in interpreting the strap of cuticle as evidence of the presence of a ligule in the original plant, so that Bode's interpretation cannot be accepted.

* Bode, H. (1929), p. 135.

† See Hirmer, M., *Handbuch der Palaobotanik*, p. 283.

‡ Walton, J., 1926.

(*Issued separately July* 8, 1931.)

XVI.—Fourier Integrals. By T. M. MacRobert.

(MS. received March 9, 1931. Read May 19, 1931.)

§ 1. *Fourier's Integral Theorem.*—In a former paper * it was shown that certain formulæ, usually obtained by means of Fourier's Integral Theorem, could also be proved by contour integration: and, indeed, Fourier's Integral Theorem can be established by this method. The functions involved must be holomorphic, so that the proof does not justify the application of the theorem to such wide classes of functions as the usual proof by means of Dirichlet Integrals. In what follows it will be assumed that all the integrals are convergent, and the changes of order of integration justified. Fourier's Integral Theorem will be proved in the following form.

If, for all real values of λ,

$$\int_p^q e^{i\lambda\rho}\phi(\rho)d\rho = f(\lambda), \qquad \text{(i)}$$

where $-\infty \leqq p < q \leqq \infty$, then

$$\int_{-\infty}^{\infty} e^{-i\lambda r} f(\lambda)d\lambda = \begin{cases} 2\pi\phi(r), \ p<r<q, \\ \quad 0, \ r<p \text{ or } r>q. \end{cases} \qquad \text{(ii)}$$

For

$$\begin{aligned} \mathrm{I} \equiv \int_{-\infty}^{\infty} e^{-i\lambda r} f(\lambda)d\lambda &= \int_{-\infty}^{\infty} e^{-i\lambda r}d\lambda \int_p^q e^{i\lambda\rho}\phi(\rho)d\rho \\ &= \int_{-\infty}^{0} e^{-i\lambda r}d\lambda \int_{\mathrm{C}_1} e^{i\lambda\zeta}\phi(\zeta)d\zeta \\ &\quad + \int_0^{\infty} e^{-i\lambda r}d\lambda \int_{\mathrm{C}_2} e^{i\lambda\zeta}\phi(\zeta)d\zeta, \end{aligned}$$

where C_1 and C_2 are contours from p to q in the ζ-plane (fig. I, a) below and above the ξ-axis respectively. (If $p = -\infty$ or $q = \infty$ the con-

* *Proc. Edin. Math. Soc.*, ser. 2, **2** (1929), 26.

tour approaches the ξ-axis asymptotically.) In both integrals $R(i\lambda\zeta)<0$. If now the order of integration be altered

$$
\begin{aligned}
I &= \int_{C_1}\phi(\zeta)d\zeta\int_{-\infty}^{0} e^{i\lambda(\zeta-r)}d\lambda \\
&+ \int_{C_2}\phi(\zeta)d\zeta\int_{0}^{\infty} e^{i\lambda(\zeta-r)}d\lambda \\
&= \int_{C_1}\frac{\phi(\zeta)d\zeta}{i(\zeta-r)} - \int_{C_2}\frac{\phi(\zeta)d\zeta}{i(\zeta-r)} \\
&= \frac{1}{i}\int_{C}\frac{\phi(\zeta)d\zeta}{\zeta-r},
\end{aligned}
$$

where C is a closed contour between p and q (fig. I, b). Hence

$$
I = \begin{cases} 2\pi\phi(r), & p<r<q, \\ 0, & r<p \text{ or } r>q. \end{cases}
$$

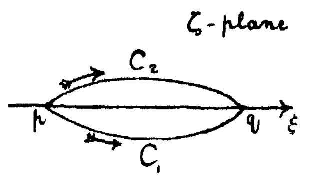


FIG. I, a.

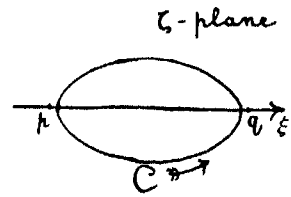


FIG. I, b.

Conversely, if

$$
\int_{-\infty}^{\infty} e^{-i\rho r}f(r)dr = \begin{cases} 2\pi\phi(\rho), & p<r<q, \\ 0, & r<p \text{ or } r>q, \end{cases} \qquad \text{(i, } a\text{)}
$$

then, for all real values of λ,

$$
\int_{p}^{q} e^{i\lambda\rho}\phi(\rho)d\rho = f(\lambda) \qquad . \quad . \quad . \quad . \quad . \quad \text{(ii, } a\text{)}
$$

For

$$
\begin{aligned}
I \equiv \int_{p}^{q} e^{i\lambda\rho}\phi(\rho)d\rho &= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\lambda\rho}d\rho\int_{-\infty}^{\infty} e^{-i\rho r}f(r)dr \\
&= \frac{1}{2\pi}\int_{0}^{\infty} e^{i\lambda\rho}d\rho\int_{C_1} e^{-i\rho\zeta}f(\zeta)d\zeta \\
&+ \frac{1}{2\pi}\int_{-\infty}^{0} e^{i\lambda\rho}d\rho\int_{C_2} e^{-i\rho\zeta}f(\zeta)d\zeta,
\end{aligned}
$$

where C_1 and C_2 are the curves of fig. I, a, with $p=-\infty$ and $q=\infty$.

Thus

$$\begin{aligned} I &= \frac{1}{2\pi}\int_{C_1} f(\zeta)d\zeta \int_0^\infty e^{-i\rho(\zeta-\lambda)}d\rho \\ &+ \frac{1}{2\pi}\int_{C_2} f(\zeta)d\zeta \int_{-\infty}^0 e^{-i\rho(\zeta-\lambda)}d\rho \\ &= \frac{1}{2\pi i}\int_{C_1} \frac{f(\zeta)d\zeta}{\zeta-\lambda} - \frac{1}{2\pi i}\int_{C_2} \frac{f(\zeta)d\zeta}{\zeta-\lambda} \\ &= \frac{1}{2\pi i}\int_{C} \frac{f(\zeta)d\zeta}{\zeta-\lambda} = f(\lambda), \end{aligned}$$

C being a closed contour about $\zeta=\lambda$.

[NOTE.—An important modification of Fourier's Integral Theorem, due to Professor H. M. Macdonald,* was established by practically the same method as that just employed to prove the theorem.]

It will now be shown that the usual Fourier-Bessel Integral Theorem can also be established by means of contour integration. Thereafter the method will be employed to derive two other Fourier-Bessel Integral Theorems and also a Fourier-Legendre Integral Theorem.

§ 2. *The Fourier-Bessel Integral Theorem.*—This theorem may be stated as follows. If $R(n) > -1$ and

$$f(\lambda) = \int_p^q \phi(\rho) J_n(\lambda\rho)\rho d\rho, \quad 0 \leqq p < q \leqq \infty, \qquad \text{(iii)}$$

then

$$\int_0^\infty f(\lambda) J_n(\lambda r)\lambda d\lambda = \begin{cases} \phi(r), & p<r<q, \\ 0, & 0<r<p \text{ or } r>q. \end{cases} \qquad \text{(iv)}$$

The following Bessel Function formulæ are required:

$$G_n(z) = \frac{\pi}{2\sin n\pi}\left\{J_{-n}(z) - e^{-in\pi}J_n(z)\right\}, \qquad \text{(v)†}$$

$$\pi i J_n(z) = G_n(z) - e^{in\pi}G_n(ze^{i\pi}), \qquad \text{(vi)‡}$$

$$G_n(z) \sim \sqrt{\left(\frac{\pi}{2z}\right)} e^{-\frac{1}{2}n\pi i + i(z+\frac{1}{4}\pi)}, \quad -\pi < \operatorname{amp} z < 2\pi, \qquad \text{(vii)§}$$

$$J_n(z) \sim \sqrt{\left(\frac{2}{\pi z}\right)} \cos(z - \tfrac{1}{4}\pi - \tfrac{1}{2}n\pi), \quad -\pi < \operatorname{amp} z < \pi, \qquad \text{(viii)||}$$

$$(\lambda^2-\mu^2)\int_a^b x U_n(\lambda x) V_n(\mu x) dx = \Big[U_n(\lambda x)\mu x V_n'(\mu x) - V_n(\mu x)\lambda x U_n'(\lambda x)\Big]_a^b, \qquad \text{(ix)¶}$$

where U_n and V_n are Bessel Functions.

* *Proc. Lond. Math. Soc.*, **35** (1902), 428.

† G.M.M., p. 23. [The letters G.M.M. refer to *Bessel Functions* by Gray, Mathews, and MacRobert.] It may be noted that $2G_n(z) = i\pi H_n^{(1)}(z)$.

‡ G.M.M., p. 57. § G.M.M., pp. 57, xiv.

|| G.M.M., pp. 57, xiv. ¶ G.M.M., p. 69.

From the Lommel Integral (ix) with J_n and G_n in place of U_n and V_n, and by means of (vii) and (viii) it follows that, if λ is real and $I(\mu)>0$,

$$(\lambda^2-\mu^2)\int_0^\infty xJ_n(\lambda x)G_n(\mu x)dx=\left(\frac{\lambda}{\mu}\right)^n, \quad \quad \quad \text{(x)}$$

while, if $I(\mu)<0$,

$$(\lambda^2-\mu^2)\int_0^\infty xJ_n(\lambda x)G_n(\mu xe^{i\pi})dx=\left(\frac{\lambda}{\mu}\right)^n e^{-in\pi} \quad \quad \quad \text{(xi)}$$

Hence

$$\begin{aligned}
I &\equiv \int_0^\infty f(\lambda)J_n(\lambda r)\lambda d\lambda = \int_0^\infty J_n(\lambda r)\lambda d\lambda\int_p^q \phi(\rho)J_n(\lambda\rho)\rho d\rho \\
&= \int_0^\infty J_n(\lambda r)\lambda d\lambda\int_p^q \phi(\rho)\frac{1}{\pi i}\left\{G_n(\lambda\rho)-e^{in\pi}G_n(\lambda\rho e^{i\pi})\right\}\rho d\rho \\
&= \frac{1}{\pi i}\int_0^\infty J_n(\lambda r)\lambda d\lambda\int_p^q \phi(\rho)G_n(\lambda\rho)\rho d\rho \\
&\quad - \frac{e^{in\pi}}{\pi i}\int_0^\infty J_n(\lambda r)\lambda d\lambda\int_p^q \phi(\rho)G_n(\lambda\rho e^{i\pi})\rho d\rho \\
&= \frac{1}{\pi i}\int_{C_2}\phi(\zeta)\zeta d\zeta\int_0^\infty J_n(\lambda r)G_n(\lambda\zeta)\lambda d\lambda \\
&\quad - \frac{e^{in\pi}}{\pi i}\int_{C_1}\phi(\zeta)\zeta d\zeta\int_0^\infty J_n(\lambda r)G_n(\lambda\zeta e^{i\pi})\lambda d\lambda \\
&= \frac{1}{\pi i}\int_{C_2}\frac{\phi(\zeta)}{r^2-\zeta^2}\left(\frac{r}{\zeta}\right)^n\zeta d\zeta - \frac{1}{\pi i}\int_{C_1}\frac{\phi(\zeta)}{r^2-\zeta^2}\left(\frac{r}{\zeta}\right)^n\zeta d\zeta,
\end{aligned}$$

by (x) and (xi). Thus

$$\begin{aligned}
I &= \frac{1}{2\pi i}\int_C \phi(\zeta)\left(\frac{r}{\zeta}\right)^n\frac{2\zeta}{\zeta+r}\,\frac{d\zeta}{\zeta-r} \\
&= \begin{cases}\phi(r), & p<r<q, \\ 0, & 0<r<p \text{ or } r>q.\end{cases}
\end{aligned}$$

[NOTE.—Formulæ (x) and (xi) do not hold at the end points of C_1 and C_2. This difficulty can, however, be got over as follows. Let C_1' and C_2' be contours obtained from C_1 and C_2 by cutting off small parts at both ends. Then the integrals along C_1 and C_2 may be defined as the limits of the corresponding integrals along C_1' and C_2' when the parts cut off tend to zero.]

For example, if $n>-1$, $m-n>0$,

$$f(\lambda)\equiv\lambda^{n-m}J_m(\lambda)=\int_0^1\phi(\rho)J_n(\lambda\rho)\rho d\rho,$$

where

$$\phi(\rho)=\frac{\rho^n(1-\rho^2)^{m-n-1}}{2^{m-n-1}\Gamma(m-n)},$$

a formula which can easily be verified by expanding $J_n(\lambda\rho)$ in series and integrating term by term. It follows that

$$\int_0^\infty \frac{J_m(\lambda)J_n(\lambda r)}{\lambda^{m-n-1}}d\lambda = \begin{cases} \dfrac{r^n(1-r^2)^{m-n-1}}{2^{m-n-1}\Gamma(m-n)}, & 0<r<1, \\ 0, & r>1, \end{cases}$$

a result due to Sonine. In this case all the integrals converge, and the change of order of integration is valid.

§ 3. *A Second Fourier-Bessel Integral Theorem.*—The Fourier-Bessel Integral Theorem considered in the previous section corresponds to the ordinary Fourier-Bessel Expansion. There is a second Fourier-Bessel Expansion of the form

$$f(\lambda) = \sum_s A_s T_n(r_s, \lambda),$$

where

$$T_n(r, \lambda) = J_n(r\lambda)G_n(ra) - G_n(r\lambda)J_n(ra),$$

and r_s is a positive zero of $T_n(r, b)$. The series vanishes for $\lambda = a$ and $\lambda = b$, and is equal to $\frac{1}{2}\{f(\lambda+0)+f(\lambda-0)\}$ for $a<\lambda<b$.*

The corresponding integral theorem is as follows. If

$$f(\lambda) = \int_p^q \phi(\rho)T_n(\rho, \lambda)\rho d\rho, \; 0 \leqq p < q, \quad . \quad . \quad . \quad . \quad \text{(xii)}$$

then

$$\int_a^\infty f(\lambda)T_n(r, \lambda)\lambda d\lambda = \begin{cases} \frac{1}{2}B_n(ra)\{\phi(r+0)+\phi(r-0)\}, & p<r<q, \\ 0, & 0<r<p \text{ or } r>q, \end{cases} \quad . \quad . \quad . \quad . \quad \text{(xiii)}$$

where

$$B_n(\kappa) = \left\{\frac{\pi}{2\sin n\pi}\right\}^2 \{J_n^2(\kappa) - 2\cos n\pi J_n(\kappa)J_{-n}(\kappa) + J_{-n}^2(\kappa)\}.$$

As was done in the previous section, this result can be established by contour integration. It is just as easy, however, to employ Dirichlet Integrals, and the results are more general. It will be assumed that $\phi(\rho)$ satisfies the usual Dirichlet conditions. The formulæ

$$xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x), \quad . \quad . \quad . \quad . \quad \text{(xiv)}$$

$$xG_n'(x) = nG_n(x) - xG_{n+1}(x) \quad . \quad . \quad . \quad . \quad \text{(xv)}$$

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* *Proc. Roy. Soc. Edin.*, **42** (1922), 90.

are required. From these and (ix) it follows that

$$\begin{aligned}
\mathrm{I} \equiv (r^2-\rho^2)\int_a^h \mathrm{T}_n(r,\lambda)\mathrm{T}_n(\rho,\lambda)\lambda d\lambda \\
= rh\mathrm{T}_n(\rho,h)\{\mathrm{J}_{n+1}(rh)\mathrm{G}_n(ra)-\mathrm{G}_{n+1}(rh)\mathrm{J}_n(ra)\} \\
-\rho h\mathrm{T}_n(r,h)\{\mathrm{J}_{n+1}(\rho h)\mathrm{G}_n(\rho a)-\mathrm{G}_{n+1}(\rho h)\mathrm{J}_n(\rho a)\}.
\end{aligned}$$

It should be noted that $(r-\rho)$ is a factor of the right-hand side of this equation. Now replace all the Bessel Functions involving h on the right-hand side by their asymptotic expansions. Then (*cf.* (vi) and (vii))

$$\begin{aligned}
\mathrm{I} = \frac{\rho-r}{2\pi\sqrt{(r\rho)}}e^{in\pi}\left\{\begin{array}{l} e^{i(\rho+r)h}\mathrm{G}_n(rae^{i\pi})\mathrm{G}_n(\rho ae^{i\pi}) \\ +e^{-i(\rho+r)h}\mathrm{G}_n(ra)\mathrm{G}_n(\rho a)\end{array}\right\} \\
+\frac{\rho+r}{2\pi\sqrt{(r\rho)}}e^{in\pi}\left\{\begin{array}{l} ie^{i(\rho-r)h}\mathrm{G}_n(ra)\mathrm{G}_n(\rho ae^{i\pi}) \\ -ie^{-i(\rho-r)h}\mathrm{G}_n(rae^{i\pi})\mathrm{G}_n(\rho a)\end{array}\right\}+\frac{\mathrm{P}}{h},
\end{aligned}$$

where P is finite for all values of h, and contains $(\rho-r)$ as a factor.

Hence

$$\begin{aligned}
\mathop{\mathrm{L}}_{h\to\infty}\int_p^q \phi(\rho)\rho d\rho\int_a^h \mathrm{T}_n(r,\lambda)\mathrm{T}_n(\rho,\lambda)\lambda d\lambda \\
= \mathop{\mathrm{L}}_{h\to\infty}\left[\begin{array}{c}\displaystyle\int_p^q \frac{\phi(\rho)\rho}{2\pi\sqrt{(r\rho)}}\,\frac{\sin(\rho-r)h}{\rho-r}e^{in\pi}\left\{\begin{array}{l}\mathrm{G}_n(ra)\mathrm{G}_n(\rho ae^{i\pi}) \\ +\mathrm{G}_n(rae^{i\pi})\mathrm{G}_n(\rho a)\end{array}\right\}d\rho \\ +\text{ integrals which }\to 0\text{ when }h\to\infty\end{array}\right] \\
= \left\{\begin{array}{l}\tfrac{1}{2}\{\phi(r+0)+\phi(r-0)\}e^{in\pi}\mathrm{G}_n(ra)\mathrm{G}_n(rae^{i\pi}),\ p<r<q, \\ \qquad 0,\ 0<r<p\text{ or }r>q,\end{array}\right.
\end{aligned}$$

from which, on changing the order of integration, the theorem follows. By means of (v) it can easily be verified that

$$e^{in\pi}\mathrm{G}_n(ra)\mathrm{G}_n(rae^{i\pi}) = \mathrm{B}_n(ra).$$

§ 4. *A Third Fourier-Bessel Integral Theorem.*—The following Integral Theorem seems to have some association with Kapteyn Series of Bessel Functions. If

$$f(\lambda) = \int_p^q \phi(\rho)\mathrm{J}_\rho(\rho\lambda)\rho d\rho,\quad 0\leqq p<q\,. \qquad . \qquad . \qquad . \qquad (\text{xvi})$$

then

$$\int_0^\infty\left(\lambda-\frac{1}{\lambda}\right)f(\lambda)\mathrm{J}_m(m\lambda)d\lambda = \left\{\begin{array}{ll}\tfrac{1}{2}\{\phi(m+0)+\phi(m-0)\}, & p<m<q, \\ \qquad 0, & 0<m<p\text{ or }m>q.\end{array}\right. \qquad . \quad (\text{xvii})$$

The theorem can be proved either by contour integration or by means of Dirichlet Integrals. The latter method is here given. The Lommel Integral

$$(m^2-\rho^2)\int_a^b\left(\lambda-\frac{1}{\lambda}\right)U_m(m\lambda)V_\rho(\rho\lambda)d\lambda$$
$$=\left[\lambda\left\{U_m(m\lambda)\frac{\partial}{\partial\lambda}V_\rho(\rho\lambda)-V_\rho(\rho\lambda)\frac{\partial}{\partial\lambda}U_m(m\lambda)\right]_a^b \quad \text{(xviii)*}$$

is required.

If $m>0$, $\rho\geqq 0$,

$$I\equiv\int_0^h\left(\lambda-\frac{1}{\lambda}\right)J_m(m\lambda)J_\rho(\rho\lambda)d\lambda$$
$$=\frac{h}{\rho^2-m^2}\{\rho J_m(mh)J_{\rho+1}(\rho h)-mJ_\rho(\rho h)J_{m+1}(mh)\}$$
$$-\frac{1}{\rho+m}J_m(mh)J_\rho(\rho h)$$

by (xiv). When the Bessel Functions on the right-hand side of this equation are replaced by their asymptotic expansions, it is found that

$$I=\frac{2}{\pi(\rho^2-m^2)\sqrt{(m\rho)}}\left\{\begin{array}{l}\rho\cos(mh-\frac14\pi-\frac12 m\pi)\cos(\rho h-\frac14\pi-\frac12(\rho+1)\pi)\\ -m\cos(\rho h-\frac14\pi-\frac12\rho\pi)\cos(mh-\frac14\pi-\frac12(m+1)\pi)\end{array}\right\}+\frac{P}{h},$$

where P remains finite for all values of h. Hence

$$I=\frac{1}{\pi(\rho^2-m^2)\sqrt{(m\rho)}}\left[\begin{array}{l}(\rho+m)\sin\{(\rho-m)(h-\frac12\pi)\}\\ -(\rho-m)\cos\{(\rho+m)(h-\frac12\pi)\}\end{array}\right]+\frac{P}{h},$$

and therefore

$$\underset{h\to\infty}{L}\int_p^q\phi(\rho)\rho d\rho\int_0^h\left(\lambda-\frac{1}{\lambda}\right)J_m(m\lambda)J_\rho(\rho\lambda)d\lambda$$
$$=\left\{\begin{array}{r}\frac12\{\phi(m+0)+\phi(m-0)\},\ p<m<q\\ 0,\ 0<m<p \text{ or } m>q)\end{array}\right\}$$

from which, on changing the order of integration, the theorem follows.

By assuming the converse of this theorem various new integrals can be evaluated. For instance, from the integral †

$$\int_0^\infty e^{-a\lambda}J_m(m\lambda)d\lambda=\frac{1}{\sqrt{(a^2+m^2)}}\left\{\frac{\sqrt{(a^2+m^2)}-a}{m}\right\}^m,$$

where $a>0$, $m\geqq 0$, it follows that

$$\int_0^\infty\frac{1}{\sqrt{(a^2+\rho^2)}}\left\{\frac{\sqrt{(a^2+\rho^2)}-a}{\rho}\right\}^\rho J_\rho(\rho\lambda)\rho d\rho=\frac{\lambda}{\lambda^2-1}e^{-a\lambda},\quad \lambda\geqq 0.$$

* G.M.M., p. 69.

† G.M.M., p. 76, ex. 14, p. 256 (35).

In particular, when $a \to 0$, these formulæ reduce to

$$\int_0^\infty J_m(m\lambda)d\lambda = \frac{1}{m}, \quad m > 0,$$

and

$$\int_0^\infty J_\rho(\rho\lambda)d\rho = \frac{\lambda}{\lambda^2 - 1}, \quad \lambda \geqq 0.$$

§ 5. *A Fourier-Legendre Integral Theorem.*—The theorem to be established is as follows. If

$$F(\lambda) = \int_p^q f(\phi)P_{\lambda-\frac{1}{2}}(\cos\phi)\sqrt{(\sin\phi)}d\phi, \quad 0 \leqq p < q \leqq \pi, \quad . \quad . \quad \text{(xix)}$$

then

$$2\int_0^\infty F(\lambda)\lambda\sin(\pi\lambda)P_{\lambda-\frac{1}{2}}(-\cos\theta)\sqrt{(\sin\theta)}d\lambda$$
$$= \begin{cases} \frac{1}{2}\{f(\theta+0)+f(\theta-0)\}, & p < \theta < q \\ 0, & 0 < \theta < p \text{ or } q < \theta < \pi \end{cases} . \quad \text{(xx)}$$

The following formulæ are required. Let

$$P(\lambda, \theta) \equiv F\left(\tfrac{1}{2}, \tfrac{1}{2}; 1+\lambda; \frac{e^{i\theta}}{2i\sin\theta}\right); \quad . \quad . \quad . \quad \text{(xxi)}$$

then

$$P_{\lambda-\frac{1}{2}}(\cos\theta) = \frac{1}{\sqrt{(2\pi\sin\theta)}}\frac{\Gamma(\lambda+\frac{1}{2})}{\Gamma(\lambda+1)}\{e^{\frac{1}{4}i\pi - i\lambda\theta}P(\lambda, -\theta) + e^{-\frac{1}{4}i\pi + i\lambda\theta}P(\lambda, \theta)\}, \quad \text{(xxii)*}$$

where $0 < \theta < \pi$. In this formula replace θ by $\pi - \theta$: then

$$P_{\lambda-\frac{1}{2}}(-\cos\theta) = \frac{1}{\sqrt{(2\pi\sin\theta)}}\frac{\Gamma(\lambda+\frac{1}{2})}{\Gamma(\lambda+1)}\{e^{\frac{1}{4}i\pi - i\lambda(\pi-\theta)}P(\lambda, \theta) + e^{-\frac{1}{4}i\pi + i\lambda(\pi-\theta)}P(\lambda, -\theta)\}. \quad \text{(xxiii)}$$

From these two formulæ it follows that

$$\frac{1}{\sqrt{(2\pi\sin\theta)}}\frac{\Gamma(\lambda+\frac{1}{2})}{\Gamma(\lambda+1)}2\cos(\pi\lambda)e^{i\lambda\theta}P(\lambda, \theta)$$
$$= e^{\frac{1}{4}i\pi}\{e^{i\pi\lambda}P_{\lambda-\frac{1}{2}}(\cos\theta) - iP_{\lambda-\frac{1}{2}}(-\cos\theta)\} \quad . \quad \text{(xxiv)}$$

and

$$\frac{1}{\sqrt{(2\pi\sin\theta)}}\frac{\Gamma(\lambda+\frac{1}{2})}{\Gamma(\lambda+1)}2\cos(\pi\lambda)e^{-i\lambda\theta}P(\lambda, -\theta)$$
$$= e^{-\frac{1}{4}i\pi}\{e^{-i\pi\lambda}P_{\lambda-\frac{1}{2}}(\cos\theta) + iP_{\lambda-\frac{1}{2}}(-\cos\theta)\}. \quad . \quad \text{(xxv)}$$

Again, two well-known formulæ † for the hypergeometric function give

$$F(\tfrac{1}{2}, \tfrac{1}{2}; 1+\lambda; z) = \frac{1}{\lambda}\left\{\frac{\Gamma(\lambda+1)}{\Gamma(\lambda+\frac{1}{2})}\right\}^2 F(\tfrac{1}{2}, \tfrac{1}{2}; 1-\lambda; 1-z)$$
$$- \frac{(1-z)^\lambda}{\sin(\pi\lambda)}F(\tfrac{1}{2}+\lambda, \tfrac{1}{2}+\lambda; 1+\lambda; 1-z)$$

* *Proc. Edin. Math. Soc.*, **41** (1923), 88.

† *Cf.* the author's *Functions of a Complex Variable*, pp. 248, 249; also *Proc. Edin. Math. Soc.*, **37** (1919), 81, 84.

and

$$F(\tfrac{1}{2}, \tfrac{1}{2};\ 1+\lambda;\ 1-z) = z^{\lambda} F(\tfrac{1}{2}+\lambda, \tfrac{1}{2}+\lambda;\ 1+\lambda;\ 1-z).$$

From these, when $z = e^{i\theta}/(2i\sin\theta)$, so that

$$1 - z = -\frac{e^{-i\theta}}{2i\sin\theta},$$

it is easily deduced that

$$P(\lambda, \theta) = \frac{1}{\lambda}\left\{\frac{\Gamma(\lambda+1)}{\Gamma(\lambda+\frac{1}{2})}\right\}^2 P(-\lambda, -\theta) - \frac{e^{i\lambda(\pi-2\theta)}}{\sin(\pi\lambda)} P(\lambda, -\theta), \quad . \quad . \quad \text{(xxvi)}$$

where the factor $e^{i\lambda\pi}$ is determined by putting $\theta = \frac{1}{2}\pi$, so that $z = \frac{1}{2}$, $1 - z = \frac{1}{2}$. It follows that

$$\begin{aligned}
&\frac{\Gamma(\lambda+1)}{\Gamma(\lambda+\frac{1}{2})} 2\sin(\pi\lambda) e^{i\lambda\theta} P(-\lambda, -\theta) \\
&\qquad = \frac{\Gamma(\lambda+\frac{1}{2})}{\Gamma(\lambda+1)} \lambda 2\sin(\pi\lambda) e^{i\lambda\theta} P(\lambda, \theta) + \frac{\Gamma(\lambda+\frac{1}{2})}{\Gamma(\lambda+1)} \lambda 2 e^{i\lambda(\pi-\theta)} P(\lambda, -\theta) \\
&\qquad = \frac{\sqrt{(2\pi\sin\theta)}}{2\cos(\pi\lambda)} \lambda \left[\begin{aligned} &2\sin(\pi\lambda)\{e^{i\pi(\lambda+\frac{1}{2})} P_{\lambda-\frac{1}{2}}(\cos\theta) - e^{\frac{1}{2}i\pi} P_{\lambda-\frac{1}{2}}(-\cos\theta)\} \\ &+ 2\{e^{-\frac{1}{2}i\pi} P_{\lambda-\frac{1}{2}}(\cos\theta) + e^{i\pi(\lambda+\frac{1}{2})} P_{\lambda-\frac{1}{2}}(-\cos\theta)\}\end{aligned}\right]
\end{aligned}$$

by (xxiv) and (xxv). Hence

$$\begin{aligned}
&\frac{\Gamma(\lambda+1)}{\Gamma(\lambda+\frac{1}{2})} 2\sin(\pi\lambda) e^{i\lambda\theta} P(-\lambda, -\theta) \\
&\qquad = \sqrt{(2\pi\sin\theta)}\, e^{-\frac{1}{2}i\pi} \lambda \{e^{i\pi\lambda} P_{\lambda-\frac{1}{2}}(\cos\theta) + i P_{\lambda-\frac{1}{2}}(-\cos\theta)\}. \quad \text{(xxvii)}
\end{aligned}$$

The theorem can be proved by contour integration in the following manner:—

$$\begin{aligned}
I &\equiv \int_0^\infty 2i\sin(\pi\lambda)\,d\lambda \int_p^q f(\phi) \left\{\begin{aligned} &e^{i\lambda(\theta-\phi-\pi)} P(-\lambda, -\theta) P(\lambda, -\phi) \\ &- e^{-i\lambda(\theta-\phi-\pi)} P(\lambda, -\theta) P(-\lambda, -\phi)\end{aligned}\right\} d\phi \\
&= \int_0^\infty d\lambda \int_p^q f(\phi) \left[\begin{aligned} &\{e^{i\lambda(\theta-\phi)} - e^{i\lambda(\theta-\phi-2\pi)}\} P(-\lambda, -\theta) P(\lambda, -\phi) \\ &+ \{e^{-i\lambda(\theta-\phi)} - e^{-i\lambda(\theta-\phi-2\pi)}\} P(\lambda, -\theta) P(-\lambda, -\phi)\end{aligned}\right] d\phi \\
&= \int_0^\infty d\lambda \int_{C_1} f(\zeta) \{e^{-i\lambda(\zeta-\theta)} - e^{-i\lambda(\zeta-\theta+2\pi)}\} P(-\lambda, -\theta) P(\lambda, -\zeta)\, d\zeta \\
&+ \int_0^\infty d\lambda \int_{C_2} f(\zeta) \{e^{i\lambda(\zeta-\theta)} - e^{i\lambda(\zeta-\theta+2\pi)}\} P(\lambda, -\theta) P(-\lambda, -\zeta)\, d\zeta \\
&= \int_{C_1} f(\zeta)\,d\zeta \int_0^\infty \{e^{-i\lambda(\zeta-\theta)} - e^{-i\lambda(\zeta-\theta+2\pi)}\} P(-\lambda, -\theta) P(\lambda, -\zeta)\, d\lambda \\
&+ \int_{C_2} f(\zeta)\,d\zeta \int_0^\infty \{e^{i\lambda(\zeta-\theta)} - e^{i\lambda(\zeta-\theta+2\pi)}\} P(\lambda, -\theta) P(-\lambda, -\zeta)\, d\lambda \\
&= \int_C \frac{f(\zeta)\,d\zeta}{i(\zeta-\theta)} \int_0^\infty e^{-\lambda}\, P\left(\frac{-\lambda}{i(\zeta-\theta)}, -\theta\right) P\left(\frac{\lambda}{i(\zeta-\theta)}, -\zeta\right) d\lambda \\
&- \int_C \frac{f(\zeta)\,d\zeta}{i(\zeta-\theta+2\pi)} \int_0^\infty e^{-\lambda}\, P\left(\frac{-\lambda}{i(\zeta-\theta+2\pi)}, -\theta\right) P\left(\frac{\lambda}{i(\zeta-\theta+2\pi)}, -\zeta\right) d\lambda \\
&= \begin{cases} 2\pi f(\theta) \displaystyle\int_0^\infty e^{-\lambda}\,d\lambda = 2\pi f(\theta), & p < \theta < q \\ \qquad\qquad\qquad\qquad 0, & 0 < \theta < p \text{ or } q < \theta < \pi. \end{cases}
\end{aligned}$$

Now, from (xxv) and (xxvii),

$$I = \int_0^\infty \frac{\lambda d\lambda}{2\cos(\pi\lambda)}\int_p^q f(\phi)\sqrt{(2\pi\sin\theta)}\sqrt{(2\pi\sin\phi)}d\phi$$

$$\times\begin{bmatrix} e^{-i\pi\lambda}\{e^{i\pi\lambda}P_{\lambda-\frac{1}{2}}(\cos\theta)+iP_{\lambda-\frac{1}{2}}(-\cos\theta)\}\{e^{-i\pi\lambda}P_{\lambda-\frac{1}{2}}(\cos\phi)+iP_{\lambda-\frac{1}{2}}(-\cos\phi)\} \\ -e^{i\pi\lambda}\{e^{-i\pi\lambda}P_{\lambda-\frac{1}{2}}(\cos\theta)+iP_{\lambda-\frac{1}{2}}(-\cos\theta)\}\{e^{i\pi\lambda}P_{\lambda-\frac{1}{2}}(\cos\phi)+iP_{\lambda-\frac{1}{2}}(-\cos\phi)\}\end{bmatrix}$$

$$=2\pi\int_0^\infty \frac{\lambda d\lambda}{2\cos(\pi\lambda)}\int_p^q f(\phi)\sqrt{(\sin\theta\sin\phi)}d\phi$$

$$\times\begin{bmatrix} -2i\sin(\pi\lambda)\{P_{\lambda-\frac{1}{2}}(\cos\theta)P_{\lambda-\frac{1}{2}}(\cos\phi)-P_{\lambda-\frac{1}{2}}(-\cos\theta)P_{\lambda-\frac{1}{2}}(-\cos\phi)\} \\ +2\sin(2\pi\lambda)P_{\lambda-\frac{1}{2}}(-\cos\theta)P_{\lambda-\frac{1}{2}}(\cos\phi)\end{bmatrix}.$$

Now equate real and imaginary parts, and get

$$\int_0^\infty 2\lambda\sin(\pi\lambda)P_{\lambda-\frac{1}{2}}(-\cos\theta)\sqrt{(\sin\theta)}d\lambda\int_p^q f(\phi)P_{\lambda-\frac{1}{2}}(\cos\phi)\sqrt{(\sin\phi)}d\phi$$

$$=\begin{cases} f(\theta),\ p<\theta<q, \\ 0,\ 0<\theta<p \text{ or } q<\theta<\pi,\end{cases}$$

from which the theorem follows, and also

$$\int_0^\infty \lambda\tan(\pi\lambda)d\lambda\int_p^q f(\phi)\sqrt{(\sin\theta\sin\phi)}d\phi$$

$$\times\{P_{\lambda-\frac{1}{2}}(\cos\theta)P_{\lambda-\frac{1}{2}}(\cos\phi)-P_{\lambda-\frac{1}{2}}(-\cos\theta)P_{\lambda-\frac{1}{2}}(-\cos\phi)\}=0.$$

It is also possible to give a proof depending on the use of Dirichlet Integrals; the following is an outline of the method. Consider the integrals

$$I_1=\int_0^h e^{i\mu\zeta}P(-\zeta,-\theta)P(\zeta,-\phi)d\zeta,$$

$$I_2=\int_0^h e^{-i\mu\zeta}P(\zeta,-\theta)P(-\zeta,-\phi)d\zeta.$$

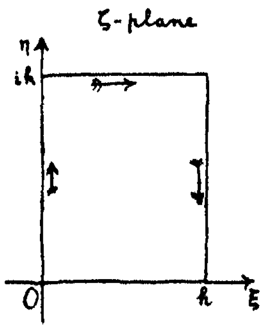

FIG. II, a.

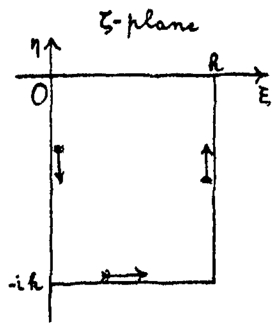

FIG. II, b.

If $\mu>0$, replace the contours in I_1 and I_2 by the contours in figs. II, a and II, b, indicated by the arrow-heads. Then

$$\begin{aligned}
\mathrm{I}_1 &= \int_0^k e^{-\mu\eta}\mathrm{P}(-i\eta, -\theta)\mathrm{P}(i\eta, -\phi)id\eta \\
&+ \int_0^h e^{-\mu k+i\mu\xi}\mathrm{P}(-ik-\xi, -\theta)\mathrm{P}(ik+\xi, -\phi)d\xi \\
&- \int_0^k e^{i\mu h-\mu\eta}\mathrm{P}(-h-i\eta, -\theta)\mathrm{P}(h+i\eta, -\phi)id\eta, \\
\mathrm{I}_2 &= -\int_0^k e^{-\mu\eta}\mathrm{P}(-i\eta, -\theta)\mathrm{P}(i\eta, -\phi)id\eta \\
&+ \int_0^h e^{-\mu k-i\mu\xi}\mathrm{P}(-ik+\xi, -\theta)\mathrm{P}(ik-\xi, -\phi)d\xi \\
&+ \int_0^k e^{-i\mu h-\mu\eta}\mathrm{P}(h-i\eta, -\theta)\mathrm{P}(-h+i\eta, -\phi)id\eta.
\end{aligned}$$

On adding I_1 and I_2 it is found that the integrals along the η-axis cancel: the others, apart from the first terms in their expansions, give integrals which $\to 0$ when h and $k \to \infty$. Hence

$$\begin{aligned}
\mathrm{J} = \mathrm{I}_1 + \mathrm{I}_2 &= \int_0^h (e^{i\mu\zeta} + e^{-i\mu\zeta})d\zeta + \epsilon \\
&= \frac{2\sin\mu h}{\mu} + \epsilon,
\end{aligned}$$

where $\epsilon \to 0$ when h and $k \to \infty$.

If $\mu < 0$, the same result can be obtained by interchanging the contours of figures II, a and II, b. Hence

$$\begin{aligned}
\mathrm{I} &= \underset{h\to\infty}{\mathrm{L}} \int_p^q f(\phi)\frac{2\sin(\theta-\phi)h}{\theta-\phi}d\phi \\
&- \underset{h\to\infty}{\mathrm{L}} \int_p^q f(\phi)\frac{2\sin(\theta-\phi-2\pi)h}{\theta-\phi-2\pi}d\phi \\
&= \begin{cases} \pi\{f(\theta+0)+f(\theta-0)\}, & p<\theta<q, \\ 0, & 0<\theta<p \text{ or } q<\theta<\pi. \end{cases}
\end{aligned}$$

(*Issued separately July* 8, 1931.)

XVII.—The Ionizing Efficiency of Electronic Impacts in Air.

By **John Thomson**, M.A., B.Sc., Ph.D., Lecturer in Natural Philosophy in the University of Glasgow.* (With Six Figures in the text.) *Communicated by* Professor E. TAYLOR JONES.

(MS. received March 12, 1931. Read June 15, 1931.)

INTRODUCTORY.

THE purpose of the present communication is to describe experiments, the object of which is the determination of the total ionization produced in air by an electron moving initially with a specified energy. Various estimates of this quantity have already been made † using widely varying electron velocities and very different types of apparatus, but the agreement between the results obtained is not such as to obviate the necessity for further research. For a given initial electron energy the number of ions produced per electron as determined by Eisl is almost one and a half times the number as determined by Lehmann and Osgood. It was thought, therefore, that a careful investigation was desirable, since it should be possible to determine the quantity concerned with much greater accuracy than is indicated by these results.

The investigation has a fundamental significance. In spite of the large number of researches directed in recent years towards an examination of the properties of the moving electron, there is still no certainty as to the exact mechanism of ionization by impact. The *minimum* electronic energy necessary for ionization is well known, but the *average* energy transfer at ionization, when an electron is completely absorbed in a gas, has not received adequate experimental treatment. The present investigation appears to indicate that the energy spent by high-speed electrons in producing one pair of ions in air, is 37 ± 2 electron volts. This value is higher than the mean of previous measurements.

EXPERIMENTAL ARRANGEMENTS.

1. *General Theory of the Method.*—It was decided to make the apparatus as simple as possible and to attack the problem in the most direct

* The work described was performed while the writer was a member of the staff of the University of Reading.

† Johnson, *Phys. Rev.*, **10**, p. 609 (1917); Anslow, *Phys. Rev.*, p. 484 (1925); Lehmann and Osgood, *Proc. Roy. Soc.*, A, **115**, p. 609 (1927); Eisl, *Ann. d. Physik*, **3**, 3, p. 277 (1929).

manner. An approximately homogeneous beam of electrons was shot into an ionization chamber. The electron current I, the average energy of the beam V (in electron-volts), and the ionization current C were measured. Hence, the energy V_0 (electron volts) spent in producing one pair of ions was found by means of the formula $V_0 = \frac{I}{C} V$.

In order that this equation should hold, it was necessary to ensure

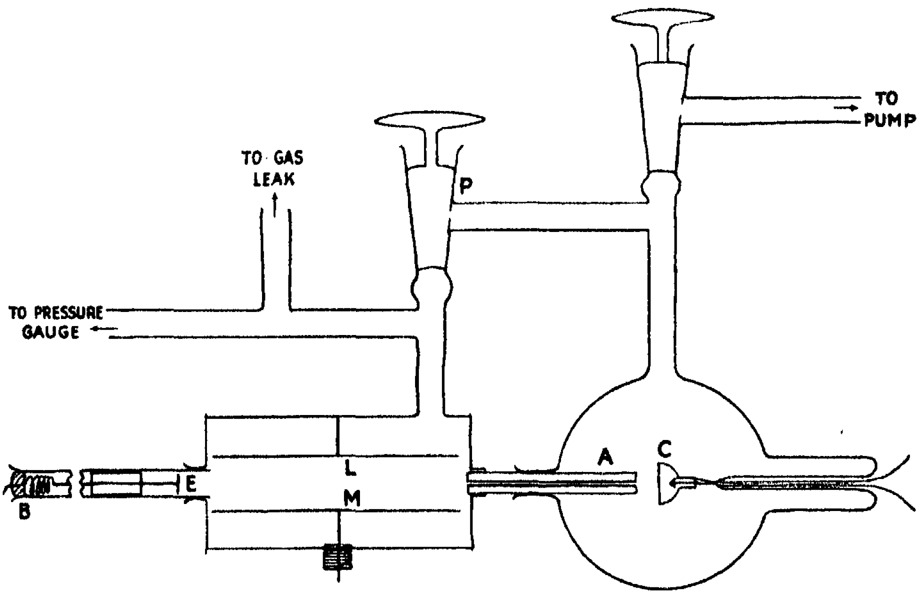

Fig. 1.

(*a*) that the electrons were totally absorbed in the gas, and (*b*) that no recombination of ions produced could take place.

2. *The Apparatus.*—Fig. 1 is a sketch of the apparatus employed. The spherical glass chamber on the right contained the elements for the production of the electron beam. C was a tungsten spiral with a hemispherical focussing cap, forming the cathode. The emitted electrons were accelerated towards the cylindrical metal anode A, which was 14 cm. long and pierced centrally by a 2-mm. diameter capillary tube. Part of the electron beam moved down the capillary tube to the metal ionization chamber on the left of the figure. This was 20 cm. long and about 10 cm. in diameter, and contained the two parallel plate electrodes L and M, arranged to give a very uniform field near the centre of the chamber. L was connected to the ionization chamber, while M was

insulated by an ebonite plug. At the end of the chamber away from the capillary tube and exactly opposite the latter, there was inserted a quartz tube about 2 cm. in diameter and 60 cm. long. This contained an aluminium rod at the end of which was a small circular electrode E. The rod was surrounded at one point by a small iron cylinder and was connected by a fine wire to the outer atmosphere at B. By means of the iron cylinder and a horse-shoe magnet it was possible to move the electrode E back and forward inside the chamber. It could be brought to within 1 mm. of the end of the capillary tube.

3. *The Vacuum System.*—The vacuum system was designed to maintain as low a pressure as possible in the chamber where the electrons were being produced, while the ionization chamber was at a very much higher pressure. The metal capillary tube formed the high air resistance, all the other connecting tubes being as wide as possible. Fig. 1 is self-explanatory. The two chambers were connected through the vacuum stop-cock P. The production chamber was connected as directly as possible to a two-stage mercury diffusion pump, while on the other side, near the ionization chamber, a gas leak and pressure gauge were incorporated. The manometer was a shortened McLeod gauge, capable of detecting a pressure of 0·0001 mm. The leak consisted of a tube of calcium chloride joined to a long, fine capillary tube which led into a large air reservoir. This reservoir could be pumped out to any desired pressure by means of an auxiliary pump. It contained phosphorus pentoxide and was always connected through a vacuum stop-cock to a tube of soda lime and to the atmosphere. Thus the amount of air leaking into the apparatus could be easily controlled with accuracy, and the air reaching the ionization chamber was reasonably free from water-vapour and carbon dioxide. When the apparatus was in use, a gas pressure of 0·1 mm. in the ionization chamber did not raise the pressure in the production chamber to 0·001 mm., the tap P being closed and the pumps working. Practically the whole pressure change took place in the capillary tube.

4. *Electrical Connections.*—The accelerating potential which energized the electrons and was applied between C and A, was varied from 200 to 400 volts. In the actual experiments, the cathode was maintained at −200 volts, but for the sake of clearness we shall suppose the cathode to be permanently earthed, and the potentials of all the conductors increased by 200 volts. The anode A was in electrical contact with the walls of the ionization chamber and the electrode L, and consequently the potential of this anode system varied between +200 and +400 volts according to experimental conditions. The insulated electrode was used

to detect the ionization current and also the electron current. Its potential could be varied between +500 volts and earth potential, and the same was true of the insulated electrode E. The heating current for the cathode C was supplied by four large accumulator cells, and a suitable series rheostat capable of delicate adjustment was included in the circuit. The electron current betweeen A and C was of the order of 100 micro-amperes and was accurately indicated on a large-scale unipivot instrument. The electron current in the ionization chamber (about 1/100th of the total electron current) and the ionization current were measured by a sensitive galvanometer. This instrument could be changed over easily from measuring the current at M to measuring the current at E. The usual precautions were taken to avoid accidental leakage currents and inductive effects.

Preliminary Experiments.

Owing to the nature of the vacuum apparatus, wax joints were necessary and baking-out was impossible. Yet, in order that tests should be carried out, it was necessary to obtain a very perfect vacuum (better than 0·0001 mm.) throughout the whole system. The outgassing of the cathode and anode surface required some days of overrunning, but eventually a stage was reached where the gas pressure could not be detected on the gauge when the apparatus was in action. Then the electron beam was produced, the accelerating potential being 200 volts, and readings were taken of the current at M with various potential differences across the ionization chamber. The curve in fig. 2 indicates the mode of variation of the current with the potential at M. The rest of the ionization chamber was at +200 volts. The almost complete absence of kinks in the curve points to the absence of gas in the apparatus, while the small positive current when the potential of M was negative with respect to the remainder of the ionization chamber is an indication of secondary electrons. For comparison with fig. 2, the corresponding curve before outgassing is shown in fig. 3. Here a number of kinks due to gas are in evidence.

A second test experiment consisted in examining the velocity of the electrons emerging from the capillary anode when the whole apparatus was evacuated. For this purpose the electrode E was placed directly in front of the end of the capillary and about 2 mm. away from it. The accelerating voltage between A and C was 400, and the current of electrons to E was measured for different values of the potential difference between E and the anode. The results are shown in fig. 4,

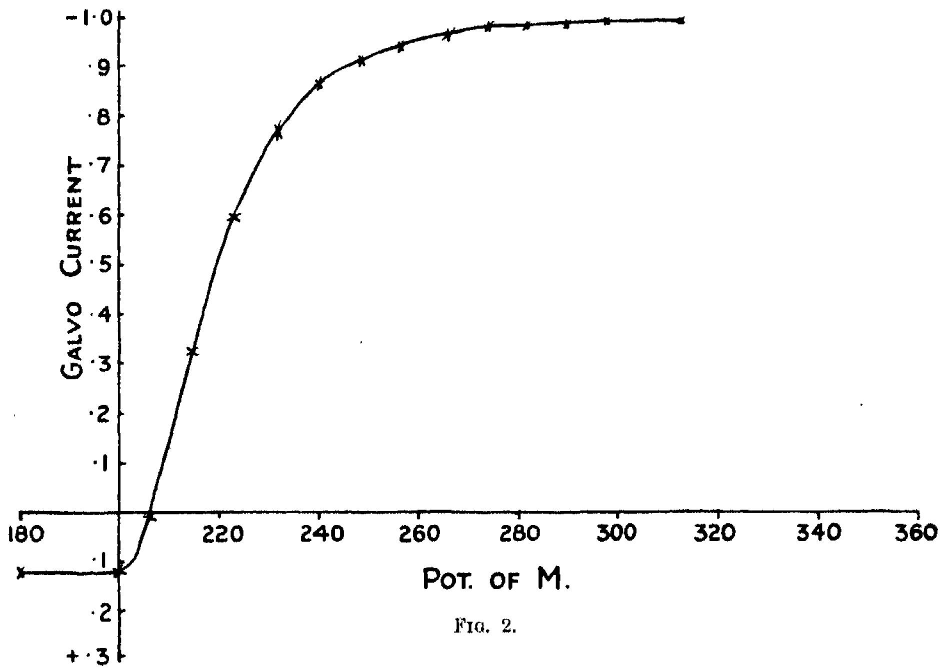
-1·0
·9
·8
·7
·6
·5
·4
·3
·2
·1
·1
·2
+·3
GALVO CURRENT
180
220
240
260
280
300
320
340
360
POT. OF M.

FIG. 2.

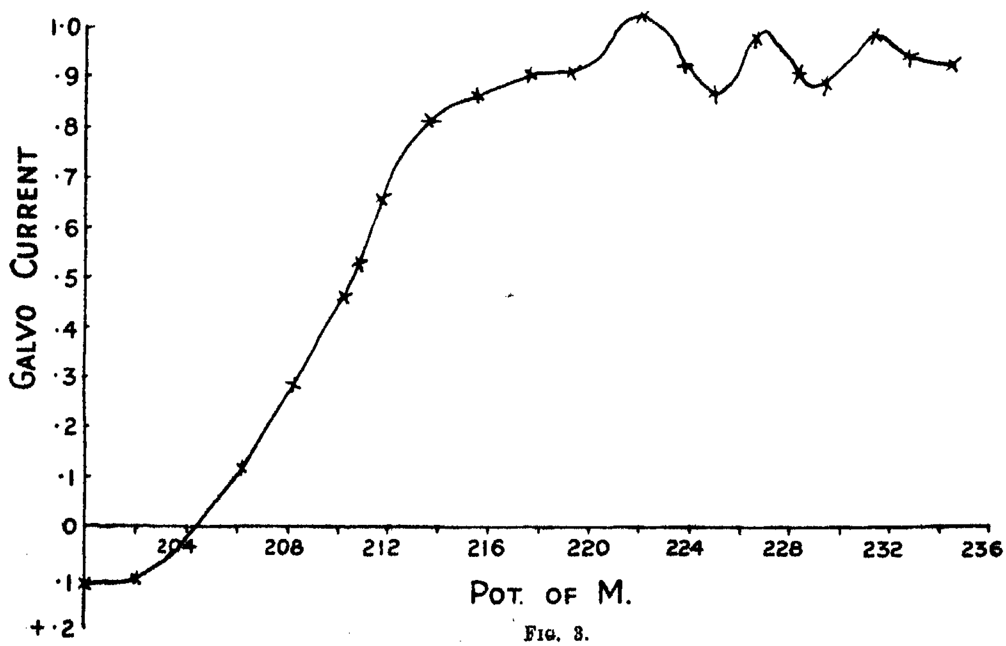
1·0
·9
·8
·7
·6
·5
·4
·3
·2
·1
0
·1
+·2
GALVO CURRENT
204
208
212
216
220
224
228
232
236
POT. OF M.

FIG. 3.

where the abscissa measures the potential of E in volts (V_E), the cathode as usual being assumed to be at earth potential, and the ordinate measures the current *to* E, in arbitrary units.

From the graph it may be seen that all the electrons emerging from the anode are collected by E when V_E is greater than 400 volts (the fixed potential of the anode, V_A). When, however, V_E is less than V_A,

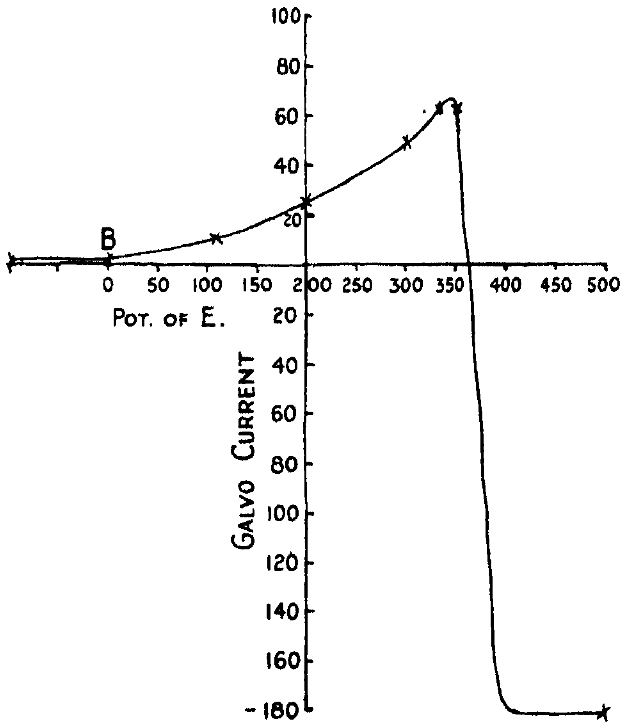

Fig. 4.

some of the secondary electrons emitted by E are collected by A, thus causing a decrease in the electron current to E. As V_E is reduced, the current actually becomes positive, showing that the loss of secondaries from E is greater than the current of absorbed primaries. Gradually, however, as the energy of the primary beam on impact with E becomes smaller (due to the increase in the retarding field between A and E) the number of secondaries per primary decreases, and, in consequence, the positive current to E also decreases.

Theoretically the curve in fig. 4 should cross the potential axis again when V_E is between 0 and 10 volts, since no secondaries are emitted

when the electron energy is small. The experiment failed to exhibit this phenomenon, however, the positive current to A persisting, even when V_E was negative This was almost certainly due to the photoelectric action of the radiation from the incandescent cathode.

In this experiment the apparatus was evacuated to a gas pressure of less than 0·0001 mm. It was assumed that the electrons emerging from A had energy corresponding to 400 volts, and the graph of fig. 4 was assumed to be typical of this energy. It was therefore used for comparison with other similar graphs obtained at higher pressures, when the energy of the electrons emerging from the capillary was not known.

Determination of the Energy spent in producing One Pair of Ions.

1. *The Initial Electronic Energy (V)*.—The first step in determining the ionizing efficiency of the electrons was to find the energy of the beam as it entered the ionization chamber at a given gas pressure. In order that the electrons should be totally absorbed in the gas in the chamber, a relatively high gas pressure was necessary, and under these circumstances a considerable fraction of the energy of the beam was absorbed in the capillary connecting tube. To measure this fraction, electrode E was brought into action, and curves similar to that in fig. 4 were obtained for a large number of gas pressures. These were then compared with fig. 4, and the voltage corresponding to B, where the electron current from E became very small and constant, was estimated as exactly as possible. Let this voltage be V_B. Then, if the accelerating potential was 200 volts, V was $200-V_B$. If the accelerating potential was 400 volts, V was $400-V_B$. A continuous graph of V against the pressure (p) was not obtained, as the procedure was too elaborate, and it was difficult to maintain the pressure in the ionization chamber at a constant value when it was less than 0·01 mm. Figs. 5A and 5B, however, exhibit the variation of the potential V with the pressure within certain limits, the accelerating potential being 200 volts in (a) and 400 volts in (b). It was found that the pressure limits indicated were determined by experimental conditions. Higher pressures were impossible, since no electrons then penetrated to the ionization chamber, and lower pressures introduced considerable uncertainty as to whether all the energy of the electrons would be absorbed in the ionization chamber. From the readings which were obtained mean values of V at four pressures were

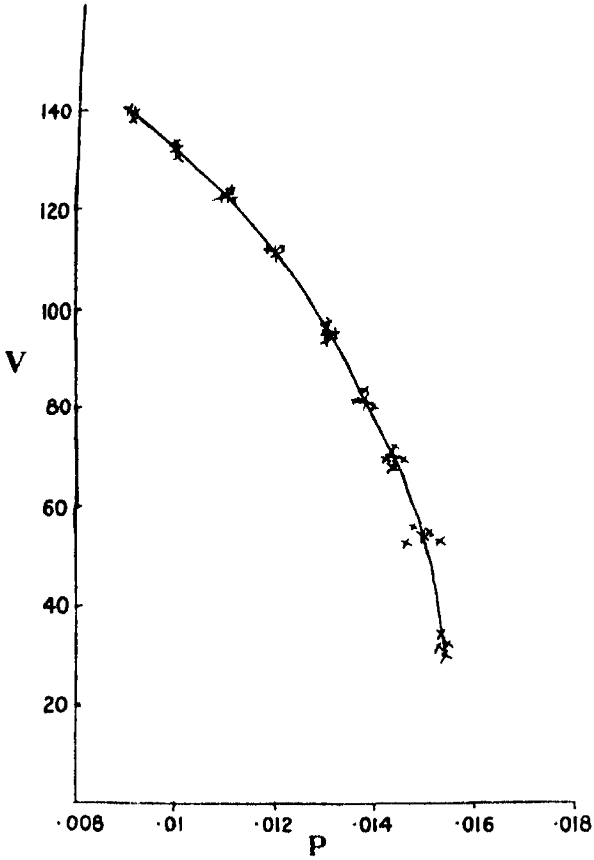

FIG. 5A.

estimated, and these four pressures were used in later observations. Table I shows results.

TABLE I.

Pressure. (mm.)	Accelerating Potential. (volts.)	V. (volts.)
0·050	400	154
0·045	400	265
0·010	200	132
0·015	200	54

2. *The Total Ionization produced.*—Having found the initial energy of the electrons in the four cases, it was now possible to determine the

optimum electric fields to be applied across LM in measuring the ionization produced. In order that no recombination of ions should take place, it was necessary to keep the potential difference larger than a certain minimum. But 54-volt electrons are easily deflected, and it

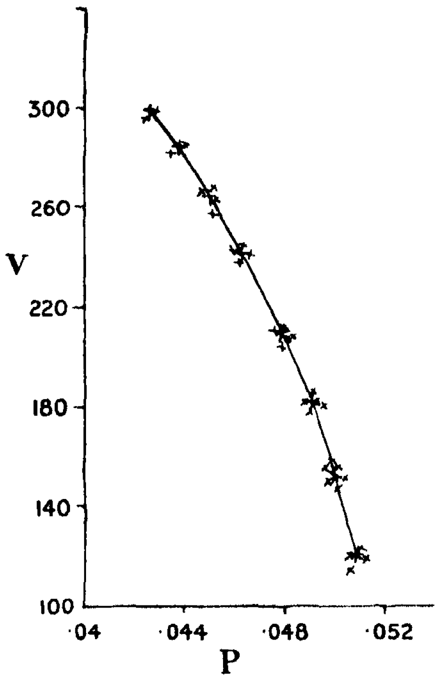

FIG. 5B.

was necessary also to ensure that the beam did not impinge on one electrode before losing its energy.

The problem resolved itself into finding the minimum potential necessary to saturate the ionization current at pressures greater than 0·010 mm. and less than 0·050 mm. This was done in the usual manner. With an accelerating potential of 200 volts, and a pressure of 0·015 mm., the potential of electrode M was varied from +200 to +150 volts, and the variation of the ionization current was observed. It was found that saturation took place with about 8 volts across LM, *i.e.* with electrode M at +192 volts. Since the potential required to produce saturation varies directly as the gas pressure, it was assumed that a potential difference of 10 volts would be sufficient for 200-volt electrons at a pressure of 0·01 mm. also. With an accelerating potential

of 400 volts the same experiment was tried, the pressure being 0·05 mm. A higher saturation potential was found—about 16 volts—and it was decided to use 20 volts in the ensuing experiments under these conditions. Thus 10–20 volts was decided upon as a satisfactory saturation potential difference for 50–265 volt electrons.

The pressure was then adjusted to 0·01 mm. by means of the gas reservoir, and a steady electron current of 40 microamperes was produced, the accelerating potential being 200 volts. Electrode M was connected to +210 volts and the current measured. Then, by means of a commutator, the potential was changed rapidly to +190 volts, and the current again measured. These readings were repeated with the usual precautions. The same experiment was performed at the other pressures with the corresponding potentials, M being then connected to different potentials to produce the required fields. Table II gives the mean of the results obtained.

TABLE II.

Pressure. (mm.)	Potential of M. (volts.)	V. (volts.)	Current.
0·015	210	54	36·5
0·015	190	54	18
0·010	220	132	69·5
0·010	180	132	52
0·050	420	154	19·5
0·050	380	154	15·5
0·045	420	265	99
0·045	380	265	86·5

As the pressure and the accelerating potential were varied, the electron current in the ionization chamber also varied enormously. By varying the heating current in the cathode, however, it was always possible to obtain a measurable current in the galvanometer. This adjustment was performed at random, with the result that the figures in the last column of Table II can only be used in pairs.

RESULTS.

From the observations recorded in Table II, I and C can be determined. The current with M positive with regard to the remainder of the ionization chamber was I+C, while the current with M negative

was C. Table III gives the the energy required to produce one pair of ions for the four cases, using the formula

$$V_0 = \frac{I}{C}V.$$

TABLE III.

V	54	132	154	265
V_0	55·5	44·4	40·0	38·3

DISCUSSION OF RESULTS.

The values obtained for V_0 indicate quite clearly a variation of the energy per ion pair with the initial velocity of the electron. This is in agreement with the fundamental theory, since the ionizing efficiency must be zero when the electron voltage is equal to the ionizing potential of the gas. Lehmann and Osgood * have shown that the total ionization N, produced in air by an electron of voltage V may be represented fairly accurately by the formula

$$N = k(V - 17) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where k is a constant, the value of which was 0·0225 in their experiments, and 17 represented the ionizing potential of air. Although the results given in Table III of the present paper are not in perfect agreement with such a formula, experimental errors are sufficient to account for the deviation. Fig. 6 exhibits the mode of variation of N with V as calculated from the experimental results, and the best straight line drawn through the points. This line is

$$N = 0{\cdot}0270(V - 17) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and hence the value of k is considerably greater than that found in Lehmann and Osgood's investigation. The present value compares more favourably with Johnson's * formula,

$$N = 0{\cdot}0275(V - 11) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

so far as gradient is concerned, but Johnson's intercept on the V-axis is much too small.

* *Loc. cit.*

Assuming equation (2) to represent the present results, and writing it to give the energy required to produce one pair of ions, we have

$$V_0 = \frac{37}{1 - \frac{17}{V}},$$

so that when $V \rightarrow \infty$, $V_0 \rightarrow 37$ volts. Such an extrapolation is not justifiable on the results of the present investigation, but it is possible

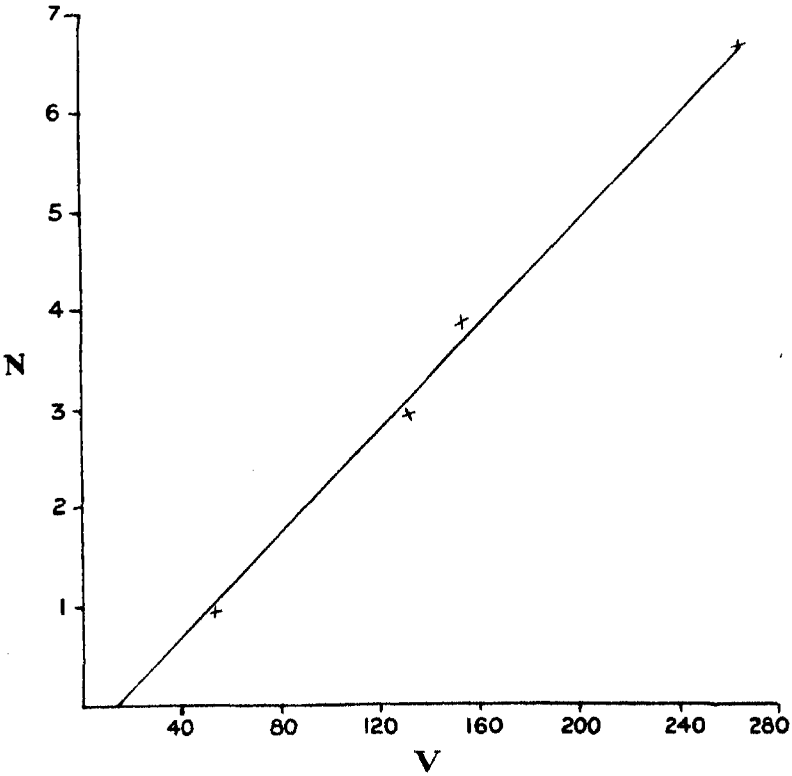

FIG. 6.

to examine its validity in the light of other researches. Lehmann and Osgood found equation (1) to hold for value of V between 200 and 1000 volts. In a recent important paper Eisl * describes an investigation similar to those under consideration, but using electrons, the accelerating

* *Loc. cit.*

potentials of which were from 5 to 60 kilovolts. Again, a linear relation between N and V is deduced, Eisl's result being

$$N = 0{\cdot}0311V \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The absence of the constant 17 in equation (4) is due, of course, to the large values of V employed. The important point to be noted is that a linear relation is characteristic of this very different electron velocity. Hence it may be safely assumed that, unless an abrupt discontinuity occurs in the N, V curve at some point between 1 and 5 kilovolts, an extrapolation from the present results is justifiable.

Yet the extrapolation does not lead to a satisfactory agreement between the various researches. Eisl's value of V_0 for fast-moving electrons is given in his paper as $32 \pm 0{\cdot}05$ volts. The value deduced from the present experiments is 37 ± 2 volts. Lehmann and Osgood's result is 45 volts, the probable error being omitted. Eisl gives a table of the values found by various observers. These range from 26–45 volts, although the present writer is not certain that a comparison is useful, since different things were measured by the various observers.

Eisl's experiments appear to have been carefully carried out, and the discrepancy between his result and that found in the present investigation is difficult to explain. The higher the velocity of the electron the more difficult it becomes to collect all the ions produced along its path. Hence, so far as gas ionization is concerned, one might expect Eisl's result to be high rather than low. On the other hand, it is difficult to ensure that stray high-velocity electrons will not collide with metal surfaces in the ionization chamber. Such collisions would, of course, give rise to large numbers of secondary electrons, and there is no doubt that the work required to ionize a gas molecule is greater than the work required to liberate an electron from a metal surface. Hence, if sufficient precautions are not taken to ensure that the electrons lose all their energy in the gas, results for high-velocity electrons may be expected to be low. It is just possible that the discrepancy between Eisl's and the writer's results is to be attributed to this cause.

Correlation of Various "Constants."

Let N, V, k, have the meaning already assigned to them. Let R be the range of an electron travelling initially with a velocity corresponding to a potential V in air at atmospheric pressure, and let c be the

ionization produced per unit path by an electron of velocity corresponding to V in air at the same pressure.

Then

$$N = \int_0^x c\,dx.$$

But by experiment

$$N = k(V - 17),$$

therefore

$$\frac{dN}{dx} = k\frac{dV}{dx} = -c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

For fast-moving electrons Whiddington's law states that

$$R = AV^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This has been verified* for ranges between 0·1 cm. and 1·5 cm. in air at atmospheric pressure.

Let the range R_0 correspond to an initial voltage V_0. Then, if x is measured from the point where the electron has energy V_0,

$$x = \frac{R_0}{V_0^2}(V_0^2 - V^2)$$

gives the relation between V and x.

Hence

$$\frac{dV}{dx} = -\frac{V_0}{2R_0}\left(1 - \frac{x}{R_0}\right)^{-\frac{1}{2}}.$$

Therefore by equation (5)

$$C = \frac{kV_0^2}{2R_0V} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

This relation between c and V has been deduced from the two experimental laws expressed by equations (2) and (6), and only holds for the region over which these two laws are true. Equation (2) appears to be true for all values of V, but Whiddington's law is meaningless when the initial energy of the electron is small. The range of an electron can only be measured algebraically along a straight line. The path of a slowly moving electron bears no relation to the range as measured.

As a consequence of this it would appear that $\frac{dN}{dx} = -c$ is also meaningless at low velocities. Yet c is a quantity which has been carefully measured by many observers for all initial electron velocities down to that corresponding to the ionizing potential of the gas.

* C. T. R. Wilson, *Proc. Roy. Soc.*, **104**, p. 192 (1923).

Finally, it may be remarked that equation (7) is in good agreement with experiment for high initial electron energies.

I wish to express my gratitude to Professor J. A. Crowther, who suggested the problem, for his many helpful criticisms; to record my indebtedness to the Research Board of the University of Reading for a grant towards the apparatus employed; and to thank Mr Burgess of the Physics Department for his excellent workmanship and for his freely given assistance in mechanical matters.

SUMMARY.

Experiments are described, the aim of which is the determination of the energy spent in producing one pair of ions when electrons of velocity corresponding to 50–270 volts are totally absorbed in air. It is found that this energy varies with the initial speed of the electron, but that it asymptotes to the value 37 ± 2 electron-volts when the energy of the electron is very great.

This value of the "volts per ion pair" is compared with those found by other experimenters. A correlation of the three electronic "constants," the range of the electron, the ionization per unit path, and the total ionization is attempted. It is concluded that, unless in the case of very fast cathode rays or β-particles, the "ionization per unit path," as measured by ionization chamber experiments, has no meaning.

NATURAL PHILOSOPHY DEPARTMENT,
THE UNIVERSITY OF GLASGOW,
March 4, 1931.

(*Issued separately August* 19, 1931.)

XVIII.—Relation between the Composition of Retortable Carbonaceous Minerals and their Yield of Crude Oil. By Professor Henry Briggs, D.Sc., Ph.D. (With Three Charts.)

(MS. received June 1, 1931. Read June 1, 1931.)

In a paper recently read before this Society * the ultimate analysis was used as a means of distinguishing between the various species of carbonaceous minerals, it being shown that, by a simple graphical method, these minerals could be classified in five groups, viz.: coals, sub-cannels, cannels, paraffin shales, and torbanites. Minerals from all these groups are past, present, or prospective raw materials for oil-distillation, for which purpose, indeed, the oil shales and torbanites are especially suitable. It is therefore of interest to inquire whether their yield of crude oil and the composition of these substances are related in a manner sufficiently definite to serve as a guide to those concerned in this branch of technology.

Such an inquiry is beset with several serious difficulties. In the first place, the oil-yield is dependent upon the process of treatment, any of these minerals giving different yields at different temperatures and with different systems of retorting. Many of the results, moreover, cannot be checked, having been extracted from publications of varying reliability and, in some instances, of considerable age. Doubt again sometimes enters when correlating the oil-yields and the analyses of the raw materials; the former may, for example, be maxima and the latter relate to average samples. And further, it is not always clear from the records whether a stated yield of oil was that obtained in actual practice from a commercial plant or from a laboratory test only.

The production derived from commercial plants being the more useful, I have endeavoured roughly to reduce laboratory results to that basis by multiplying them by the factor 0·8. I am aware that that figure is too large in connection with bituminous coals and, if anything, too small for the German retinite-lignites under present-day conditions. Where so many other variables enter, however, it seemed preferable to adopt a simple uniform factor throughout.

* "The Classification and Development of Carbonaceous Minerals," *Proc. Roy. Soc. Edin.*, li, 1931, p. 54.

The effects of different treatments may be studied in relation to the group bearing the numbers 1, 3, 4, 5, 6, 7, 8, 8*a*, and 26 in the Appendix and on the accompanying charts. The data concerning these nine coals were

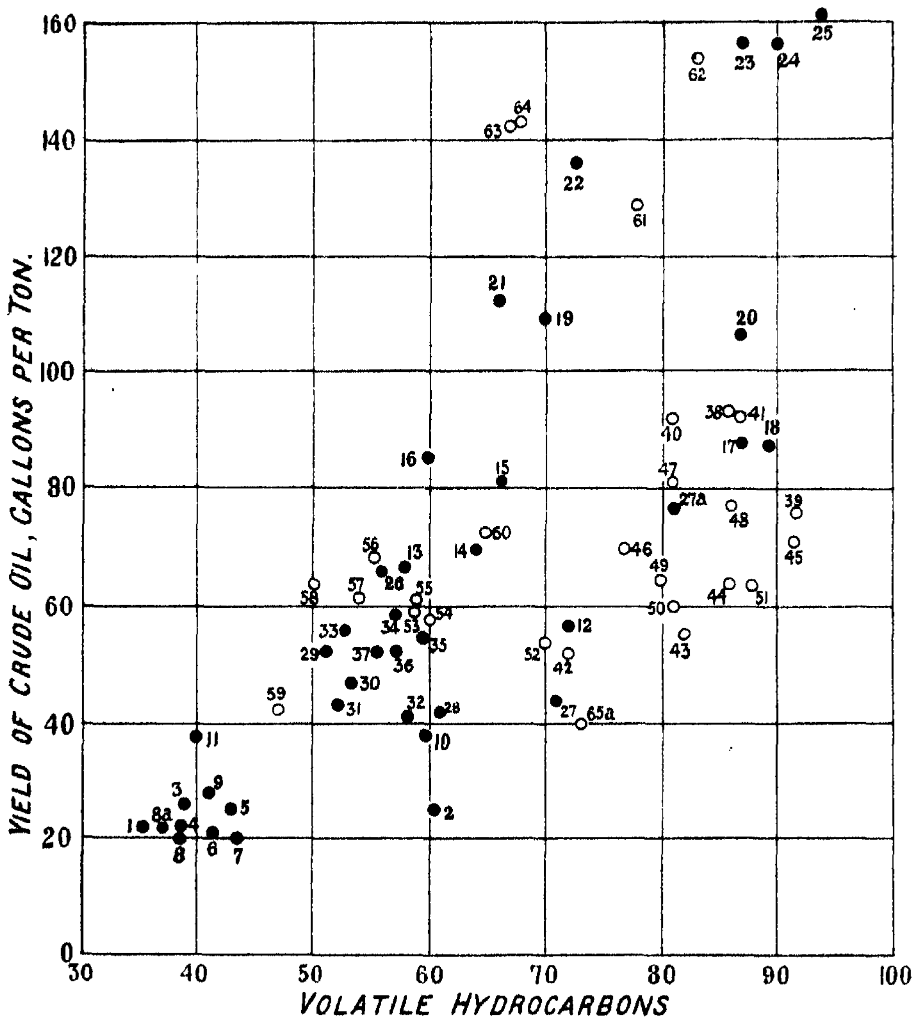

FIG. 1.—Oil-yield and percentage of volatile hydrocarbons, the minerals being regarded as free from ash and moisture.

obtained, under test-conditions and by means of almost as many low-temperature retorting processes, by the Fuel Research Board.

The Scottish oilshales—a numerous and equally trustworthy series—were retorted under the same, or nearly the same, conditions; yet they exhibit considerable irregularities in their positions on the charts. It

would therefore seem that intrinsic differences in the chemical make-up of the organic constituents of these substances affect the oil-yield as much

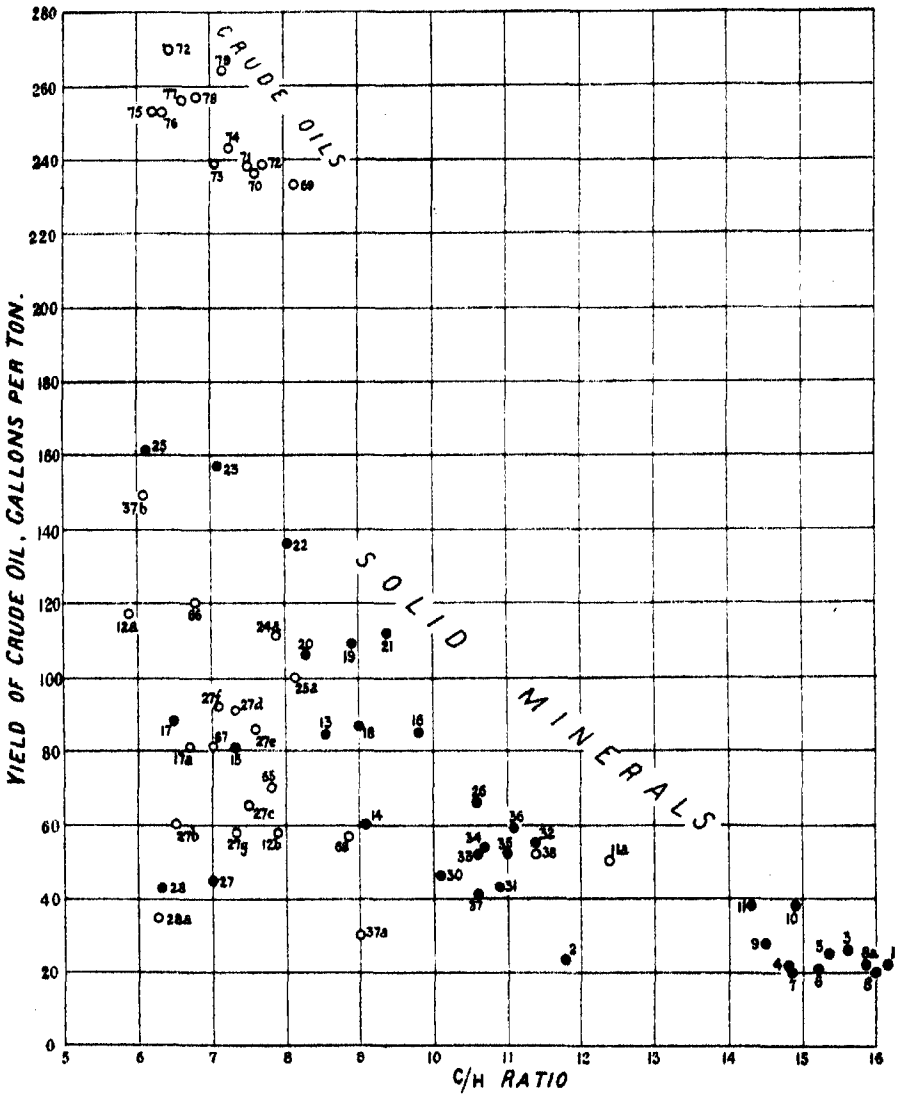

FIG. 2.—Oil-yield and carbon-hydrogen ratio, the minerals being regarded as free from ash, moisture, sulphur, and nitrogen.

as, if not more than, any differences of treatment that are likely to be met.

The Appendix furnishes the names and sources of the 84 solid minerals and 11 crude petroleums whose analyses were made use of in constructing the charts. The designation, in brackets, given to the

majority of the solid minerals refers to the classification proposed in my last paper (*loc. cit.*).

All the proportions set forth on the charts, figs. 1, 2, and 3, are expressed in terms of their respective "true coal" values. The oil-yields, for

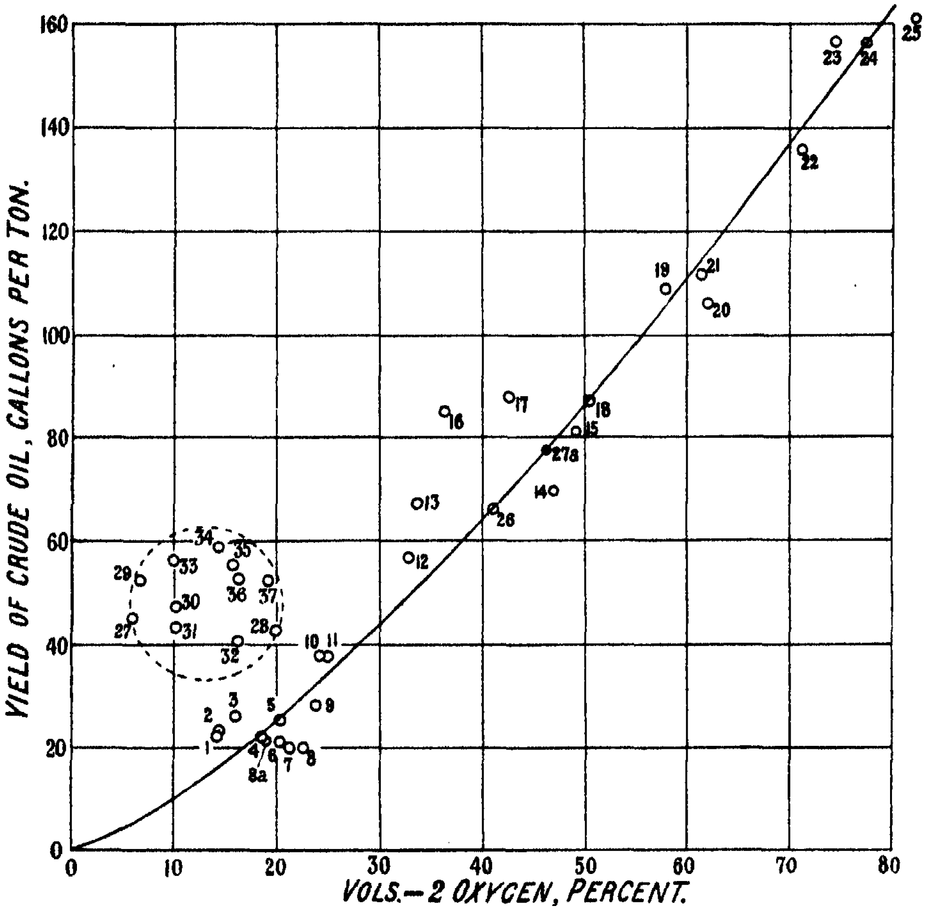

FIG. 3.—Relation between oil-yield and the volatile hydrocarbons reduced by twice the Oxygen percentage, the minerals being regarded as free from impurities.

example, are the actual yields multiplied by the factor 100 ÷ (100 − *percentage of impurities*); similarly, the volatile hydrocarbons are computed to the basis:

Volatile Hydrocarbons + *Fixed Carbon* = 100 per cent.

while the oxygen percentages employed in calculating the abscissæ of fig. 3 are those derived from the ultimate analyses after re-expressing those analyses in the form

$C + H + O = 100$ per cent.

The minerals indicated by black circles in figs. 1 and 2 are those, 39 in number, for which I possess both the ultimate and the proximate analyses as well as the yield of crude oil. These are the only substances that could be included in fig. 3 since, for the construction of that chart, both forms of analysis are required.

That a general relation exists between the oil-yield and the proportion of volatile matter is obvious on the face of it, as it is from the volatile hydrocarbons that the oil is derived. Fig. 1 shows, however, that that relation is most indefinite, and that it is useless to attempt to express it in any numerical fashion.

J. B. Robertson, in a paper entitled "Chemical Examination of the Organic Matter in Oil Shales," read before this Society in 1914, indicated that the lower the proportion *carbon* ÷ *hydrogen* the higher the oil-yield from Scottish shales.* His opinion receives general support from fig. 2, though, again, the relationship appears to be too intangible to be of much value as a guide.

Having observed that a high oxygen proportion is detrimental to oil-production, I decided to try graphing the yield against the volatile hydrocarbons adjusted in different ways for the amount of oxygen the mineral contains. Eventually it was found that when the volatiles diminished by twice the oxygen percentage are plotted as abscissæ (fig. 3), the various points close in to a smooth curve in a manner reasonably satisfactory, having regard to all the difficulties stated in the preamble. There is, however, a group of minerals, represented by the points within the circle, fig. 3, which do not conform with the rest. One of these, No. 27, is a shale from New Zealand which has remained in a peat-like stage of evolution; another, No. 28, though Cretaceous in age, is lignitous in character, while the others of the group are Tertiary retinite-lignites from Saxony. Though Bovey Tracy lignite (No. 2)—a Tertiary deposit—takes up a position not far from the curve, it seems advisable to regard the empirical rule stated below as applying only to minerals of higher rank than lignites.

The equation of the curve is

$$Y = 0{\cdot}47\,(V - 2O)^{\frac{4}{3}},$$

in which Y is the yield of crude oil in British Imperial gallons per ton, V the percentage of volatile hydrocarbons, and O the percentage of oxygen in the mineral retorted. As already stated, the quantities on both sides of the equation are expressed on the basis of mineral free from impurities (ash, moisture, and sulphur).

* *Proc. Roy. Soc. Edin.*, xxxiv, 1914, p. 10.

Given the proximate and ultimate analyses of one of these retortable carbonaceous substances, then, the formula furnishes an indication of the approximate oil-yield to be anticipated under ordinary working conditions.

APPENDIX.

List of Oil-yielding Minerals.

No.	*Description.*
1.	Gas Coal, Brynna Colliery, Llanharan, South Wales (bituminous coal).
2.	Bovey Tracy Coal, Devonshire (lignite).
3.	Top Hard Coal, Kirkby Colliery, Notts (bituminous coal).
4.	Blended coal, Seaton Delaval Colliery, Northumberland (bituminous coal).
5.	Ell Coal, Bankend Colliery, Lanarkshire (bituminous coal).
6.	Virgin Coal, Clydesdale Colliery, Lanarkshire (bituminous coal).
7.	Hawkhill Coal, Tullygarth Colliery, Clackmannanshire (bituminous coal).
8.	Dalton Main Colliery, Yorkshire (bituminous coal).
8*a*.	6-ft. seam, Kinneil Colliery, Linlithgowshire (bituminous coal).
9.	Wigan Cannel, Lancashire (sub-cannel).
10.	Gordon House Cannel, Co. Durham (sub-cannel).
11.	Leeswood Smooth Cannel, Flintshire (sub-cannel).
11*a*.	Leeswood Shale (cannel).
12.	Camps Shale, Linlithgowshire (oilshale).
12*a*.	Do. do. (inclined part) (oilshale).
12*b*.	Do. do. (flat part) (oilshale).
13.	Capeldrae Cannel, Fife (oilshale).
14.	Blackstone, Kimmeridge, Dorset (oilshale).
15.	Main Bed, Corton, Dorset (torbanite).
16.	Lesmahagow Cannel, Lanarkshire (cannel).
17.	Broxburn Shale, Linlithgowshire (oilshale).
17*a*.	Do. do. do.
18.	Estonian Oilshale (calcareous shale).
19.	Leeswood Curly Cannel, Flintshire (oilshale).
20.	Methil Brown Shale, Fife (oilshale).
21.	Albertite, New Brunswick (oilshale).
22.	Stellarite, Nova Scotia (torbanite).
23. 24.	Brown Boghead, Linlithgowshire (torbanite).
24*a*.	Black Boghead, do. do.
25.	Joadja Creek Shale, N.S.W. (torbanite).
25*a*.	Commonwealth Shale, N.S.W. (oilshale).
26.	Cannel, blended with about 10 per cent. bituminous coal, Welbeck Colliery, Notts (cannel).
27.	Waiapu Shale, N. Island, New Zealand (oilshale, equivalent to peat in rank).

No.	*Description.*
27*a*.	Dunnet Shale, Linlithgowshire (oilshale).
27*b*.	Do. do. top foot.
27*c*.	Do. do. 2nd foot.
27*d*.	Do. do. 3rd foot.
27*e*.	Do. do. 4th foot.
27*f*.	Do. do. 5th foot.
27*g*.	Do. do. 6th foot.
28.	Inferior Shale, Corton, Dorset (oilshale, lignitous in rank).
28*a*.	Inferior Shale over Blackstone, Kimmeridge, Dorset.
29.	
30.	
31.	
32.	
33.	Oil-yielding lignites, of Tertiary age, Saxony * (retinite-lignites).
34.	
35.	
36.	
37.	
37*a*.	Waldau lignite, Saxony (retinite-lignite).
37*b*.	Pyropissite, Saxony (torbanite).
38.	Fells Shale, Linlithgowshire (oilshale).
39.	Broxburn Grey Shale, Linlithgowshire (oilshale).
40.	Broxburn Curly Shale, do. do.
41.	Broxburn Main Shale, do. do.
42.	Pumpherston No. 1 Shale, do. do.
43.	Pumpherston No. 2 Shale, do. do.
44.	Pumpherston No. 3 Shale, do. do.
45.	Pumpherston No. 4 Shale, do. do.
46.	Pumpherston No. 5 Shale, do. do.
47.	Dunnet Shale, Pentland, Midlothian (oilshale).
48.	Broxburn Shale, do. do. do.
49.	Munduran Creek Shale, Queensland.
50.	Whangaroa Shale, New Zealand.
51.	Turfa de Marahú, Brazil.
52.	Brazilian Oilshale.
53.	Hartley Retorting Shale, N.S.W. (oilshale).
54.	Wardend Cannel, Lanarkshire.
55.	Capertee Retorting Shale, N.S.W. (oilshale).
56.	Arniston Cannel, Midlothian.
57.	Pirnie Cannel, Fife.
58.	Canneloid Shale, Brora, Sutherlandshire.
59.	Capeldrae Shale, 2nd quality, Fife.
60.	Ouachita River Coal, Arkansas, U.S.A. (brown coal).
61.	Stellar Shale, Nova Scotia (oilshale).
62.	Best Hartley Kerosene Shale, N.S.W. (torbanite).
63.	Breckenridge Coal, Kentucky, U.S.A. (torbanite ?).
64.	
65.	Trinidad Asphalt (torbanite).

* Particulars kindly supplied by Professor Frank, Berlin.

No.	*Description.*
65a.	Ostaschkow Shale, Volga Region, Russia.
66.	Grand Valley Shale, Colo., U.S.A. (torbanite).
67.	Nevada Shale, U.S.A.
68.	Ione Shale, Cal., U.S.A. (cannel).
69.	Borneo Crude Oil.
70.	California Heavy Crude.
71.	Mexico Crude.
72.	Roumania Crude.
73.	Caucasia Heavy Crude.
74.	Java Crude.
75.	Pennsylvania Crude.
76.	Caucasia Light Crude.
77.	Rangoon Crude.
78.	East Galicia Crude.
79.	Burmah Crude.

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XIX.—Photochemical Measurements of Light Intensity in Two Common Vegetation Types in Tropical Africa, by Means of the Improved Eder-Hecht Photometer.

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Introduction.

So far as we are aware, no measurements of light-intensity in terms of photochemical units are published for Tropical Africa,† hence a concise account of readings taken by means of the Eder-Hecht photometer in 1930–1931 at Kikori Ecological Station (4° 21′ S., 35° 19′ E.), situated at an elevation of 1238 metres (4062 ft.), on the western edge of the Massai Steppe, Tanganyika Territory, should be of interest to climatologists, biologists, and medical men.

At the outset we would record it as our view that the nature of the light—qualitative as well as quantitative—in Tropical Africa deserves far wider and deeper study than it has hitherto received, for it seems that the influences of Tropical African light, together with those of the high altitudes common within the interior of Tropical Africa, are not unlikely to constitute in their joint action a limiting factor to the development of a really virile European population permanently domiciled in these regions. While the temperature rarely exceeds 100° F. (37·8° C.), while the humidity is continuously very low or very high for short periods only, at all times during hours 9–17 the light—direct and diffused—has a marked action upon the nervous system of the European.

The objects of the present paper are to record the high photochemical values registered under full exposure to insolation and under the light canopy of that very widely distributed woodland type, the *Berlinia-Brachystegia-Other Spp. Community*,‡ or "Miombo," to show how the Bunsen-Roscoe values are correlated with duration of sunshine and with the readings of the Livingston radio-atmometer, and to bring to the

* Mr W. H. Potts, Senior Entomologist, assisted from time to time in this work.

† Dorno (1927) has published data for Assuan (24° 8′ N.), just beyond the tropics proper.

‡ *Vide* Phillips, 1930 : 213 for an account of this community.

attention of biologists the usefulness of the Eder-Hecht photometer in the study of habitats in nature.

STATIONS AND METHODS.

At Kikori measurements were made at one and the same spot at a height of 4 feet in the following stations:—

(1) *Station B:* seasonal swamp, or "mbuga"; a non-climax, grass-sub-shrub community, experiencing at photometer height full insolation (Phillips, 1930 : 203).

(2) *Station H: Berlinia-Brachystegia-Other Species Woodland*, with the delicate canopy normal to this community. Both stations were representative of great areas of their respective types in Tropical Africa.

The improved model of the Eder-Hecht Dauer-Graukeilphotometer was employed, direct sunshine being registered by means of standardised Campbell-Stokes instruments at the same level. As but one Eder-Hecht photometer was available this was placed on alternate days in one station and on the others in the second station. In each station a standardised Livingston (1915) radio-atmometer was placed near the photometer and the sunshine recorder. Observations were taken over the period 7–17·5 hrs., the radio-atmometric ones over the period 9–17 hrs.

As the Eder-Hecht photometer so far has a very limited use within the British Empire, and as no descriptions in English are known to us, a short account of the nature and use of this instrument is given:

THE EDER-HECHT GRAUKEILPHOTOMETER.

First described by Hecht (1918), shortly after in greater detail by Eder (1919, 1920, 1921), the instrument has been greatly improved by Dorno (1925, 1928) and Kopmüller (1930).

The photometer is a photochemical one,* based upon the Bunsen-Roscoe (1862) *law* that darkenings of silver-chloride papers of like sensitiveness correspond to the products of light-intensity and time of exposure. In the improved pattern the essential parts are:

(i) A wedge of opalescent diffusing glass, 15·5 × 2·4 cm., set in a metal frame. From one end to the other the wedge increases in depth in *logarithmic* progression.

(ii) Immediately behind the wedge, supported at its extremities by two 1-mm.-deep rubber rests, is a thin strip of plain glass bearing a celluloid scale ranging from 0 to 152 mm., by 2 mm.

* The optical centre is in the blue-violet-ultraviolet portion of the spectrum.

graduations; the lower scale values are opposite the shallower portions of the wedge. In the instance of an instrument with a *wedge constant* 0·305 *—the pattern used in the investigations under description—each scale graduation is to the one above it as 1 : 1·157—as regards the value of the Eder-Hecht Relative Light-intensity unit registered by the 2 mm. strip of wedge covering that particular graduation of the scale.

(iii) A strip of Davos silver-chloride photographic paper, its sensitive face towards the scale and the opalescent wedge, is placed upon this scale.*

(iv) A rubber-lined, tight-fitting, metal back is clamped above the paper, making the chamber containing the scale and the paper light-tight except for the illumination penetrating the opalescent wedge.

The instrument is loaded in yellow lamplight, is placed in a dark case, taken to the site to be studied, and there removed from the container. It is placed so that the length of the wedge runs approximately true N.-S. and so that the wedge surface is horizontal. A record of the time of exposure and of removal is kept; in a 0·305 wedge-constant instrument the duration of exposure may range from a few minutes to the whole of the day, according to the light-intensity, or to the objects of the investigator.†

On being examined in yellow light, the paper is seen to bear a photographic impression of the scale. The ultimate *continuous* cross line shown upon this impression, on holding the paper at about 30° to the incident light, is marked as corresponding to the intensity of the illumination existing within the site studied. By means of special "Relative Light-intensity" tables calculated by Hecht (1918: 42-44) it is possible, knowing the ultimate scale reading plus the correction factor for the paper,‡ to determine the corresponding Eder-Hecht "R.L.-i." unit (which bears the ratio 1·092 : 1 to the Bunsen-Roscoe unit).§ The "R.L.-i." or

* Wedges and papers are standardised at the Dorno Physikalisch-Meteorologisches Observatorium, Davos, Switzerland, whence the photometer and supplies of paper may be purchased at the cost of production and standardising.

† Three types of wedge are made: 0·305, 0·401, 0·188—for short or long, very long, and very short exposures respectively; there are also papers of different sensitiveness.

‡ Supplied by the Dorno Laboratorium.

§ The B.-R. unit is that light-intensity which in 1 sec. upon "normal" B.-R. paper develops a shade equivalent to the B.-R. "normal shade"—produced by the mixing of 1000 parts chemically pure zinc oxide and 1 part pure lampblack, heated to incandescence in absence of air; the pulverised grey powder being spread by means of a gelatinous solution in an even coat on white Bristol-board.

Bunsen-Roscoe values so obtained are divided by the time of exposure in seconds.

The strips are best fixed by immersion for 8 minutes in a gold-toning-fixing bath of medium strength, thereafter being thoroughly washed, and dried in the dark.

Measurements made at Kikori: 1930-31.

Reliable daily measurements are available for the period April 1930–March 1931, but on grounds of space it is impossible to give these in detail. Suffice it to record (1) *the monthly mean*; (2) *absolute maximum* and *absolute minimum* B.-R. and R.L.-i. units; (3) several examples of the daily data for each of the stations.

(1) Data as to the mean monthly, absolute maximum, and absolute minimum Bunsen-Roscoe units per second, and mean hours of sunshine, Stations B and H are given in Table I.

Table I.

	Station B.					Station H.				
Month.	No. of Observations.	Bunsen-Roscoe Units per Second.			Mean Hours' Sunshine for Same Dates.	No. of Observations.	Bunsen-Roscoe Units per Second.			Mean Hours' Sunshine for Same Dates.
		Mean.	Absolute Max.	Absolute Min.			Mean.	Absolute Max.	Absolute Min.	
April .	10	·057	*·089*	·026	3·6	11	·013	·019	·007	0·4
May .	12	·031	·070	·013	3·4	12	·008	·015	·004	0·46
June .	13	·021	·041	*·004*	4·7	11	·007	·013	*·001*	0·8
July .	13	·023	·053	·013	4·6	12	·008	·013	*·001*	1.3
August .	13	·022	·030	·013	3·5	13	·009	·013	·006	0·8
Sept. .	10	·033	·054	·008	4·8	10	·014	·030	·005	2·5
October	10	·032	·042	·022	6·1	8	*·019*	·026	·010	4·2
Nov. .	11	·044	·086	·027	5·1	13	·015	·026	·009	2·38
Dec. .	13	·035	·055	·025	6·8	10	·012	·019	·006	1·3
Jan. .	12	·036	·054	·020	7·1	12	·013	*·036*	·007	1·3
Feb. .	11	·036	·054	·021	7·1	11	·010	·013	·004	1·9
March .	11	·042	·051	·025	5·2	11	·010	·013	·008	1·0
Year .	139	·034	·089	·004	5·16	134	·011	·036	·001	1·52

It is plain that the B.-R. values are high in both stations, but especially high in the fully exposed one.

(2) For sake of giving a better impression of the relative greatness of the values shown at Kikori, we give in Table II, in terms of Eder-Hecht "R.L.-i." units, *mean monthly* data for the *daily sums* of

TABLE II.

Eder-Hecht Relative Light-intensity: Mean Daily Sums.								Hours' Sunshine.				
1930–1. Months. Kikori.	Kikori. 4° 21′. Stn. B.	Kikori. 4° 21′. Stn. H.	1925–6. Months. Dorno.	Rio de Janeiro. 22° S.	Assuan. 24° 8′ N.	Davos. 46° 48′.	Kew. 51° 27′.	Kikori. Stn. B.	Kikori. Stn. H.	Rio de Janeiro.	Davos.	Kew.
1930.			1925.									
April .	1888	416	April .	335	..	624	176	3·6	0·4	7·5	4·2	4·3
May .	1015	271	May .	325	..	714	268	3·4	0·46	6·5	6·1	6·4
June .	646	270	June .	288	..	829	487	4·7	0·8	5·8	7·3	9·0
July .	729	254	July) .	298	..	625	283	4·6	1·3	5·6	4·6	6·1
Aug. .	729	299	Aug. .	468	..	460	144	3·5	0·8	8·7	5·9	4·6
Sept. .	971	446	Sept. .	337	..	477	84	4·8	2·5	5·2	4·5	4·3
Oct. .	975	646	Oct. .	292	..	..	29	6·1	4·2	4·9	..	2·7
Nov. .	1362	508	Nov. .	397	..	222	19	5·1	2·38	5·2	3·0	2·6
Dec. .	1113	361	Dec. .	424	566	185	13	6·8	1·3	8·5	1·9	1·7
1931.			1926.									
Jan. .	1139	445	Jan. .	427	544	202	..	7·1	1·3	4·8	2·0	..
Feb. .	1187	321	Feb. .	467	695	293	..	7·1	1·9	7·1	3·5	..
March	1373	335	March	286	904	503	..	5·2	1·0	6·8	3·7	..

TABLE III.

Eder-Hecht Relative Light-intensity Units: Absolute Maxima and Minima.									
1930–31. Months. Kikori.	Kikori. Station B. Max.	Kikori. Station B. Min.	Kikori. Station H. Max.	Kikori. Station H. Min.	1925–26. Months. Dorno.	Maxima. Rio de Janeiro.	Maxima. Assuan.	Maxima. Davos.	Maxima. Kew.
1930.					1925.				
April .	2986	843	637	238	April .	513	..	925	416
May .	2249	419	480	136	May .	502	..	1107	678
June .	1117	118	481	207	June .	549	..	1117	..
July .	1700	419	419	103	July .	630	..	1073	633
August	971	419	419	207	August	612	..	1107	292
September .	1700	238	971	180	September .	579	..	637	192
October	1286	637	843	316	October	510	..	..	63
November .	2631	843	843	316	November .	756	..	(296)	41
December .	1700	843	419	238	December .	704	685	256	24
1931.					1926.				
January	1700	637	1286	136	January	759	637	316	..
February .	1700	733	419	136	February .	756	843	517	..
March	1700	843	419	238	March	497	971	685	..

illumination for the two Kikori stations and for several stations recorded by Dorno (1927 : 370).

In Table III are supplied comparative data for the same stations and months in terms of Eder-Hecht-unit *absolute maximum* daily sums. Absolute minima are given for the two Kikori stations only. (See Table II.)

(3) The following detailed examples are given as being of interest:—

(i) *Station B:* Daily data for months with lowest (May 1930) and highest (December 1930) mean daily duration of sunshine. (See Table IV.)

(ii) *Station H:* Daily data for months with *least leaf canopy* (October 1930) and with very dense leaf canopy and at the same time the greatest number of hours' sunshine under full exposure (December 1930). (See Table V.)

Noteworthy it is that while December 1930 had the highest amount of direct sunshine (6·8 hours) under full exposure, the canopy in Station H was so dense that the direct sunlight reaching the photometer was *slight.* That there is a *strong positive* correlation between direct sunlight and the value of the B.-R. units in Station H is indicated by the fact that the photometric values are fairly low (0·012). In October, when the community has a very open canopy, due to defoliation following seasonal drought, the sunshine under full exposure (Station B) was great (6·1 hours), while as much as 4·2 hours per diem sunshine were experienced on the site of the photometer in Station H. The instrument accordingly registered a very high value—0·019 B.-R. units per second.

(4) In respect to the comparison of the light-intensities under full exposure and in the Berlinia woodland, as the data were collected upon different days in the two stations a *strict* comparison is rendered impossible. At the same time it is possible to base a *gross* comparison upon the mean monthly B.-R. units:

Over the period April 1930–March 1931 the approximate ratio of light in the open to the light under canopy of the Berlinia was as 0·034 : 011, or about 3 : 1. During the period of lesser foliage—September-November—the ratio of B to H was as 0·036 : 0·016, or 2·25 : 1, whereas during the remainder of the year (December to August), B : H was as 0·033 : 0·01, or 3·3 : 1.

While it is true that vegetation canopies absorb more completely the red, blue-violet-ultraviolet rays than the orange, yellow, and green, they do not do so to such a degree as to lower to any great extent light

values within the Berlinia stand measured by photographic paper largely sensitive to the blue-violet-ultraviolet portion of the solar spectrum.

TABLE IV.

May 1930.	Relative Light-intensity Eder-Hecht Units.	Total B.-R. Units.	B.-R. Units per Sec.	Hours' Sun-shine.	Dec. 1930.	Relative Light-intensity Eder-Hecht Units.	Total B.-R. Units.	B.-R. Units per Sec.	Hours' Sun-shine.
1	2249	2456	·067	6·0	1	971	1060	·035	6·2
3	..	..	..	..	3	843	921	·026	8·2
6	2249	2456	·070	5·0	5	1117	1220	·036	4·9
8	971	1060	·031	2·0	8	1117	1220	·035	5·9
10	733	800	·022	1·0	10	1117	1220	·032	5·5
13	637	696	·020	1·6	12	971	1060	·028	3·7
15	553	604	·017	1·2	15	1700	1856	·053	9·8
17	481	525	·015	0·6	17	1117	1220	·037	9·4
20	1700	1856	·053	8·7	22	843	921	·027	5·7
22	733	800	·023	1·6	24	843	921	·026	7·1
26	971	1060	·031	3·3	26	843	921	·025	7·4
28	481	525	·015	1·5	29	1286	1404	·040	9·6
30	419	458	·013	2·2	31	1700	1856	·053	5·8
Mean	1015	..	·031	3·4	Mean	1113	..	·035	6·85

TABLE V.

Oct. 1930.	Relative Light-intensity Eder-Hecht Units.	Total B.-R. Units.	B.-R. Units per Sec.	Hours' Sun-shine.	Dec. 1930.	Relative Light-intensity Eder-Hecht Units.	Total B.-R. Units.	B.-R. Units per Sec.	Hours' Sun-shine.
4	481	525	·015	1·4	4	419	458	·012	2·6
7	553	604	·014	3·7	6	238	260	·006	0·2
9	553	604	·017	3·2	9	274	299	·009	1·8
11	843	921	·026	5·5	11	419	458	·012	1·4
14	843	921	·026	6·6	13	274	299	·008	0·9
18	316	345	·010	3·1	16	419	458	·013	1·3
21	843	921	·025	4·4	20	419	458	·012	1·3
23	733	800	·022	5·7	23	419	458	·013	1·4
					27	364	398	·019	0·5
					30	364	398	·015	1·5
Mean	646	..	·019	4·2	Mean	361	..	·012	1·3

While the delicate leaf-mosaic permits abundant entrance of rays of longer wave-length, it also allows the passage of a very considerable amount of light of short wave-length that has not had to pass through, or to be reflected from, the foliage. Under a dense canopy such as is

found in many African subtropical and tropical evergreen forests,* the error due to a relatively high absorption of the blue-violet-ultraviolet rays by the foliage is greater—but still not sufficiently great to vitiate data collected by photochemical methods.†

In connection with the relative light-values in Stations B and H, it is not without interest to record that a comparison based upon radio-atmometric readings taken at 4 feet in very strongly insolated Station G, and in Station H, during the period October 1929–September 1930, and over the hours 9–17, shows the reduction in total light in Station H to be as 1·29 : 0·56, or as 1 : 2·3, the standard error of the mean within Station G being *plus-or-minus* 0·0439 and in Station H *plus-or-minus* 0·025. These radio-atmometer figures, 2·3 : 1, bear a close resemblance to those from the Eder-Hecht photometer for April 1930–March 1931 (3 : 1), suggesting that the relative values shown by the two methods are approximately comparable; this point is referred to in greater detail later.

Correlation between the Bunsen-Roscoe Units per Second and the Number of Hours' Sunshine per Diem as recorded by the Campbell-Stokes Method.

The relations between the B.-R. values per second and the hours of sunshine per diem, for the period April 1930–March 1931, were decided by the determination of the correlation coefficient, "r." In Station B, the value of "r" is plus ·45, with P = ·07 ‡—that is, "r" is *significant.* In Station H, "r" is plus ·55, with P = ·07—that is, "r" is fairly strongly significant. In this connection it is interesting to note that the total insolation as indicated by the radio-atmometer between 9 and 17 hours, in Station H, April–September 1930, bears to the hours of sunshine between the same times on the same days a relation expressed by "r" = plus ·62, with P = ·05.

In other words, the correlation of the B.-R. units per second is stronger (plus ·55 : plus ·45) in the Berlinia woodland than under full exposure. This possibly is due to the fact that the direct sunlight is much less abundant in the Berlinia than in the open, hence the increase in the B.-R. units more directly corresponds with the increase in direct sunlight than does the B.-R. value in the open, where there is always strong *diffused*

* *Vide* Phillips (1931: and his papers cited therein) for data collected by means of Clements's stop-watch photometer based on the photographic paper principle.

† Based on a few spectroscopic observations carried out in the Knysna subtropical evergreen forest by Phillips in 1927.

‡ P: probability of correlation arising by random sampling from an uncorrelated population.

light even when the *direct* sunlight is intercepted by clouds. The ratio of *diffused* to *direct* sunlight at Kikori, as measured by Clements's photometer, is as 75 per cent. : 25 per cent., or 3 : 1.

That there is not a stronger correlation between the B.-R. values and sunshine under full exposure supports the popular observation based upon personal physical feelings, that days of abundant diffused light and sparse direct sunlight are as trying as, or even more trying than, days of abundant direct sunlight to the human nervous system—principally the nerves of head, neck, and eye. Actually the mean number of hours' sunshine over the year at Kikori is very low in proportion to the amount theoretically possible. Thus:

Table VI.—Comparison of Actual–Possible Hours' Sunshine: Kikori.

Date.	Possible Hours.*	Actual Mean Hours for the Current Month based on the Whole Month's Observations.	Ratio. Possible : Actual.†
	Hr. Min.		
March 21, 1931	12 6	3·2	3·8 : 1
June 21, 1930	11 52	3·2	3·7 : 1
Sept. 21, ,,	12 6	4·6	2·6 : 1
Dec. 21, ,,	12 25	7·1	1·7 : 1

* Calculated from *Nautical Almanac*, 1931.

† As the Campbell-Stokes recorders fail to register sunshine either very early or very late in the day, these ratios err slightly on the high side; they should be reduced by perhaps 8 per cent.

These data are supported by German observations that the mean cloudiness for these regions lies near $\frac{6}{10}$ths (Heidke, 1923). That despite the low direct sunshine the B.-R. values should be so high, again supports the supposition that the unpleasant effects of light in these regions to the European—especially the not strongly pigmented individual—is due in no small measure to the short wave-lengths.

Correlation between the Bunsen-Roscoe Units per Second and the Difference between Black and White Livingston Atmometers (Radio-atmometer) at the same Level.

Livingston (1915) has outlined the use of the "Radio-atmometer," comprised of a black (celloidin-lampblack) and a white spherical non-absorbent porcelain atmometer of the same dimensions, both suitably standardised. It is desirable here to outline the nature of the action of the black and the white spheres.

The *black* sphere, in its measurement of the whole evaporating power of the air, records the impress of (i) about 90 per cent. of the visible rays; (ii) practically all the invisible rays; (iii) much of the visible and non-visible rays reflected from the earth's surface—or air heat. It thus registers over 90 per cent. of the total insolation. On the other hand, the *white* sphere registers (i) about 10 per cent. of the visible; (ii) practically all the invisible rays; (iii) much of the air heat. As atmospheric and electrostatic pressure, electric and electromagnetic density, humidity, and wind—factors integrated with light and heat in controlling the evaporating power of the air—act upon black and white spheres to a practically equivalent degree, the *difference* between the black and the white spheres is a fairly constant measure, approximating rather less than 80 per cent. of the total *light* of the locality. This measure is in terms of cubic centimetres of water. Acting over periods of several to many hours, the radio-atmometer gives a useful general quantitative impression of the intensity of the total illumination.

While the Eder-Hecht photometer registers the intensity of the blue-violet-ultraviolet end of the solar spectrum, and the radio-atmometer about 80 per cent. of the *total* direct and diffused light, it is logical to expect that there should be some correlation between the B.-R. values and those values shown by the radio-atmometer. When the B.-R. and radio-atmometer values for the months April–September 1930 are plotted against one another, the following correlations are exhibited:—

(1) *Station B:* "r" = plus ·35, with P = 0·05; that is, "r" strongly *significant.* (2) *Station H:* "r" = plus ·62, with P = 0·2; that is, "r" has some slight significance.

There is in Station G—almost as much exposed as Station B—a positive correlation of ·74 (P of 0·01) between the data of the radio-atmometer and the number of hours' sunshine, while in Station H "r" = plus ·62 (P of 0·05).

Unfortunately no radio-atmometer readings are available for the period 9 to 17 hours for the months October 1930–March 1931, the only measurements being those referring to 24-hour periods, that is including the evening, night, and early morning, as well as the period of high illumination between 9 and 17 hours. As wind and humidity, *inter alia*, influence the radio-atmometer values during these periods even more than they do during the day, it is not surprising to find that in both Stations B and H there is no correlation between the B.-R. units per second and the radio-atmometer values for the 24 hour-period. The presence in B, for considerable periods, of large amounts of standing or flowing water

in the months February–July 1930, and again in January–March 1931, further vitiates the radio-atmometric readings as *comparative* values of light-intensity.

Conclusions.

From the year's measurements with the Eder-Hecht instrument it is possible to demonstrate the high value of the blue-violet-ultraviolet rays in both fully exposed and lightly canopied sites at Kikori. It has been possible to correlate the B.-R. values per second with hours of sunshine, and at the same time to correlate the same values with the readings of the radio-atmometer. Despite a low proportion of direct sunshine, very high B.-R. values have been recorded, arguing that the unpleasant effects of the tropical light upon cloudy days are largely due to waves of short wave-length.

We are satisfied as to the utility of the Eder-Hecht photometer in ecological studies—especially when its data are supported by those obtained from the radio-atmometer. Its cheapness (45 Swiss francs), its simplicity, its convenient size, its efficiency should win for this instrument a much wider use among field ecologists.

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We desire to thank Professor C. Dorno and Professor W. Moriköfer, successive Directors of the Dorno Observatorium, Davos, for assistance in correspondence and for gifts of relevant literature.

Summary.

(1) A year's measurement of the light-intensity in an exposed site ("mbuga") and in a Berlinia woodland community, Kikori (4° 21′ S.), Tanganyika, by means of the Eder-Hecht Graukeilphotometer is described.

(2) The Bunsen-Roscoe units per second are shown to be very high, even under canopy of the Berlinia; they are very much greater than measurements recorded by Dorno for Assuan and Rio de Janeiro.

(3) While there is a positive correlation between the B.-R. values and the hours of direct sunshine, it is also clear that during cloudy periods (less than 50 per cent. of possible sunshine is experienced at Kikori and on the Central Plateau of East Africa generally) the B.-R. values are also very high.

(4) There is a positive correlation between the values of the Eder-Hecht photometer (in terms of blue-violet-ultraviolet rays), and the

readings of the Livingston radio-atmometer measuring the *total* light-intensity of the same stations in terms of cubic centimetres of water.

(5) In virtue of the peculiar effects of the light of Tropical Africa upon the European nervous system, it is urged that quantitative and qualitative studies of the light in those regions should be investigated.

(6) Field biologists should make greater use of this efficient, cheap, simple, and portable instrument.

REFERENCES.

BUNSEN, R., and H. ROSCOE (1862), "Meteorologische Lichtmessungen," in *Poggendorff's Annalen*, cxvii.

DORNO, C. (1925), "Über die Verwendbarkeit von Eders Graukeilphotometer im meteorologischen Dienst, usw.," *Met. Zeitschr.*

DORNO, C. (1927), "Parallelmessungen der photochemischen Ortshelligkeit zwischen nördl. Polarkreis und südl. Wendekreis mittels Eders Graukeilphotometer," *Met. Zeitschr.*

DORNO, C. (1928), "Ratschlage zum Gebrauch des Graukeilsphotometers noch modifiziert für meteorologische Zwecke von C. Dorno," cyclograph issued by the Davos Observatorium.

EDER, J. M. (1919), in *Zitz. Ber. d. Ak. d. Wiss. i. Wien*, Abt. IIa, Bd. cxxviii; also in *Photographische Korrespondenz*, S. 141. *Mitteilungen der osterreichischen Techn. Versuchsamtes in Wien*, Nr. 4.

EDER, J. M. (1920), *Ein neues Graukeilphotometer*, W. Knapp, Halle.

EDER, J. M. (1921), "Die Messung der Lichstrahlen Dosen in der Therapie," a. d. *Wiener Medizinischen Wochenschrift*, Nr. 45.

HECHT, W. (1918), "Das Graukeilphotometer im Dienste der Pflanzenkultur," *Zitz. Ber. d. Ak. d. Wiss. i. Wien.*

HEIDKE, P. (1923), "Das Klima," in *Obst E. Mitt. d. Geogr. Gesellschaft Hamburg*, xxxv.

KOPMÜLLER, A. (1930), "Verbessertes Graukeilphotometer," a. d. *Zeitschr. f. Wiss. Baderkunde*, Heft 11, Berlin.

LIVINGSTON, B. E. (1915), "Atmometry and the Porous Cup Atmometer. IV. (The Radio-atmometer)," *The Plant World*, xviii, pp. 143–149.

PHILLIPS, J. F. V. (1930), "Some Important Vegetation Communities in the Central Province of Tanganyika Territory," *Journ. Ecol.*, xviii, pp. 193–234.

PHILLIPS, J. F. V. (1931), *Forest Succession and Ecology in the Knysna Region*, Union S. Africa Bot. Survey Memoir.

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XX.—Note on Cayley's Elimination-problem involving Superfluous Data. By Sir Thomas Muir, F.R.S.

(MS. received May 5, 1931. Read June 1, 1931.)

1. The given set of equations in question is

$$\left\|\begin{matrix} x^4 & x^3y & x^2y^2 & xy^3 & y^4 \\ a_1 & b_1 & c_1 & d_1 & e_1 \\ a_2 & b_2 & c_2 & d_2 & e_2 \\ a_3 & b_3 & c_3 & d_3 & e_3 \end{matrix}\right\| = 0,$$

and the problem was solved by Cayley in the *Quarterly Journal of Mathematics*, viii, pp. 183–185.* The method was peculiar and is still interesting. He introduced three additional unknowns, deduced a fresh set of five equations, from which he then proceeded to eliminate four unknowns, namely, the new three and x/y.

The eliminant he obtained in the form of a 3-line compound determinant, the elements of which are procurable from the peculiar 4-by-6 array

$$\begin{matrix} a_1 & a_2 & a_3 & b_1 & b_2 & b_3 \\ b_1 & b_2 & b_3 & c_1 & c_2 & c_3 \\ c_1 & c_2 & c_3 & d_1 & d_2 & d_3 \\ d_1 & d_2 & d_3 & e_1 & e_2 & e_3 \end{matrix}$$

by the operation of dropping two columns, his own abridged mode of writing it being

$$\left|\begin{matrix} 4123 & 1426+1453 & 1456 \\ 5123 & 2453+2156 & 2456 \\ 6123 & 3156+3426 & 3456 \end{matrix}\right|$$

where 4123 stands for the determinant whose columns are columns 4, 1, 2, 3 of the array.

2. It is this eliminant that here concerns us, mainly because of two observations which strike one almost immediately on looking at it. In the first place we note the want of attractiveness about its form, and in the second place the incompatibility of this with the simple symmetry

* *Coll. Math. Papers*, vi, pp. 40–42.

of the data. In fact one is inclined to go farther and prophesy a resultant symmetrically dependent on the given array of coefficients

$$\begin{matrix} a_1 & b_1 & c_1 & d_1 & e_1 \\ a_2 & b_2 & c_2 & d_2 & e_2 \\ a_3 & b_3 & c_3 & d_3 & e_3 . \end{matrix}$$

In any case there is need for more knowledge about the problem. It does not stand alone, being only one of a type; and Cayley, when he tackled the next of the same kind

$$\left\| \begin{matrix} x^5 & x^4y & \cdot\,\cdot\,\cdot\,\cdot\,\cdot & xy^4 & y^5 \\ a_1 & b_1 & \cdot\,\cdot\,\cdot\,\cdot\,\cdot & e_1 & f_1 \\ \cdot & \cdot & \cdot\,\cdot\,\cdot\,\cdot\,\cdot & \cdot & \cdot \\ a_4 & b_4 & \cdot\,\cdot\,\cdot\,\cdot\,\cdot & e_4 & f_4 \end{matrix} \right\| = 0,$$

could hardly feel encouraged at the outcome.

3. At the same time it must not be forgotten that he was somewhat hampered by the circumstances under which he came to take the problem up. An equivalent of it had originated with Sylvester during the period of the latter's heated enthusiasm about "double-determinants": and Cayley, who could not share this enthusiasm and doubtless wished to calm his friend, found it expedient to solve his own problem in the way he did, because doing so involved showing that it and Sylvester's were identical. However this may be, the interesting fact remains that *The eliminant of*

$$\left\| \begin{matrix} x^4 & x^3y & x^2y^2 & xy^3 & y^4 \\ a_1 & b_1 & c_1 & d_1 & e_1 \\ a_2 & b_2 & c_2 & d_2 & e_2 \\ a_3 & b_3 & c_3 & d_3 & e_3 \end{matrix} \right\| = 0$$

is the same as the eliminant of

$$\frac{a_1u + a_2v + a_3w}{b_1u + b_2v + b_3w} = \frac{b_1u + b_2v + b_3w}{c_1u + c_2v + c_3w} = \frac{c_1u + c_2v + c_3w}{d_1u + d_2v + d_3w} = \frac{d_1u + d_2v + d_3w}{e_1u + e_2v + e_3w}.$$

4. In proceeding to attempt afresh the solution of Cayley's problem one naturally thinks first of the superfluity of data. We recall the fact* that, if of the five vanishing determinants we make a proper choice of two, the three others are negligible, and the further fact that such a choice is the first and the last, namely,

$$\left| \begin{matrix} x^4 & x^3y & x^2y^2 & xy^3 \\ a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{matrix} \right| = 0, \qquad \left| \begin{matrix} x^3y & x^2y^2 & xy^3 & y^4 \\ b_1 & c_1 & d_1 & e_1 \\ b_2 & c_2 & d_2 & e_2 \\ b_3 & c_3 & d_3 & e_3 \end{matrix} \right| = 0,$$

* *Hist.*, ii, p. 15.

as they have three columns in common. Then if we denote the ten 3-line determinants

$$|\, a_1b_2c_3 \,|,\ |\, a_1b_2d_3 \,|,\ |\, a_1b_2e_3 \,|,\ \ldots\ldots\ldots,\ |\, c_1d_2e_3 \,|$$

of the given 3-by-5 array of coefficients by

$$1',\ 2',\ 3',\ \ldots\ldots\ldots,\ 10'$$

these two equations are seen to be equivalent to

$$7'x^3 - 4'x^2y + 2'xy^2 - 1'y^3 = 0$$
$$10'x^3 - 9'x^2y + 8'xy^2 - 7'y^3 = 0\,;$$

and we thence at once have the resultant

$$\begin{vmatrix} \cdot & \cdot & 7' & -4' & 2' & -1' \\ \cdot & \cdot & 10' & -9' & 8' & -7' \\ \cdot & 7' & -4' & 2' & -1' & \cdot \\ \cdot & 10' & -9' & 8' & -7' & \cdot \\ 7' & -4' & 2' & -1' & \cdot & \cdot \\ 10' & -9' & 8' & -7' & \cdot & \cdot \end{vmatrix} = 0 \quad . \quad . \quad . \quad \text{(I)}$$

It has the disadvantage of containing an irrelevant factor $7'7'$, which is a little troublesome to remove. I have, however, gone through the necessary labour, obtaining the cofactor in the form of a term in $7'7'7'7'$, a term in $7'7'$, and a term independent of $7'$, the last somewhat lengthy. It at least establishes the existence of an eliminant of the material, if not of the elegance, desired.

5. The next mode of seeking a solution that occurs to one is to take the problem in Sylvester's form (§ 3 above), and apply a general theorem given by me in the *Transac. R. Soc. S. Africa,* ii, pp. 373–380, the appropriate case of which affirms that *the eliminant of*

$$\frac{a_1x + b_1y + c_1z}{\alpha_1x + \beta_1y + \gamma_1z} = \frac{a_2x + b_2y + c_2z}{\alpha_2x + \beta_2y + \gamma_2z} = \frac{a_3x + b_3y + c_3z}{\alpha_3x + \beta_3y + \gamma_3z} = \frac{a_4x + b_4y + c_4z}{\alpha_4x + \beta_4y + \gamma_4z}$$

is

$$\begin{vmatrix} |\,\alpha_1\beta_2\gamma_3\,| & \sum |\,a_1\beta_2\gamma_3\,| & \sum |\,a_1b_2c_3\,| & |\,a_1b_2c_3\,| \\ |\,\alpha_2\beta_3\gamma_4\,| & \sum |\,a_2\beta_3\gamma_4\,| & \sum |\,\alpha_2b_3c_4\,| & |\,a_2b_3c_4\,| \\ |\,\alpha_3\beta_4\gamma_1\,| & \sum |\,a_3\beta_4\gamma_1\,| & \sum |\,\alpha_3b_4c_1\,| & |\,a_3b_4c_1\,| \\ |\,\alpha_4\beta_1\gamma_2\,| & \sum |\,a_4\beta_1\gamma_2\,| & \sum |\,\alpha_4b_1c_2\,| & |\,a_4b_1c_2\,| \end{vmatrix} = 0.$$

Making in this the appropriate substitution we see at once that the first and fourth columns become simply

$$\begin{matrix} 7' & & 1' \\ 10' & \text{and} & 7' \\ 9' & & 4' \\ 8' & & 2'\,; \end{matrix}$$

but that the second and third are found to contain determinants of a hybrid character which require examination. Thus the first element of the second column is

$$\begin{vmatrix} a_1 & b_2 & b_3 \\ b_1 & c_2 & c_3 \\ c_1 & d_2 & d_3 \end{vmatrix} + \begin{vmatrix} b_1 & a_2 & b_3 \\ c_1 & b_2 & c_3 \\ d_1 & c_2 & d_3 \end{vmatrix} + \begin{vmatrix} b_1 & b_2 & a_3 \\ c_1 & c_2 & b_3 \\ d_1 & d_2 & c_3 \end{vmatrix}$$

where each determinant has a column from $| a_1 b_2 c_3 |$ and two columns from $| b_1 c_2 d_3 |$. Such hybrids however are not unknown, and we find we have only to expand them in terms of a_1, b_1, c_1, d_1 and complementaries to reduce their sum to $| a_1 c_2 d_3 |$ *i.e.* $4'$. This or a similar reduction being possible in every case, our two columns emerge in almost as simple a form as the previous two, and we are gladdened with the result

$$\begin{vmatrix} 7' & 4' & 2' & 1' \\ 10' & 9' & 8' & 7' \\ 9' & 6'+8' & 7'+5' & 4' \\ 8' & 7'+5' & 3'+4' & 2' \end{vmatrix}.$$

This is readily seen to take by transposition of rows and of columns the still more pleasing form

$$\begin{vmatrix} 1' & 2' & 4' & 7' \\ 2' & 3'+4' & 5'+7' & 8' \\ 4' & 5'+7' & 6'+8' & 9' \\ 7' & 8' & 9' & 10' \end{vmatrix} \quad . \quad . \quad . \quad . \quad . \qquad \text{(II)}$$

which is perhaps most helpfully viewed as the persymmetric

$$\begin{vmatrix} 1' & 2' & 4' & 7' \\ 2' & 4' & 7' & 8' \\ 4' & 7' & 8' & 9' \\ 7' & 8' & 9' & 10' \end{vmatrix} \text{ surmounted by } \begin{vmatrix} 3' & 5' \\ 5' & 6' \end{vmatrix}.$$

6. Having thus attained the desire we had on starting we naturally now wish for more, namely, a simpler mode of reaching our end: in fact, we want something still more, namely, four equations that are homogeneous and linear in four unknowns and have for their determinant our just-found eliminant. And seeing that we have already (§ 4) two such equations with eliminands x^3, x^2y, xy^2, y^3, we are not unreasonable in anticipating the existence of other two. A little examination confirms this. We have only to take Cayley's first and second equations

$$0 = \begin{vmatrix} x^4 & x^3y & x^2y^2 & xy^3 \\ a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{vmatrix}, \quad \begin{vmatrix} x^4 & x^3y & x^2y^2 & y^4 \\ a_1 & b_1 & c_1 & e_1 \\ a_2 & b_2 & c_2 & e_2 \\ a_3 & b_3 & c_3 & e_3 \end{vmatrix} = 0$$

perform division by x^2/y on the one and by x on the other, thus obtaining on development

$$7'x^2y - 4'xy^2 + 2'y^3 - 1'\frac{y^4}{x} = 0$$

$$8'x^3 - 5'x^2y + 3'xy^2 \qquad - 1'\frac{y^4}{x} = 0$$

whence subtraction gives us another of our desires, namely,

$$8'x^3 - (5' + 7')x^2y + (3' + 4')xy^2 - 2'y^3 = 0.$$

And quite similarly from his fourth and fifth equations we deduce

$$9'x^3 - (6' + 8')x^2y + (5' + 7')xy^2 - 4'y^3 = 0,$$

and so have all we want.

7. The law of formation of the new eliminant continues simple of application in the higher cases. For example, in the other case investigated by Cayley (see § 2 above), if we denote the primary minors $|\ a_1b_2c_3d_4\ |$, ... , $|\ c_1d_2e_3f_4\ |$ of its array of coefficients by their numbers in order 1 to 15, we have only to write these numbers column after column in the natural order as follows:—

$$\begin{matrix} 1 & 2 & 4 & 7 & 11 \\ & 3 & 5 & 8 & 12 \\ & & 6 & 9 & 13 \\ & & & 10 & 14 \\ & & & & 15 \end{matrix} ;$$

then complete the square in accordance with axisymmetry: and finally carry out the required *per*symmetry,—a proceeding which ought to have the same effect as imposing

$$\begin{matrix} 4 & 7 & 11 \\ 7 & 8+11 & 12 \\ 11 & 12 & 13 \end{matrix}$$

on the 3-line central minor of the axisymmetric square just reached.

8. The result thus got is

$$\begin{vmatrix} 1 & 2 & 4 & 7 & 11 \\ 2 & 3+4 & 5+7 & 8+11 & 12 \\ 4 & 5+7 & 6+8+11 & 9+12 & 13 \\ 7 & 8+11 & 9+12 & 10+13 & 14 \\ 11 & 12 & 13 & 14 & 15 \end{vmatrix} ;$$

and it is curious and useful to note that, if we delete from it all the numbers of the last column wherever they occur, we have left

$$\begin{array}{cccc} 1 & 2 & 4 & 7 \\ 2 & 3+4 & 5+7 & 8 \\ 4 & 5+7 & 6+8 & 9 \\ 7 & 8 & 9 & 10; \end{array}$$

that is to say, the square of the case below, if the numbers be now given the meanings proper to that case.

Further, we can similarly derive the next lower case by deleting the new 7, 8, 9, 10 wherever they occur,—a curious way of reaching the theorem that *the eliminant of*

$$\left\| \begin{array}{cccc} x^3 & x^2y & xy^2 & y^3 \\ a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \end{array} \right\| = 0$$

is

$$\left| \begin{array}{ccc} 1 & 2 & 4 \\ 2 & 3+4 & 5 \\ 4 & 5 & 6 \end{array} \right| = 0$$

where 1, 2, 3, 4, 5, 6 *stand for* $|a_1b_2|$, $|a_1c_2|$, ...

9. In conclusion, it seems desirable and not out of place to put on record a result obtained in dealing with the hybrid determinants of § 5. To this end it will be convenient to introduce a notation for families of such hybrids, writing

$$H(\phi, \psi)$$

for "the sum of the hybrids originating in ϕ and ψ": so that as an example we have from § 5

$$H(|a_1b_2c_3|, |b_1c_2d_3|) \equiv \left| \begin{array}{ccc} a_1 & b_2 & b_3 \\ b_1 & c_2 & c_3 \\ c_1 & d_2 & d_3 \end{array} \right| + \left| \begin{array}{ccc} b_1 & a_2 & b_3 \\ c_1 & b_2 & c_3 \\ d_1 & c_2 & d_3 \end{array} \right| + \left| \begin{array}{ccc} b_1 & b_2 & a_3 \\ c_1 & c_2 & b_3 \\ d_1 & d_2 & c_3 \end{array} \right|$$
$$= |a_1c_2d_3|.$$

In specifying the parents, if we may so call ϕ and ψ, a little extra care is necessary, because we have to remember that the equality of two determinants does not at all imply identity in parenthood: for example, although $|a_1b_2c_3|$ and $|c_1a_2b_3|$ are in the ordinary sense equal, not to say identical, yet the substitution of the latter for the former leads to a different result, namely,

$$H(|c_1a_2b_3|, |b_1c_2d_3|) = |b_1a_2d_3|.$$

10. Our theorem on the subject is:—*If from an* n-*by*-(n + 2) *array we take any two of its* n-*column determinants,* ϕ *and* ψ *say; and*

from them form a new series of determinants by substituting for one of ψ's rows the corresponding row of ϕ, then the sum of the said new series is equal to a sum of not more than n *of the series to which ϕ and ψ belong.*

For example, if n be 4, and ϕ be denoted in Cayley's manner by $hklm$ where h, k, l, m are the numbers of its columns in the given array, and ψ be similarly denoted by $xyzw$, the sum of the series of hybrids is equal to

$$hyzw + xkzw + xylw + xyzm.$$

Of course of these four determinants one or more may vanish on account of identity of two columns, for example

$$\begin{aligned} \mathrm{H}(1235,\ 2346) &= 1346 + 2246 + 2336 + 2345, \\ &= 1346 + 2345, \end{aligned}$$

this identity of columns being a consequence of the fact that we have only six (*i.e.* $n+2$) columns to choose from, and ϕ and ψ between them require eight (*i.e.* $2n$).*

* Cf. *Transac. R. Soc. S. Africa*, xviii, pp. 301-303

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XXI.—Studies in the Scottish Marine Fauna.—The Crustacea of the Sandy and Muddy Areas of the Tidal Zone.

By **R. Elmhirst,** F.L.S., Superintendent, Millport Marine Biological Station. *Communicated by* A. C. Stephen, B.Sc. (With Two Figures.)

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Introduction.

While collecting material for the study of the Molluscs and Polychætes of the sandy and muddy areas of the tidal zone (4, 5, and 6), Mr A. C. Stephen has usually preserved the Crustacea taken in his samples. For the opportunity of examining these crustaceans I am much indebted to him: they, together with a number of others, from standard ¼ square metre samples sieved in the Clyde area, have provided the material on which the present paper is based. Some areas have been sampled only once, and in these merely a very general idea of the crustacean fauna is possible, but in the proximity of the Millport Marine Laboratory a rather more detailed investigation has been made. The stations at which samples were taken are fully listed and described by Stephen (5), p. 301, Appendix I.

Most of the species taken belong to the order Amphipoda and the results are perhaps best seen by dealing with the species separately. The three genera most prevalent in the tidal sands all belong to the family Haustoriidæ, but *Corophium* is the dominant genus in muddy places. The nomenclature used is that of Stebbing in Das Tierreich, Amphipoda, 1906.

Notes on the Dominant Species.

Amphipoda.

Table I shows that twenty species occur, most of them casually, in Kames Bay, Millport. However, fig. 1 shows that there is a strong tendency for the prevalent species to inhabit definite zones and to have their "maximum concentration at some particular level" just as was found for the molluscs and polychætes (Stephen (5), p. 296).

Bathyporeia pelagica. In Kames Bay this species is most frequent near high-water mark, especially in summer, but it also occurs in small

numbers down to about two fathoms. In Loch Gilp (fig. 2 and Table II) it occurs at one station just beyond low-water mark. This striking difference of position seems to be due to marked differences in the ground and fresh-water inflow in the two areas. Samples from Loch Riddon, Turnberry, Prestwick, North Berwick, Carnoustie, Kilconquhar, St Andrews, Nairn, Aberdeen, and Barra, all show this species as present in small numbers.

B. guilliamsoniana. In Kames Bay, Loch Gilp, Balloch Bay, and Fairlie Sands this species is the dominant low-water form, beginning about mid-tide level and extending seawards to about four fathoms with a

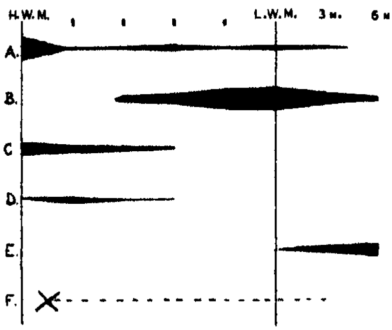

FIG. 1.—Kames Bay. Figure showing the distribution and relative frequency of various species at each station.

A. *Bathyporeia pelagica.*
B. *B. guilliamsoniana.*
C. *Haustorius arenarius.*
D. *Eurydice pulchra.*
E. *Ampelisca lævigata.*
F. *Pontocrates norvegicus.*

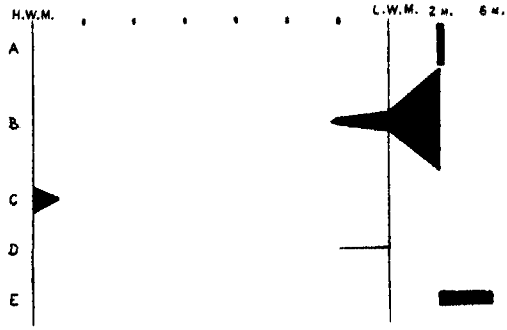

FIG. 2.—Loch Gilp.

A. *Bathyporeia pelagica.*
B. *B. guilliamsoniana.*
C. *Corophium volutator.*
D. *C. crassicorne.*
E. *Ampelisca lævigata.*

definite maximum about low water. The sudden cessation of this species about one fathom in Loch Gilp is very striking. In Loch Riddon, on the Ayrshire coast, at Nairn, St Andrews, and Barra this species occurs in rather larger numbers than *B. pelagica.*

B. pilosa. Apparently absent from the Clyde Sea area, but occurs in small numbers at Aberlady, St Andrews, Kilconquhar, and Nairn, and in considerable numbers near high-water mark in the last locality. The occurrence of this Baltic species in the east coast estuaries is interesting.

B. robertsoni. Occurs from August to October at all stations in Kames Bay. The fact that all the specimens are immature males would seem to confirm the view held in turn by Meinert (2), Blanc (1), and Stebbing (3) that *robertsoni* is the immature male of *pelagica.* The whole question of the species of *Bathyporeia* needs careful and experimental investigation; there is considerable support for the view that

there is one rather variable species showing the form *pilosa* in brackish areas, *pelagica* in clean sand in the upper part of the intertidal area, and *guilliamsoniana* about low-water mark and in sandy shallows.

Haustorius arenarius. Usually confined to the upper half of the intertidal area and prefers the rather loose sands which often occur high up the beach as a result of wind- and wave-action. Comparing fig. 1, C with fig. 1, B, p. 296, Stephen (5), it will be seen that this species has a distribution coinciding with that of *Nerine foliosa.* It also occurs in small numbers on Ayrshire coast, Balloch Bay, Carnoustie, Nairn, and Aberdeen.

Urothoe marina. Present in small numbers in Kames Bay and Loch Gilp. Frequent in Balloch Bay and at Turnberry; it prefers clean, rather coarse sands where there are some seaweed-clad boulders. In such situations it is often associated with *Corophium crassicorne* and *B. guilliamsoniana.*

Phoxocephalus holbolli. Occurs occasionally from mid-tide to a few fathoms: Balloch Bay, Kames Bay.

Ampelisca. This genus is represented by *A. lævigata* which begins near low-water mark and extends to a few fathoms, where it is succeeded by *A. tenuicornis* (Table II). Taken in Kames Bay, Loch Gilp, Fairlie Sands, and Aberdeen area.

Pontocrates norvegicus. Usually present in clean sands in small numbers: Kames Bay, Ayrshire coast, Carnoustie, Nairn, St Andrews, and Aberdeen. While never plentiful in Kames Bay, night townettings reveal that this species ranges up to high-water mark as a common night tidal-migrant, when it is found in numbers (*e.g.* 286 on 12th Dec. 1930) associated with *Gammarus locusta* (9) and *Schistomysis spiritus* (54), both species which occur usually beyond low-water mark. This nightly migration is indicated by × in fig. 1.

Corophium. C. crassicorne is a typical low-water mark species occurring in Balloch Bay, Fairlie Sands, Loch Gilp, and other parts of the Clyde not included in the areas sampled. The closely allied *C. volutator* is a typical inhabitant of muddy area in the brackish high-water region—Loch Gilp, Balloch Bay, Fairlie Sands, loch heads, and estuarine waters generally.

Isopoda.

Eurydice pulchra. In Kames Bay sand this species has the same distribution as the Polychæte *Eteone flava* (*cf.* fig. 1, D and fig. 1, C, p. 296, Stephen (5)). In general it occurs about and above mid-tide in the Clyde, Aberlady, Lossiemouth, Nairn, and Vatersay. In winter it seeks shelter in

the shallow water beyond low-water springs. In summer large numbers follow the flotsam at the edge of the tide.

Cumacea.

Very few cumaceans occurred in the sand samples, and those only in the Clyde.

Iphinoe trispinosa. Balloch Bay, low-water mark; Kames Bay, low-water mark to 6 metres, and Hunterston Sands.

Lamprops fasciata. Balloch Bay, mid-tide to low-water mark; Loch Gilp, low-water mark to 1 fathom.

Pseudocuma cercaria. Occurs in Kames Bay, particularly in the region round large boulders attracted by the debris which gathers there, but was not present in any of the sand samples.

Mysidacea.

Neomysis vulgaris. Present in pools in Loch Gilp, and occurs in swarms at loch heads—West Loch Tarbert, Loch Gilp, Loch Riddon, and the brackish area behind Fairlie Sands, etc.

Schistomysis spiritus. A common tidal-migrant into tidal sandy areas; see under *Pontocrates.*

Macromysis flexuosus. Also occurs as a tidal migrant.

Decapoda.

Crangon vulgaris. Rare in Kames Bay but common on Fairlie Sands, and present in all parts of the Clyde. Also taken at Aberdeen, Tiree, and Barra.

Portumnus latipes. Small ones taken at Gullane, Aberlady, and St Andrews.

Portunus arcuatus. A few taken near high-water mark at Barnbougle Castle. Also occurs on the shore at Fairlie and Balloch Bay.

Carcinus mœnas. Though never actually present in any sieved samples, this species frequents all sandy areas in the Clyde, and is often found buried some inches deep in the sand.

TABLE I.—KAMES BAY. SEPTEMBER 1926 TO OCTOBER 1927.

Station Nos.	1.		2.		3.		4.		5.		6.		7.		8.	
No. of Samples Taken.	6.		6.		6.		7.		7.		5.		5.		4.	
Bathyporeia pelagica	4	106	1	2	3	6	5	43	4	12	3	25	2	9	..	..
B. guilliamsoniana	..	..	..	..	2	19	6	49	6	116	5	93	5	44	4	15
B. robertsoni	1	1	2	2	..	..	..	..	2	2	..	..	1	1	..	..
Haustorius arenarius	5	55	2	17	6	34	2	4	..	..	..	..	..	..	..	..
Urothoe marinus	..	..	..	..	1	1	..	..	..	..	..	..	..	..	..	..
Phoxocephalus holbolli	..	..	..	..	..	..	1	1	..	..	..	..	..	..	..	..
Ampelisca lævigata	..	..	..	..	..	..	..	..	..	..	2	6	3	19	3	49
A. macrocephala	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	2
Perioculodes longimanus	..	..	..	..	..	..	..	..	..	..	..	..	1	1	..	..
Pontocrates norvegicus	..	..	3	6	1	1	3	7	4	7	3	8	2	4	..	..
Calliopius rathkei	..	..	..	..	..	..	..	..	..	..	..	..	1	1	..	..
Paratylus swammerdami	..	..	..	..	..	..	..	..	1	1	1	1	2	16	1	1
Gammarus locusta	..	..	..	..	..	..	..	..	1	1	..	..	1	1	..	..
Microdeutopus	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	1
Podocerus	..	..	..	..	..	..	..	..	..	..	..	..	2	6	..	..
Siphonœcetes	..	..	..	..	..	..	..	..	..	..	..	..	1	1	..	..
Eurydice pulchra	2	2	6	16	5	5	1	1	..	..	..	..	..	..	..	..
Idothea linearis	..	..	..	..	..	..	..	..	..	..	..	..	1	1	1	1
Crangon vulgaris	..	..	..	..	..	..	1	1	..	..	..	..	..	..	..	..
Portunus depurator	..	..	..	..	..	..	..	..	..	..	..	..	1	3	..	..

TABLE II.—LOCH GILP.

	Nos. per ¼ sq. m.								Actual Nos. Dredged.		
	Stations.								1 fm.	2½ fm.	4½ fm.
	1.	2.	3.	4.	5.	6.	7.	8.			
Corophium volutator	25	..	..	..	..	..	..	..	..	..	..
C. crassicorne	..	..	..	..	..	..	1	..	2	..	..
Bathyporeia guilliamsoniana	..	..	..	..	..	..	7	19	103	..	..
B. pelagica	..	..	..	..	..	..	..	..	40	..	..
Gammarus locusta	..	..	..	..	..	..	1	..	9	..	..
Urothoe marina	..	..	..	..	..	..	..	1	..	..	..
Ampelisca lævigata	..	..	..	..	..	..	..	..	9	9	..
A. tenuicornis	..	..	..	..	..	..	..	..	..	9	99
Phoxocephalus holboli	..	..	..	..	..	..	..	..	..	..	6
Perioculodes longimanus	..	..	..	..	..	..	..	..	13	..	..
Lamprops fasciata	..	..	..	..	..	..	1	..	3	..	..

Table III.—Ayrshire Coast.

	Ayrshire Coast—1. Lendalfoot, etc. 8 stations.		Ayrshire Coast—2. Turnberry. 4 stations.		Ayrshire Coast—3. Prestwick. 5 stations.		Ayrshire Coast—4. Barassie. 2 stations.	
Bathyporeia guilliamsoniana .	1	3	2	7	1	1	1	5
B. pelagica	..	..	2	7	2	2	..	..
Urothoe marina . . .	..	..	4	42	1	1	..	..
Haustorius arenarius . .	..	..	1	1	4	5	1	1
Pontocrates norvegicus . .	..	..	..	..	1	1	..	..

Table IV.—Nairn Area.

	1. W. of Nairn. 9 stations		2. Sands E. of Nairn Bar.								3. Lossie-mouth. 2 stations.	
			L.W.M. 5 stations.		¼ way up Beach. 1 station.		¾ way up Beach. 1 station.		H.W.M. 1 station.			
Bathyporeia guilliamsoniana	2	12	1	1	..	..	..	..	..	..	1	1
B. pelagica	4	5	..	..	..	..	..	..	..	..	..	..
B. pilosa	2	2	3	7	1	2	..	..	1	44	..	..
Haustorius arenarius . .	..	..	1	2	..	..	..	..	..	..	2	5
Pontocrates norvegicus .	1	1	..	..	..	..	..	..	..	..	1	1
Eurydice pulchra . .	..	..	..	..	..	..	..	..	..	..	2	3

Table V.—Aberdeen Area.

	3 stations.	
Bathyporeia pelagica	1	3
Haustorius arenarius	2	4
Pontocrates norvegicus	3	27
Ampelisca (damaged)	1	1

Summary.

1. An attempt is made to indicate the quantitative distribution of the Crustaceans inhabiting certain Scottish intertidal areas.

2. Certain areas in the Clyde, notably Cumbrae, show a rich Amphipodan fauna which exhibits zoning by species.

3. *Bathyporeia guilliamsoniana* is the dominant amphipod on the

Tellina-Nephthys grounds (clean sand) and *Corophium volutator* is indicative of a muddy to brackish environment.

REFERENCES.

(1) Blanc, H., 1884. *Nova Acta K. Leop-Carol. Akad. Naturf.*, Bd. xlvii, pp. 39–96.

(2) Meinert, F. V. A., 1877. *Naturh. Tidsskr.*, Bd. xi, pp. 57–248.

(3) Stebbing, T. R. R., 1888. "*Challenger*" *Reports*, vol. xxix, p. 334.

(4) Stephen, A. C., 1928. *Proc. Roy. Phys. Soc. Edin.*, vol. xxi, pp. 205–216.

(5) Stephen, A. C., 1929. *Trans. Roy. Soc. Edin.*, vol. lvi, pp. 291–306.

(6) Stephen, A. C., 1930. *Ibid.*, pp. 521–535.

Fig. 1 is a supplement to	Fig. 1, p. 296,	vol. lvi,	*Trans.*	*Roy.*	*Soc.*	*Edin.*
„ 2 „ „ „	„ 1, p. 525,	„	„	„	„
Table I „ „ „	Table III, p. 304,	„	„	„	„
„ II „ „ „	„ III, p. 523,	„	„	„	„
„ III „ „ „	„ II, p. 303,	„	„	„	„
„ IV „ „ „	„ V, p. 305,	„	„	„	„
„ V „ „ „	„ VI, p. 305,	„	„	„	„

The tables show the number of samples taken at each station and the frequency per ¼ square metre is indicated by the figures in the columns; the left-hand column shows the number of times the species are present in the samples, and the right-hand column gives the total number of individuals taken. Loch Gilp, Table II., was only visited once.

(*Issued separately November* 23, 1931.)

XXII.—An Operational Method for the Solution of Linear Partial Differential Equations. By W. O. Kermack and W. H. McCrea.

(MS. received July 21, 1931. Read November 2, 1931.)

Introduction.

1. A general method for the solution of differential equations by definite integrals has recently been given by Professor E. T. Whittaker.* It is briefly that, if a contact transformation from variables (q, p) to (Q, P) be given by $Q = \mathbf{Q}(q, p)$, $P = \mathbf{P}(q, p)$, and if this transforms an expression $G(Q, P)$ into $F(q, p)$, then the solutions of the differential equations

$$G(Q, d/dQ)\phi = 0 \quad . \quad . \quad . \quad . \quad . \quad (1.1)$$

$$F(q, -d/dq)\psi = 0 \quad . \quad . \quad . \quad . \quad . \quad (1.2)$$

are connected by a relation of the form

$$\psi(q) = \int \chi(q, Q)\phi(Q)dQ. \quad . \quad . \quad . \quad . \quad . \quad (1.3)$$

Here $\chi(q, Q)$ is a solution of the pair of equations

$$Q\chi = \mathbf{Q}(q, -\partial/\partial q)\chi, \qquad \partial\chi/\partial Q = \mathbf{P}(q, -\partial/\partial q)\chi. \quad . \quad . \quad . \quad (1.4)$$

The present authors have discussed the theory of the method in two papers,† and in particular have given rules for writing equations (1.4) in compatible form and for the derivation of $F(q, p)$ from $G(Q, P)$.

The theorem enunciated in the present paper derives a solution of equation (1.2) from the solution of (1.1) by a method which involves only differentiation. This solution, though different in form, is in fact closely related to the solution in definite integrals given by Professor Whittaker. We give two methods of proof, neither of which claims to be complete. There is much more that requires investigation about the nature of the functions for which the result holds and the paths of integration used to obtain it, and about its degree of generality when obtained. We do not attempt this investigation, but pass on to applications of the general theorem.

As it appears, Maclaurin's Theorem is a very simple special case of the new theorem. The latter may be regarded as a generalisation of

* Whittaker, *Proc. Edinburgh Math. Soc.*, **2**, II (1931), p. 189.

† Kermack and McCrea, *ibid.*, pp. 205, 220. These will be referred to as Papers I and II.

Maclaurin's Theorem. The new theorem also yields very readily the result recently given by R. A. Fisher relating to the transformation of moment generating functions.* Further, it also turns out to have considerable bearing on the theory of generating functions in general.

The General Theorem.

2. Theorem.—*Consider a contact transformation from the variables* $(q_1, q_2, \ldots q_n, p_1, p_2, \ldots p_n)$ *to the variables* $(Q_1, Q_2, \ldots Q_n, P_1, P_2, \ldots P_n)$ *derived from a function* $W(q_1, q_2, \ldots q_n, P_1, P_2, \ldots P_n)$, *so that*

$$Q_r = \frac{\partial W}{\partial P_r} \qquad p_r = \frac{\partial W}{\partial q_r} \qquad (r = 1, 2, \ldots n) \quad . \quad . \quad . \quad (2.01)$$

and suppose that the variables Q_r, P_r when expressed in terms of q_r, p_r are denoted by $\mathbf{Q}_r(q_1, q_2, \ldots q_n, p_1, p_2, \ldots p_n)$, $\mathbf{P}_r(q_1, q_2, \ldots q_n, p_1, p_2, \ldots p_n)$. *Suppose a set of* n *compatible partial differential equations for the function* $\phi(Q_1, Q_2, \ldots Q_n)$ *be given in the form*

$$G_r(Q_1, Q_2, \ldots Q_n, P_1, P_2, \ldots P_n)\phi = 0 \qquad (r = 1, 2, \ldots n) \quad . \quad (2.02)$$

where $P_r = \dfrac{\partial}{\partial Q_r}$. *Then the* n *equations obtained by substituting* $\mathbf{Q}_r(q_1, q_2, \ldots p_n)$ *for* Q_r, *and* $-\mathbf{P}_r(q_1, q_2, \ldots p_n)$ *for* P_r *giving*, say,

$$F_r(q_1, q_2, \ldots q_n, p_1, p_2, \ldots p_n)\psi = 0 \qquad (r = 1, 2, \ldots n) \quad . \quad (2.03)$$

where $p_r = -\dfrac{\partial}{\partial q_r}$ *are compatible and have the solution*

$$\psi(q_1, q_2, \ldots q_n) = e^{-W(q_1, q_2, \ldots, q_n, -\partial/\partial Q_{10}, \ldots -\partial/\partial Q_{n0})}\phi(Q_1 \ldots Q_n) \quad (2.04)$$

where the suffix zero means that after performing the operations indicated on the right-hand side the variables $Q_1, Q_2, \ldots Q_n$ are put equal to zero.

It will be sufficient to write out the proof for a single pair of variables (q, p) or (Q, P). We may write then G for G_r, F for F_r.

The general theorem of Paper I shows that the solution of the equation

$$F(q, p)\psi = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.05)$$

is of the type

$$\psi(q) = \int_C \chi(q, Q)\phi(Q)dQ \quad . \quad . \quad . \quad . \quad . \quad (2.06)$$

where C is a suitable contour and where $\chi(q, Q)$ is the solution of the pair of equations

$$\left[Q - \frac{\partial W}{\partial P}\right]\chi = 0 \qquad \left[p - \frac{\partial W}{\partial q}\right]\chi = 0 \quad . \quad . \quad . \quad (2.07)$$

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* R. A. Fisher, *Proc. London Math. Soc.*, **30** (1930), p. 199. *Cf.* §10, p. 226.

These we shall write as

$$Q\chi = W_P\left(q, \frac{\partial}{\partial Q}\right) \cdot \chi \qquad -\frac{\partial \chi}{\partial q} = W_q\left(q, \frac{\partial}{\partial Q}\right)\chi, \qquad \text{(2.08)}$$

using suffixes to denote derivatives of W. The solution is obtained at once by means of a Laplace transformation. For they are satisfied by

$$\chi(q, Q) = \int_\Gamma \rho(q, s) e^{-sQ} ds \qquad \text{(2.09)}$$

for a suitable path of integration Γ, if the function $\rho(q, s)$ satisfies the equations obtained from (2.08) by writing $+\frac{\partial}{\partial s}$ for Q and $-s$ for $\frac{\partial}{\partial Q}$, namely

$$\frac{\partial}{\partial s}\rho = -W_s(q, -s)\rho \qquad -\frac{\partial}{\partial q}\rho = W_q(q, -s)\rho.$$

We see at once that

$$\rho(q, s) = e^{-W(q, -s)}$$

and hence

$$\chi(q, Q) = \int_\Gamma e^{-W(q, -s) - sQ} ds \qquad \text{(2.10)}$$

$$= \int_\Gamma e^{-W(q, d/dQ)} e^{-sQ} ds \qquad \text{(2.11)}$$

introducing an operational form.

Substituting in (2.06) we have

$$\psi(q) = \int_C dQ \int_\Gamma ds \phi(Q) e^{-W(q, d/dQ)} e^{-sQ}. \qquad \text{(2.12)}$$

We now suppose that $e^{-W(q, d/dQ)}$ can be expanded in the form

$$e^{-W(q, d/dQ)} = \sum_{n=0}^{\infty} a_n(q)(d/dQ)^n \qquad \text{(2.13)}$$

and we consider in (2.12) the term in

$$\int_C \phi(Q) \left(\frac{d}{dQ}\right)^n e^{-sQ} dQ,$$

assuming that the order of integration may be interchanged. Integrating by parts n times we obtain

$$\int_C \phi(Q)\left(\frac{d}{dQ}\right)^n e^{-sQ} dQ = \left[\phi.\left(\frac{d}{dQ}\right)^{n-1} e^{-sQ} - \phi'.\left(\frac{d}{dQ}\right)^{n-2} e^{-sQ} + \ldots\right]_C + (-)^n \int_C e^{-sQ} \phi^{(n)} d\phi.$$

Hence, summing over all values of n, we obtain the formal result

$$\int_C \phi e^{-W(q, d/dQ)} e^{-sQ} dQ = \int_C e^{-sQ} e^{-W(q, -d/dQ)} \phi dQ \qquad \text{(2.14)}$$

together with the sum of the integrated parts. We shall now assume that the contour C and the function ϕ are such that the integrated part vanishes.

This is the case, for example, if ϕ and its derivatives have no branch points inside C, and have no singularities on C.

We now have

$$\psi(q) = \int_\Gamma ds \int_C e^{-sQ} e^{-W(q,\ -d/dQ)} \phi dQ$$
$$= \int_C \frac{[-e^{-sQ}]_\Gamma}{Q} e^{-W(q,\ -d/dQ)} \phi dQ$$
$$= \left\{ [-e^{sQ}]_\Gamma \,.\, e^{-W(q,\ -d/dQ)} \phi \right\}_{Q=0}$$
$$= \text{const.}\quad e^{-W(q,\ -d/dQ_0)} \phi(Q), \qquad . \quad . \quad . \quad (2.15)$$

provided the point $Q=0$ lies inside the contour C, and provided the function $e^{-W(q,\ -d/dQ)}\phi(Q)$ has no other singularities in or on C, and provided the contour Γ is not closed. Since the equation (2.05) is linear, we may omit the constant factor. We have now obtained the required result.

3. An alternative proof depends on the following Lemma.

Lemma.—If K(d/dt) *is a given function of the operator* d/dt *and if* f(t) *is any function of* t, satisfying appropriate differentiability and other conditions, *then the result of* K *operating on* f(t) *is given by*

$$K(d/dt)f(t) = \int\int K(s) f(\tau) e^{s(t-\tau)} ds d\tau, \qquad . \quad . \quad . \quad . \quad (3.1)$$

when the integrals are taken along suitable contours, as described below.

Let $f(t)$ be the Laplace transform of a function $g(s)$ with respect to a contour D, so that

$$f(t) = \int_D g(s) e^{st} ds. \qquad . \quad . \quad . \quad . \quad . \quad . \quad (3.2)$$

Then we have

$$g(s) = \int_{D'} f(\tau) e^{-s\tau} d\tau, \qquad . \quad . \quad . \quad . \quad . \quad . \quad (3.3)$$

where D′ is the conjugate contour.

From (3.2), provided the integral converges in a suitable manner,

$$K(d/dt)f(t) = \int_D K(d/dt) g(s) e^{st} ds$$
$$= \int_D K(s) g(s) e^{st} ds$$
$$= \int_D ds \int_{D'} d\tau K(s) f(\tau) e^{s(t-\tau)}, \qquad . \quad . \quad . \quad (3.4)$$

from (3.3). This is the Lemma. The form we require is derived by letting t tend to zero, assuming the integral uniformly convergent in t in the neighbourhood of $t=0$, giving

$$K(d/dt_0)f(t) = \int_D \int_{D'} K(s) f(\tau) e^{-s\tau} d\tau ds. \qquad . \quad . \quad . \quad (3.5)$$

Writing now the value of χ given by (2.10) in (2.06) we have

$$\psi(q)=\int_C\int_\Gamma e^{-W(q,\,-s)}\phi(Q)e^{-sQ}ds\,dQ. \qquad (3.6)$$

Comparing (3.5), (3.6) we see that by taking

$$K(s)=e^{-W(q,\,-s)} \qquad (3.7)$$

the right-hand sides become identical, provided the contours C and Γ may be made to agree with D and D′ respectively. This identification gives immediately

$$\psi(q)=K\left(\frac{d}{dt_0}\right)\phi(t)=e^{-W(q,\,-d/dQ_0)}\phi(Q), \qquad (3.8)$$

which is the required theorem.

The difficulty of this method is to justify the identification of the contours. The first method seems to make more evident the kind of condition the contours must satisfy.

It is not easy to state the degree of generality of the result. The order of the differential equation for ψ is in general different from that of the equation for ϕ. The equations (2.01) may possess more than one solution, in which case, corresponding to a given $W(q, P)$ and $F(q, p)$, there may exist more than one differential equation for a function ϕ. On the other hand the operation $e^{-W(q,\,-d/dQ_0)}$ may remove a certain number of arbitrary constants in ϕ, as will certainly happen if F is of order lower than G.

4. *An Extension of the Theorem.*—If in the theorem we write

$$W(q, P)=U(q, P)+aP \qquad (a \text{ constant}) \qquad (4.1)$$

the transformation becomes

$$Q=\partial U/\partial P+a, \qquad p=\partial U/\partial q, \qquad (4.2)$$

and in place of (2.15) we find

$$\begin{aligned}\psi(q)&=\int_\Gamma ds\int_C e^{-s(Q-a)}e^{-U(q,\,-d/dQ)}\phi(Q)dQ\\ &=\text{const.}\left[e^{-U(q,\,-d/dQ)}\phi(Q)\right]_{Q=a} \qquad (4.3)\end{aligned}$$

where the expression (4.3) is evaluated at $Q=a$. This is a slightly more general result.

5. *Generalisation.*—Much of the theory of Papers I and II can be taken over directly into the present work.

Let us suppose we have two conjugate functions $R(q,p)$, $S(q,p)$ such that $RS-SR=1$. Then if we form the equations

$$[P-R(q,\ -\partial/\partial q)]\sigma=0 \qquad (5.1)$$

$$[\partial/\partial P-S(q,\ -\partial/\partial q)]\sigma=0 \qquad (5.2)$$

it follows from Paper II, Theorem XIII, that they are compatible and have the solution $\sigma(q, \mathrm{P})$ uniquely defined, apart from a constant factor. Then the proposition is that *if we substitute in any differential equation*

$$\mathrm{G}(\mathrm{Q}, d/d\mathrm{Q})\phi = 0 \qquad (5.3)$$

S$(q, -d/dq)$ *for* Q, *and* R$(q, -d/dq)$ *for* P, *then the solution of the resulting equation is given by*

$$\psi(q) = \sigma(q, \mathrm{P}_0)\phi(\mathrm{Q}). \qquad (5.4)$$

Further, if R and S are conjugate functions of q, p, then it follows by the method of Paper I, § 4.1, that the equations

$$[\Pi \mathrm{P} \Pi^{-1} - \varpi \mathrm{R}(q, p)\varpi^{-1}]\sigma^* = 0 \qquad (5.5)$$

$$[\Pi \mathrm{Q} \Pi^{-1} - \varpi \mathrm{S}(q, p)\varpi^{-1}]\sigma^* = 0 \qquad (5.6)$$

with $p = -\partial/\partial q$, $\mathrm{Q} = \partial/\partial \mathrm{P}$, and where $\Pi(\mathrm{Q}, \mathrm{P})$, $\varpi(q, p)$ are any functions of Q, P and q, p respectively, are compatible and have the solution

$$\sigma^* = \varpi \Pi \sigma(q, \mathrm{P}) \qquad (5.7)$$

and that (5.5), (5.6) may be taken as defining the transformation from (q, p) to (Q, P). The proofs follow the methods of the previous papers.

A simple example will serve to illustrate the point. Take $\Pi = e^{a\mathrm{P}}$; then we have

$$\Pi \mathrm{P} \Pi^{-1} = \mathrm{P} \quad \text{and} \quad \Pi \mathrm{Q} \Pi^{-1} = e^{a\mathrm{P}} \mathrm{Q} e^{-a\mathrm{P}} = \mathrm{Q} + a \,. \qquad (5.8)$$

and

$$\sigma^* = e^{a\mathrm{P}}\sigma(q, \mathrm{P})$$

Then the theorem gives

$$\begin{aligned} \psi(q) &= \sigma^*(q, \mathrm{P}_0)\phi^*(\mathrm{Q}) \\ &= [\sigma(q, \mathrm{P})e^{+a\mathrm{P}}\phi^*(\mathrm{Q})]_{\mathrm{Q}=0} \\ &= \sigma(q, \mathrm{P}_0)\phi^*(\mathrm{Q} + a) \\ &= [\sigma(q, \mathrm{P})\phi^*(\mathrm{Q})]_{\mathrm{Q}=a}. \end{aligned} \qquad (5.9)$$

Here $\phi^*(\mathrm{Q})$ is the function which leads to $\psi(q)$ on the *new* transformation obtained by using equations (5.8). This result is identical with (4.3).

Special Cases.

6. Take $\mathrm{W} = q\mathrm{P}$. Then the transformation (2.01) reduces to

$$\mathrm{Q} = q, \qquad \mathrm{P} = p \qquad (6.1)$$

i.e. the identical transformation, and the general result (2.04) becomes

$$\psi(q) = e^{q\,d/d\mathrm{Q}_0}\psi(\mathrm{Q}) \qquad (6.2)$$

which is just the symbolic expression of *Maclaurin's Theorem*.

If we again take $\mathrm{W} = q\mathrm{P}$, but use the theorem in the form (4.3), we obtain

$$\psi(q) = [e^{(q-a)d/d\mathrm{Q}}\psi(\mathrm{Q})]_{\mathrm{Q}=a}$$

which, by an obvious change of variables, becomes

$$\psi(x+h)=e^{hd/dx}\psi(x),$$

and this is *Taylor's Theorem.*

The new theorem may, therefore, be looked upon as a kind of generalisation of Maclaurin's or Taylor's Theorem. The latter give, under certain conditions, an expansion of a given function $\psi(q)$ in a series of powers of q; the new theorem gives, under certain conditions, an expansion in a *series of functions of* q.

To obtain this expansion, it is not in general necessary to solve the differential equation for $\phi(Q)$. For if this equation is of order n, we may in general give ϕ, ϕ', . . . $\phi^{(n-1)}$ arbitrary values at $Q=0$. Then by differentiating the equation $(m-n)$ times, we can express $\phi^{(m)}$, for $Q=0$, in terms of these values. This is all that is required for evaluating the right-hand side of (2·04).

7. *Proof of a Theorem due to R. A. Fisher.*—If $f(x)$ is the *frequency function* for a variable x, then $f(x)dx$ is the probability that x lies in the range $(x, x+dx)$, and the corresponding *moment generating function* $\phi(Q)$ is defined as

$$\phi(Q)=\int_{-\infty}^{\infty}e^{Qx}f(x)dx. \qquad (7.1)$$

It is therefore just the Laplace-transform of $f(x)$. Hence if we write y for d/dx and P for $-d/dQ$ the contact transformation from (x, y) to (Q, P) is given by

$$x=-P, \qquad y=Q. \qquad (7.2)$$

If now we change the variable from x to ξ where

$$\xi=T(x), \qquad x=S(\xi), \qquad (7.3)$$

then the frequency function in ξ is $f\{S(\xi)\}S'(\xi)$. Hence the corresponding moment generating function is $\psi(q)$, where

$$\begin{aligned}\psi(q)&=\int_{-\infty}^{\infty}e^{q\xi}f\{S(\xi)\}S'(\xi)d\xi\\&=\int_{-\infty}^{\infty}e^{qT(x)}f(x)dx \qquad (7.4)\end{aligned}$$

on changing the variable from ξ to x. If we write p for $-d/dq$ it follows that the contact transformation from (x, y) to (q, p) is obtained by taking the function $W(q, Q)$ of Paper I to be $qT(Q)$, which gives

$$T(x)=-p, \qquad y=qT'(x). \qquad (7.5)$$

Therefore the contact transformation * from (q, p) to (Q, P) is, from (7.2), (7.5),

$$p = -T(-P), \qquad Q = qT'(-P). \qquad (7.6)$$

These relations are obviously derivable by equations (2.01) from a function $W(q, P)$ given by

$$W(q, P) = -qT(-P). \qquad (7.7)$$

Hence by the theorem of this paper we have

$$\begin{aligned}\psi(q) &= e^{-W(q, -d/dQ_0)}\phi(Q)\\ &= e^{qT(d/dQ_0)}\phi(Q). \qquad (7.8)\end{aligned}$$

This is precisely Fisher's rule (*loc. cit.*) for the transformation of the moment generating function.

8. As a further special form of W, consider that given by

$$e^{-W(q, -P)} = \sum_{n=0}^{\infty} a_n(q)P^n = H(q, P), \quad \text{say}. \qquad (8.1)$$

Take $\phi(Q) = e^Q$, and $\psi(q)$ becomes by (2.04)

$$\begin{aligned}\psi(q) &= \left[\sum_{n=0}^{\infty} a_n(q)P_o{}^n\right]e^Q\\ &= \sum_{n=0}^{\infty} a_n(q) \qquad (8.2)\\ &= H(q, 1) \qquad (8.3)\end{aligned}$$

using the usual result that $f(P)e^{aQ} = e^{aQ}f(a)$ for constant a for any function $f(P)$ of the operator P.

Now the transformation is given by

$$W(q, P) = -\log H(q, -P) \qquad (8.4)$$

from which we obtain P as a function of q, p in the usual way as

$$P = P(q, p). \qquad (8.5)$$

But the differential equation satisfied by $\phi(Q) = e^Q$ is

$$(P - 1)\phi = 0.$$

Whence it follows from the rules of Paper I that the differential equation satisfied by $\psi(q)$ given by (8.2) is

$$\{P(q, p) + 1\}\psi = 0. \qquad (8.6)$$

We have here *a method for obtaining a differential equation satisfied by a given series of functions* $\sum a_n(q)$.

* Once we have everything expressed as contact transformations it does not matter if we sometimes interpret P (say) as $-d/dQ$ and sometimes as d/dQ, provided we follow consistently the rules of Paper I in going from one differential equation to another.

9. The theorem allows us formally to reduce the solution of any differential equation (order m, say)

$$F(q, p)\psi = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (9.1)$$

to a simple quadrature, together with the solution of a problem in non-commutative algebra. For suppose the corresponding ϕ equation is

$$(P - 1)\phi = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (9.2)$$

then the contact transformation must give

$$P = F(q, p) - 1. \quad . \quad . \quad . \quad . \quad . \quad . \quad (9.3)$$

This is an algebraic (non-commutative) equation for p giving m roots of the form

$$p = \mathrm{p}(q, P). \quad . \quad . \quad . \quad . \quad . \quad . \quad (9.4)$$

This transformation can then be derived from a function $W(q, P)$, where

$$\frac{\partial W}{\partial q} = \mathrm{p}(q, P) \quad . \quad . \quad . \quad . \quad . \quad . \quad (9.5)$$

giving

$$\begin{aligned} W(q, P) &= \int \mathrm{p}(q, P)dq + u(P) \\ &= w(q, P) + u(P), \quad \text{say} \quad . \quad . \quad . \quad . \quad (9.6) \end{aligned}$$

where $u(P)$ is an arbitrary function of P, and w is given by the quadrature indicated. But (9.2) gives $\phi = e^{Q}$, and hence

$$\begin{aligned} \psi(q) &= e^{-W(q_{\bullet} \, -d/dQ_{\bullet})}e^{Q} \\ &= e^{-W(q_{\bullet} \, -1)} \\ &= \text{const.}\, e^{-w(q_{\bullet} \, -1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9.7) \end{aligned}$$

which is the required formal solution. It is also formally *complete*, since to the m roots (9.4) there will correspond m values of w.

The method is not of practical value, since the difficulty of solving (9.3) will be the same as that of solving the original differential equation (9.1) by a substitution of the form $\psi(q) = e^{\chi(q)}$.

10. *Generating Functions.*—Suppose a set of functions $a_n(q)$ to be defined by the generating function $e^{U(q, t)}$ so that

$$e^{U(q, t)} = \sum_{n=0}^{\infty} \frac{a_n(q)}{n\,!} t^n.$$

We proceed to show how, *given the generating function, we can find the differential equation and recurrence relation satisfied by* $a_n(q)$, and conversely, *given the differential equation and recurrence relation, we can find the generating function.*

Consider the function $\phi(Q)=Q^n$. It satisfies the two conjugate equations (see below)

$$(QP-n)\phi=0 \qquad (10.1)$$

$$(Q-e^{d/dn})\phi=0. \qquad (10.2)$$

Now apply the transformation derived from

$$W(q, P)=-U(q, -P) \qquad (10.3)$$

so that

$$\begin{aligned}\psi(q)&=e^{-W(q, -d/dQ_0)}\phi(Q)\\ &=e^{U(q, d/dQ_0)}Q^n\\ &=\left[\sum_{m=0}^{\infty}\frac{a_m(q)}{m\,!}\left(\frac{d}{dQ_0}\right)^m\right]Q^n\\ &=a_n(q). \qquad (10.4)\end{aligned}$$

Hence, transforming equations (10.1) and (10.2) by means of the contact transformation defined by (10.3), we obtain the required differential and difference equations satisfied by the function $a_n(q)$.

Conversely, starting from these two equations, which we may write in the general case as

$$\left.\begin{aligned}\sigma(q, p, n)\psi=0\\ \tau(q, p, n, e^{d/dn})\psi=0\end{aligned}\right\} \qquad (10.5)$$

where σ, τ are given functions, we first solve formally for $n, e^{d/dn}$, obtaining relations of the form

$$n=\lambda(q, p) \qquad (10.6)$$

$$e^{d/dn}=\mu(q, p). \qquad (10.7)$$

We now require these to be the equations obtained by the transformation of those of the form (10.1), (10.2). Consequently we set

$$\left.\begin{aligned}-PQ=\lambda(q, p)\\ Q=\mu(q, p)\end{aligned}\right\} \qquad (10.8)$$

noting that for the first we have to take the *adjoint* of QP as indicated. Solving these algebraic (non-commutative) equations we obtain

$$Q=\mathbf{Q}(q, P), \qquad p=\mathrm{p}(q, P),$$

giving the equations for the function $W(q, P)$ as

$$\frac{\partial W}{\partial P}=\mathbf{Q}(q, P) \qquad \frac{\partial W}{\partial q}=\mathrm{p}(q, P).$$

These now give $W(q, P)$ by simple quadratures. Writing finally

$$U(q, P)=-W(q, -P) \qquad (10.9)$$

we obtain as the required generating function $e^{U(q, t)}$.

We mean here by conjugate equations those of the type considered in Paper II, Theorem XIII. We hope in a forthcoming paper to discuss the

significance of such pairs of equations for sets of functions. In particular we shall derive the orthogonal properties. The present results should be compared with those of Ferrar * on generating functions. He connects them up with the differential and difference equations but does not consider this conjugacy between the equations.

11. One further special form of $W(q, P)$ may be noted. If $W(q, P)$ is quadratic in q, P then the transformation is given by equations linear in q, p, Q, P, *and thus soluble by elementary algebra.*

12. *The Infinitesimal Transformation.*—If we take

$$W(q, P) = qP - \epsilon\theta(q, P) \quad . \quad . \quad . \quad . \quad . \quad (12.1)$$

when θ is some function of q, P, we obtain

$$Q = q - \epsilon\frac{\partial\theta}{\partial P}(q, P),$$

$$p = P - \epsilon\frac{\partial\theta}{\partial q}(q, P).$$

If now we neglect powers of ϵ higher than the first we may write p for P in θ and we find

$$Q = q - \epsilon\frac{\partial\theta}{\partial p}(q, p), \quad . \quad . \quad . \quad . \quad . \quad . \quad (12.2)$$

$$P = p + \epsilon\frac{\partial\theta}{\partial q}(q, p), \quad . \quad . \quad . \quad . \quad . \quad . \quad (12.3)$$

which are the usual equations for an *infinitesimal contact transformation.*

Take $\phi(Q)$ to be the solution of a given differential equation

$$G(Q, P)\phi = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (12.4)$$

Then by our theorem the solution of the transformed equation is

$$\begin{aligned} e^{-W(q, -d/dQ_0)}\phi &= e^{+\epsilon\theta(q, -P_0)+qP_0}\phi \\ &= [1 + \epsilon\theta(q, -P_0)]\phi(Q+q) \\ &= [1 + \epsilon\theta(q, p)]\phi(q) \quad . \quad . \quad . \quad (12.5) \end{aligned}$$

working to the first power in ϵ and noticing that $-P\phi(Q+q) = p\phi(Q+q)$. *Hence the transformation is equivalent to operating with the function* $[1 - \epsilon\theta(q, p)]$.

This, however, is precisely our former result [Paper II (14.6)] that

$$\varpi(q, p) = (1 + \epsilon\theta) \quad . \quad . \quad . \quad . \quad . \quad . \quad (12.6)$$

for the latter gives, in the notation of that paper,

$$\begin{aligned} \psi(q) &= \int(1 + \epsilon\theta)\delta(Q - q)\phi(Q)dQ \\ &= (1 + \epsilon\theta)\phi(q) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12.7) \end{aligned}$$

which is identical with (12.5).

* Ferrar, *Proc. Edinburgh Math. Soc.* (2), II (1930), p. 71.

EXAMPLES AND APPLICATIONS.

13. As a first elementary example designed to show some characteristic points in the method consider the case

$$W(q, P) = P^2/q, \qquad (13.1)$$

so that

$$Q = 2P/q \qquad (13.2)$$
$$p = -P^2/q^2. \qquad (13.3)$$

Now take the equation for $\phi(Q)$ to be

$$(P^2+1)\phi = 0 \qquad (13.4)$$

giving

$$\phi = Ae^{iQ} + Be^{-iQ} \qquad (13.5)$$

where A, B are arbitrary constants.

Corresponding to (13.4) the equation for $\psi(q)$, using (13.3), becomes

$$(q^2p - 1)\psi = 0, \qquad (13.6)$$

and our theorem gives for the solution

$$\begin{aligned}\psi(q) &= e^{-W(q, -d/dQ_0)}\phi(Q)\\ &= e^{-1/q(d/dQ_0)^2}(Ae^{iQ}+Be^{-iQ})\\ &= Ce^{1/q} \qquad (13.7)\end{aligned}$$

where $C = A + B$. This is obviously the correct solution of (10.6). We may notice first that the method has yielded a solution expansible in negative powers of q, when obviously the ordinary solution in series fails in the neighbourhood of the origin. Secondly, the number of arbitrary constants has automatically adjusted itself to the orders of the equations.

14. As a second example take

$$W = qP + \tfrac{1}{2}\log q \qquad (14.1)$$

then we find

$$Q = q, \qquad P = p - \tfrac{1}{2}q^{-1}, \qquad (14.2)$$

giving

$$\begin{aligned}P^2 &= (p - \tfrac{1}{2}q^{-1})(p - \tfrac{1}{2}q^{-1})\\ &= p^2 - q^{-1}p - \tfrac{1}{4}q^{-2} \qquad (14.3)\end{aligned}$$

Hence, if we take as the equation for ϕ,

$$(P^2+1)\phi = 0 \quad \text{giving} \quad \phi = A\cos Q + B\sin Q \quad (A, B \text{ const.}) \qquad (14.4)$$

then the equation for ψ is

$$(p^2 - q^{-1}p + 1 - \tfrac{1}{4}q^{-2})\psi = 0 \qquad (14.5)$$

which is just Bessel's equation of order $\tfrac{1}{2}$.

The theorem gives

$$\begin{aligned}\psi(q) &= e^{-\frac{1}{2}\log q + qP_0}(A\cos Q + B\sin Q)\\ &= q^{-\frac{1}{2}}[A\cos(Q+q) + B\sin(Q+q)]_{Q=0}\\ &= q^{-\frac{1}{2}}(A\cos q + B\sin q) \qquad (14.6)\end{aligned}$$

This is of course the well-known solution of (14.5) since we have

$$J_{\frac{1}{2}}(q)=\sqrt{\frac{2}{\pi q}}\sin q,\qquad J_{-\frac{1}{2}}=\sqrt{\frac{2}{\pi q}}\cos q.$$

15. *Hermite Polynomials* $H_n(q)$.—These are defined by the generating function

$$e^{U(q,\,t)}=e^{-t^2+2tq}=\sum_{n=0}^{\infty}\frac{H_n(q)}{n\,!}t^n, \qquad (15.1)$$

Corresponding to (10.3) we define a transformation by means of

$$W(q,P)=-U(q,-P)=P^2+2Pq \qquad (15.2)$$

whence

$$Q=p+2q,\qquad P=\tfrac{1}{2}p\,. \qquad (15.3)$$

Transforming the equation (10.1), *i.e.*

$$(QP-n)\phi=0,$$

according to our usual rules we obtain

$$[(p+2q)(-\tfrac{1}{2}p)-n]\psi=0$$

or

$$(p^2+2qp+2n)\psi=0 \qquad (15.4)$$

which is *Hermite's differential equation* for $H_n(q)$.

Transforming the equation (10.2), *i.e.*

$$(Q-e^{d/dn})\phi=0,$$

we obtain

$$(p+2q-e^{d/dn})\psi=0 \qquad (15.5)$$

giving

$$\begin{aligned}p\psi&=(e^{d/dn}-2q)\psi\\ p^2\psi&=(e^{d/dn}-2q)p\psi+2\psi\\ &=(e^{2d/dn}-4qe^{d/dn}+4q^2+2)\psi.\end{aligned}$$

Hence, substituting in (15.4) we have

$$(e^{2d/dn}-2qe^{d/dn}+2(n+1))\psi=0$$

or, putting

$$\psi(q)=H_n(q),$$

$$H_{n+2}-2qH_{n+1}+2(n+1)H_n=0 \qquad (15.6)$$

which is the *recurrence relation* for $H_n(q)$.

It is clear now how we could work backwards from (15.4) and (15.6) to derive (15.1); but we shall instead take a fresh example.

16. *Laguerre Polynomials* $L_n(q)$.—In this case the differential equation and recurrence relation are respectively

$$(qp^2+(q-1)p+n)\psi=0\,. \qquad (16.1)$$

and

$$L_{n+1}-(2n+1-q)L_n+n^2L_{n-1}=0\qquad(n\geqslant 1). \qquad (16.2)$$

We write (16.2) as

$$\{e^{2d/dn}-(2n+3-q)e^{d/dn}+(n+1)^2\}\psi=0. \quad . \quad . \quad . \quad (16.3)$$

We have now to solve (16.1), (16.3) for n and $e^{d/dn}$ and then make the resulting equations transform into the forms (10.1), (10.2). From (16.1) we therefore take

$$PQ=qp^2+(q-1)p. \quad . \quad . \quad . \quad . \quad . \quad (16.4)$$

We then write (12.3) as

$$\{e^{2d/dn}-e^{d/dn}(2n+1-q)+(n+1)^2\}\psi=0$$

so that we may substitute $n\psi$ from (16.1). Doing this and then writing Q for $e^{d/dn}$ and making use of (16.4), we obtain

$$Q^2+Q(2PQ+q-1)+(PQ-1)^2=0. \quad . \quad . \quad . \quad (16.5)$$

We have now to solve (16.4), (16.5) for Q, p. We have immediately from (16.5)

$$Q=-\frac{q}{(1+P)^2}+\frac{1}{(1+P)}. \quad . \quad . \quad . \quad . \quad (16.6)$$

Substituting in (16.4) we get

$$-\frac{qP}{(1+P)^2}+\frac{P}{(1+P)}=q(p^2+p)-p,$$

which clearly has the solution

$$p=-\frac{P}{(1+P)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (16.7)$$

The function W(q, P) is consequently given by

$$\left.\begin{aligned}\frac{\partial W}{\partial P}&=-\frac{q}{(1+P)^2}+\frac{1}{(1+P)}\\ \frac{\partial W}{\partial q}&=-\frac{P}{(1+P)},\end{aligned}\right\}$$

and hence

$$W(q,\ P)=-\frac{qP}{(1+P)}+\log(1+P) \quad . \quad . \quad . \quad (16.8)$$

giving

$$U(q,\ P)=-\frac{qP}{(1-P)}-\log(1-P).$$

Therefore the generating function sought is

$$e^{U(q,\ t)}=\frac{e^{-qt/(1-t)}}{1-t},$$

which is the known result for the Laguerre polynomials.

(Issued separately January 6, 1932.)

XXIII.—An X-ray Examination of *d*-Mannitol and *d*-Mannose.

By **George W. McCrea**, B.Sc., A.I.C., Ph.D. *Communicated by* Professor J. Kendall, F.R.S. (With One Plate.)

(MS. received July 25, 1931. Read November 2, 1931.)

Among those sugar compounds which are readily obtained in the form of good crystals, *d*-mannitol has been shown to have a comparatively simple structural formula. An X-ray examination of *d*-mannitol was therefore undertaken, and the results thus obtained were compared with those obtained in a similar examination of *d*-mannose, the corresponding sugar, since it was thought that such an examination would give some indication of the molecular structures of these compounds in the crystalline state.

d-Mannitol.

The structural formula of *d*-mannitol, $C_6H_{14}O_6$, has been arrived at by chemical means, and shows two possible configurations for the molecule, which are given in Formulæ I and II.

$$
\begin{array}{cccccccc}
 & & & & H & H & & \\
 & H & H & H & O & O & H & \\
H- & C- & C- & C- & C- & C- & C & -H \\
 & O & O & O & H & H & O & \\
 & H & H & H & & & H & \\
\end{array}
\qquad\qquad
\begin{array}{cccccccc}
 & & & & H & H & H & \\
 & H & H & H & O & O & O & \\
H- & C- & C- & C- & C- & C- & C & -H \\
 & O & O & O & H & H & H & \\
 & H & H & H & & & & \\
\end{array}
$$

I. II.

Numbering the carbon atoms in each formula from left to right as 1 to 6, it will be seen that the hydroxyl group attached to carbon atom 6 has the choice of two positions. It was believed that this hydroxyl group was free to rotate, and thus had no fixed position. From a study of the iso-propylidene derivatives and the methylated hexitols derived from them, Irvine and Paterson (*Jour. Chem. Soc.*, 1914, **105**, 898) found that the terminal primary hydroxyl group of the *d*-mannitol molecule does not appear to possess free rotation. A comparison of the reactive powers of the hydroxyl groups attached to the pairs of carbon atoms 1 and 2, and 5 and 6, made by Irvine and Steele (*Jour. Chem. Soc.*, 1915, **107**, 1221) shows that while the groups 1 and 2 behave as if they lie in proximity, the hydroxyl groups in the pairs 3 and 4, and 5 and 6 behave as if they were thrust apart. This is in agreement with the

structure shown in Formula I, and this configuration has been accepted as that of *d*-mannitol.

X-ray Examination.

Groth (*Chem. Krystal.*, 3, 431) records measurements made on two different crystalline forms of *d*-mannitol. Those values found for the β-form are quoted, since this type of crystal, obtained from the crystallisation of water solutions, was examined. These measurements are due to Zepharovich.

Crystal Class.	Orthorhombic bisphenoidal.
Unit Cell.	$a = 8{\cdot}66$ A.U.
	$b = 16{\cdot}58$ A.U.
	$c = 5{\cdot}50_1$ A.U.
Axial Ratio.	$a : b : c = 0{\cdot}5121 : 1 : 0{\cdot}6577$. (Groth.)
	$a : b : c = 0{\cdot}523 : 1 : 0{\cdot}332$. (X-ray analysis.)

Number of Molecules per Unit Cell. 4.

Calculated Density. 1·522 grms. per c.c.

Space-group.—A series of oscillation photographs was made about each axis of the crystal, and an examination of these showed that the odd order reflections from the (hOO), (OkO), and (OOl) planes were absent. Normal reflections from nearly two hundred planes were observed. The only abnormal spacings belong, therefore, exclusively to the axial zones, and the Bravais lattice is then the simple lattice Γ_0. The crystal thus belongs to the space-group Q_4 (Astbury and Yardley, *Phil. Trans.*, A, **224**, 221) and the four molecules in the unit cell are asymmetric.

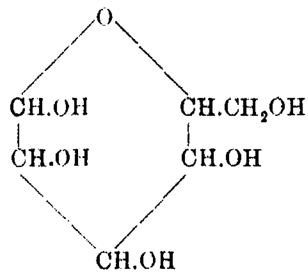
Structure of the Molecule.—Chemical knowledge points to the molecule of *d*-mannitol having a structure based on that of the zigzag carbon chain of the hydrocarbons and fatty acids. If the presence of such a structure is assumed, then according to the X-ray data, the *d*-mannitol molecule has the structure represented in Formula I, which is asymmetric, while that of Formula II possesses a diad axis of symmetry. Further evidence of a chain structure is obtained from consideration of the sides of the unit cell. Construction of scale drawings of a zigzag chain molecule show that the length of the molecule corresponds to the length of the a axis of the crystal, namely, 8·66 A.U. A. Müller (*Proc. Roy. Soc.*, A, **127**, 417) has shown that the molecule of hexane possesses a zigzag carbon chain, and that in the crystal it lies along the a axis of length 8·55 A.U. The similarity of the length of the hexane and *d*-mannitol molecules seems to indicate a similarity of structure.

The results of the X-ray examination of *d*-mannitol thus confirm those

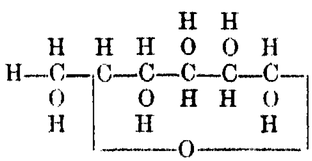
obtained by chemical experiment, and the structure of the molecule is that represented in Formula I.

d-MANNOSE.

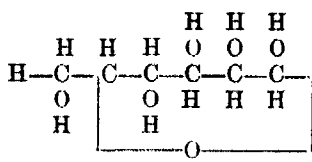
d-Mannose, $C_6H_{12}O_6$, is known to exist in three forms, *α*, *β*, and *γ*, each differing in the internal arrangement of the atoms. A pyran ring structure is now attributed to the normal or *α*- and *β*-forms, while the labile or *γ*-form has a furan ring structure (Goodyear and Haworth, *Jour. Chem. Soc.*, 1927, **2**, 3136). The normal sugar is therefore a tetrahydroxy-tetrahydro-pyran or mannopyranose.



In order to explain the difference between the *α*- and *β*-forms of *d*-mannose, Haworth and Hirst (*Jour. Chem. Soc.*, 1928, **1**, 1221) make use of a possible cis- and trans- arrangement in which the hydroxyl groups attached to carbon atoms 5 and 6 are affected (*vide*, p. 1 of this paper).



Trans- or *α*-*d*-mannose.



Cis- or *β*-*d*-mannose.

H. S. Isbell (*Bur. Stand. J. Res.*, 1930, **5**, 1179) has shown that the epimeric difference in the molecular rotation between 4-glucosido-*α*-mannose and cellobiose approximates to the difference of that between *α*-*d*-mannose and *α*-*d*-glucose. This suggests that the ring structures of *α*-*d*-mannose and 4-glucosido-*α*-mannose are identical. Such a result is directly opposed to the work of Hudson and others, who have put forward the view that a change in the ring structure of *d*-mannose occurs during methylation.

4-Glucosido-*α*-methyl mannoside has been proved by Haworth and others (*Jour. Chem. Soc.*, 1930, p. 2664) to have a pyran ring structure. Hence if Haworth's view that no change in the ring structure of *d*-mannose

occurs during methylation be accepted, the molecule of *d*-mannose must possess a pyran ring structure.

For the purpose of the X-ray examination, crystals of *d*-mannose were obtained by a method suggested by Sir James Irvine, of St Andrews, in a private communication. Conditions were such that the α-form was obtained.

X-ray Examination.

Groth (*Chem. Krystal.*, **3**, 441) records measurements, due to Mohr, from only one form of crystal of *d*-mannose.

Crystal Class.	Orthorhombic bisphenoidal.	
Unit Cell.	$a = 5{\cdot}53$ A.U.	
	$b = 17{\cdot}66$ A.U.	
	$c = 7{\cdot}59$ A.U.	
Axial Ratio.	$a : b : c = 0{\cdot}319 : 1 : 0{\cdot}826.$	(Groth.)
	$c : b : a = 0{\cdot}313 : 1 : 0{\cdot}429.$	(X-ray data.)

Number of Molecules per Unit Cell. 4.

Calculated Density. 1·602 grms. per c.c.

Space-group.—The examination of a series of oscillation photographs showed that the odd order reflections from the (hOO), (OkO), and (OOl) planes were absent. The reflections observed from over one hundred and fifty planes showed no other abnormal spacings. The crystal, thus containing the simple lattice Γ_0, belongs to the space-group Q_4 (Astbury and Yardley, *loc. cit.*), and the four molecules in the unit cell are asymmetric.

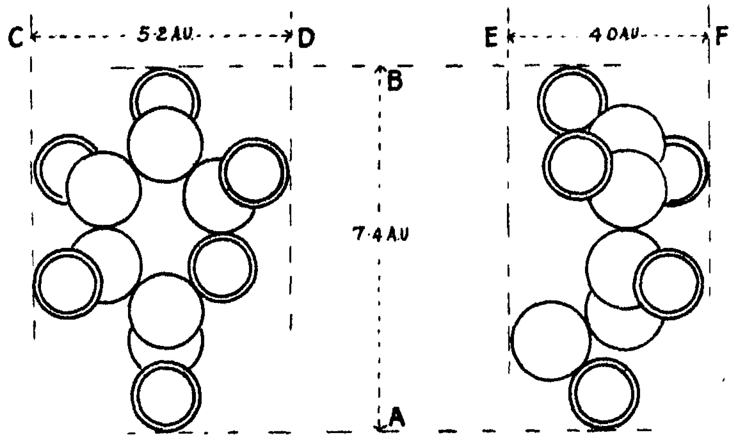
Structure of the Molecule.—The results of the X-ray examination given above yield no direct evidence of a particular structure for the *d*-mannose molecule, since the pyran ring, the furan ring, and the chain structures are equally asymmetric. A comparison of the results obtained from *d*-mannitol with those obtained from *d*-mannose, and a further consideration of the data obtained in the X-ray examination of *d*-mannose, give an indication of the probable structure of the *d*-mannose molecule.

Discussion of the Results from *d*-Mannitol and *d*-Mannose.

Chemically *d*-mannitol differs from *d*-mannose in that the hydroxyl group attached to carbon atom 6 of the *d*-mannitol molecule (see Formula I) is replaced by an aldehyde group in the *d*-mannose molecule. It has been shown that *d*-mannitol possesses a chain structure, and if *d*-mannose were also to have a chain structure it is to be expected that the X-ray photographs would be similar. Reproductions of photo-

graphs taken with rotation about the b axis of d-mannitol, and about the b axis of d-mannose are given in figs. 1 and 2 (plate) respectively. The very different intensity distributions exhibited in these photographs show that it is very unlikely that d-mannose has a chain structure.

A ring structure of some nature may thus be expected. There are two ways in which the atoms comprising the pyran ring may be arranged. They may lie all in one plane with a zigzag side chain, or the ring may be "puckered" with a straight side chain. It is considered that the ring with all the atoms in one plane will be under less strain and will thus be more



Front view. Side view.

Single ring—Carbon atom. Double ring—Oxygen atom.

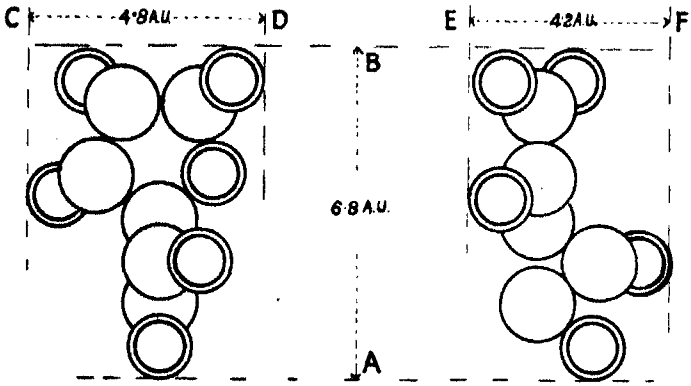
FIG. 3.—Pyranose ring.

stable. Such an arrangement has been found in some benzene compounds (K. Lonsdale, *Trans. Faraday Soc.*, 1929, **25**, 352). In the case of a furan ring structure a "puckered" ring is very unlikely, since such an arrangement would give a molecule which would be under very considerable strain.

Scale drawings of these pyran and furan ring structures are to be found in figs. 3 and 4. Consideration of the dimensions of the two structures and the comparison of these with the dimensions of the unit cell give a further indication of the structure of d-mannose.

It will be noted from the X-ray data that the a axis of d-mannitol corresponds in length to the c axis of d-mannose, while the c axis of d-mannitol corresponds to the a axis of d-mannose. In this examination the greatest length of the crystal was taken as the c axis and the shortest length as the b axis. It appears that the d-mannose crystal had become more elongated along the a axis than had the d-mannitol crystal.

Comparison of the intensity distribution of the reflected X-ray beams on the corresponding rotation photographs of *d*-mannitol and *d*-mannose does not show such a marked difference as is found in the *b* axis photographs. This shows that the *d*-mannose molecule lies along the *c* axis of the crystal.



Front view. Side view.

FIG. 4.—Furanose ring.

The dimensions of a *d*-mannose molecule having a pyran ring and of one having a furan ring structure are given with the corresponding X-ray data in Table I.

TABLE I.

	Length *CD*.	Length *EF*.	Length *AB*.
Pyran ring . .	5·2 A.U.	4·0 A.U.	7·4 A.U.
Furan ring . .	4·8 A.U.	4·2 A.U.	6·8 A.U.
X-ray data . .	$a = 5{\cdot}53$ A.U.	$b = 17{\cdot}66$ A.U. $= 4 \times 4{\cdot}42$ A.U.	$c = 7{\cdot}59$ A.U.

The length *AB* of the pyran ring structure is such that a molecule having this structure would fill up more of the available length along the *c* axis of the *d*-mannose crystal than would a molecule having a furan ring structure, whose length *AB* is less than that of the pyran ring structure.

From Table II, which gives the volume occupied by one molecule of each type and that occupied by one molecule in the unit cell, it will be seen that the volume of the molecule with the pyran ring structure is in better agreement with the volume occupied by the molecule in the unit cell than

is that of the furan ring structure. The values given in Table II are calculated from the measurements recorded in Table I.

TABLE II.

	Pyran Ring Structure.	Furan Ring Structure.	X-ray Data.
Volume per molecule . .	154 A.U.^{3}	137 A.U.^{3}	178 A.U.^{3}

Since the volume V, the mass M, and the density D of a substance are related by the equation $M = V.D$, the volume varies inversely as the density, if the mass remains constant. Now the mass of the *d*-mannose molecule remains constant no matter what the structure of the molecule may be. Hence a unit cell made up of molecules having a pyran ring structure would have a density in better agreement with that found in the X-ray examination than would a unit cell made up of molecules having a furan ring structure.

The results of the X-ray examination of *d*-mannose thus indicate that a pyran ring structure is probable in the molecule, and so tend to corroborate previous chemical examinations.

In concluding, the author wishes to express his thanks to Sir James Walker, F.R.S., and to Professor James Kendall, F.R.S., for their very valuable criticism and advice. The author's thanks are due also to the Department of Scientific and Industrial Research, and to the Carnegie Trust for grants which enabled him to carry out this research.

ADDENDUM.

Miss Thora C. Marwick has recently published a paper entitled "An X-ray Study of *d*-Mannitol, Dulcitol, and *d*-Mannose" (*Proc. Roy. Soc.*, A, 1931, **131**, 621), a summary of the results obtained being given in *Nature* (1931, **127**, 11). The results obtained are in very good agreement with those set out above, and the work was carried out simultaneously. (See *Nature*, 1931, **127**, 162.)

SUMMARY.

The X-ray examination of *d*-mannitol and *d*-mannose gives the following results:—

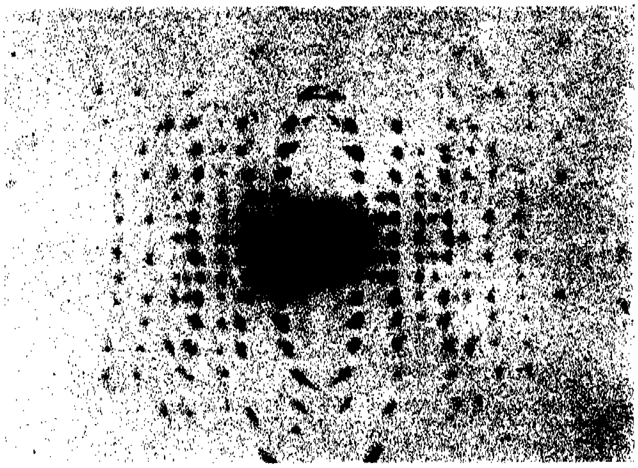


FIG. 1.—Rotation photograph about the *b* axis of *d*-Mannitol.

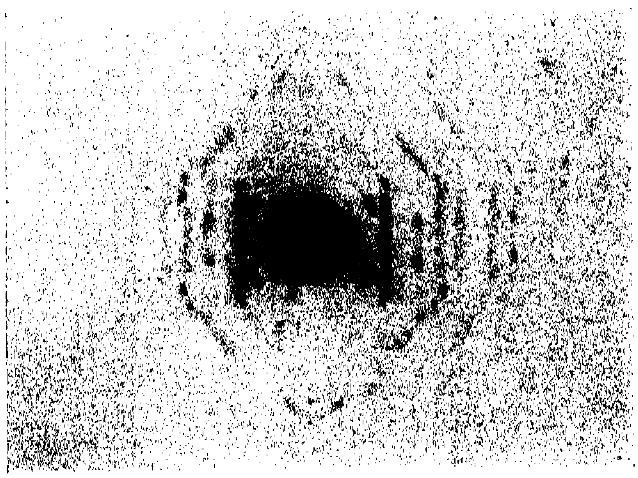


FIG. 2.—Rotation photograph about the *b* axis of *d*-Mannose.

d-*Mannitol.*—Unit cell, $a = 8{\cdot}66$ A.U., $b = 16{\cdot}58$ A.U., $c = 5{\cdot}501$ A.U.; Space-group, Q_4; Calculated density, 1·522 grms. per c.c.; Number of molecules per unit cell, 4.

The X-ray data show that the greatest length of the molecule corresponds to the a axis, and that the molecules have a long chain configuration.

d-*Mannose.*—Unit cell, $a = 5{\cdot}53$ A.U., $b = 17{\cdot}66$ A.U., $c = 7{\cdot}59$ A.U.; Space-group, Q_4; Calculated density, 1·602 grms. per c.c.; Number of molecules per unit cell, 4.

The X-ray results show that a manno-pyranose ring structure is probable in the *d*-mannose molecule, and that the molecule has its greatest length along the c axis.

(*Issued separately January* 11, 1932.)

OBITUARY NOTICES.

Archibald Barr, D.Sc., LL.D., F.R.S.

By the death of Professor Archibald Barr on 5th August 1931, at the age of seventy-six years, Engineering Science lost one of its most distinguished figures. He was outstanding as a teacher of Engineering, as an inventor, and as an authority on the design and running of laboratories and workshops.

Professor Barr was born in Paisley and received his early education in Paisley Grammar School. He then proceeded to Glasgow University and entered the classes of Engineering, then presided over by James Thomson, brother of Lord Kelvin. He also attended Kelvin's Natural Philosophy classes. He served his engineering apprenticeship with Messrs A. F. Craig and Company, of Paisley. In 1876, at the age of twenty-one, he was appointed James Young Assistant to Professor James Thomson, and somewhat later received the degree of Doctor of Science of the University of Glasgow. To Professor James Thomson's teaching and example he owed a vast debt, a debt which he never failed to acknowledge and which was handed on by him to generations of his own students.

In 1884 Dr Barr was appointed Professor of Engineering at the Yorkshire College, now the University of Leeds. Then he became the colleague of Professor William Stroud, who occupied the Chair of Physics in that Institution. The two men were strongly attracted to one another, and the partnership was formed which was to result in so much constructive thought, invention, and achievement. While at Leeds Professor Barr took an active part in the collection of funds for new engineering laboratories. These laboratories and workshops he designed and completed.

In 1889 Professor Barr was appointed to succeed his old teacher in the Chair of Engineering at Glasgow University. He occupied this Chair for twenty-four years, retiring in the summer of 1913. He performed great work for the University of Glasgow as a teacher and organiser. His lectures were models of clearness, and he was regarded by his pupils with pride and affection. He initiated, and took a foremost part in, a movement which resulted in the sum of £40,000 being available for the establishment of new laboratories at Glasgow. These, the James Watt Laboratories, were built and equipped under his supervision. On his retiral in 1913 Dr Barr received the degree of LL.D. of the University, and in 1915 he was

presented with two portraits painted by Mr G. Fiddes Watt. One of these is in the possession of Dr Barr's family, the other in that of the University. Some time after his retirement from his Chair, Professor Barr's scientific work was recognised by his admission into the Royal Society as a Fellow.

The history of the Barr and Stroud Rangefinder is somewhat as follows: A War Office advertisement appeared in the *Engineer* of 24th May 1888 and *Engineering* of the following day. With Dr Stroud, Dr Barr designed and constructed an instrument which proved the most successful in the competitive trials. In 1891 the British Admiralty issued a similar advertisement, but specially invited Professors Barr and Stroud to submit a rangefinder for tests. The results were so satisfactory that the inventors were requested to tender for the supply of six instruments. In these early forms of the instrument the mechanical work was carried out by James White in Glasgow, and the optical parts were constructed by Adam Hilger in London. The assembly and adjustment work was effected in the attics of Dr Barr's house in Dowanhill, Glasgow. A small works was established in Byres Road, near the University, and later the present factory at Anniesland was built. The magnitude of the work carried out may be judged from the fact that of one type of rangefinder alone, the Anniesland works constructed 27,000 during the War.

Dr Barr filled many posts of honour. In 1901 he supervised the first Motor Car Reliability Trials held in Scotland. In the same year he served as Convener of the Engineering Committee of the Glasgow International Exhibition. In 1910 he acted as Chairman of the Committee which organised the first Aviation meeting in Scotland. In the course of his career he acted as President of the Institution of Engineers and Shipbuilders in Scotland, of the Scottish Aeronautical Society, of the Optical Society of London, of the Royal Philosophical Society of Glasgow, and on more than one occasion of the Engineering Section of the British Association.

He was elected a Fellow of the Society in 1921.

J. G. G.

Raymond Keiller Butchart, B.Sc., Ph.D.

RAYMOND KEILLER BUTCHART, the only son of the late R. K. Butchart of Dundee, was born in 1888. After his early training in the Morgan Academy and in the Dundee High School he proceeded to University College, Dundee, in the University of St Andrews, where he graduated B.Sc. in 1913.

He then carried on some post-graduate work, and acted as student-assistant in mathematics until December 1914, when he was offered an appointment at Wilson College, Bombay, almost at the moment when a Commission as Lieutenant in the 14th Royal Scots was also within his reach. He consulted me, and all that I could do, without urging him in any direction, was to say, as I had already said to others, that in the country's need every not incapacitated man should offer himself to some branch of the services for national defence; and that no such man, if he were spared, would be neglected by his fellow-countrymen when he returned to his native land.

Without hesitation he accepted the Commission and was trained at the Stobs Camp. In due course he went out and did good service as Brigade Signalling Officer.

Towards the end of the War he was very dangerously wounded on the open field. His leg had to be amputated well above the knee. He was sent home to one of the great West End mansions that had been placed by their owners at the disposal of the authorities as military hospitals.

There I found him lying, and, as by a fortunate chance the assistant in the mathematical department was resigning in order to qualify for medical work, I was able to offer Captain Butchart the vacant post. He accepted it, and for nine years discharged its duties with remarkable fidelity. In his spare time he continued his researches, and in July 1921 they gained for him the Ph.D. of his university.

In the same year he married Jean Ainslie Broome, daughter of the late W. W. Broome of Bo'ness.

By this time the increasing number of students and of classes had made a junior assistant necessary, and Dr Butchart became Senior Assistant and University Lecturer.

His work was always carefully prepared: and his interest in it

never flagged. He met his serious disability in a fine spirit. The short daily walk to and from his house, up and down an incline to the tramway line that took him to and from the College, must have been a serious strain upon his body and his nerves; his friends were amazed at his quiet endurance of a daily exertion that, especially in winter, gave obvious proof of its severity.

His administrative gifts were of a high order; he was the trusted adviser of the students as Treasurer of their Union, as well as in many other capacities. He was respected and loved by all the members of the College.

Perhaps I may be permitted to say that his devotion to the University was only equalled by his loyalty to his chief. During his long service his conscientious work and his discriminating counsel were of inestimable value; the spirit of happiness and goodwill that he diffused throughout the department rendered its administration a most happy duty.

In March 1928 Dr Butchart accepted the Professorship of Mathematics at the Raffles College in Singapore. His accounts of his work there were most interesting. He was extremely happy, and the climate, about which his friends had felt some anxiety, seemed to suit him. While starting and developing his Department he won the esteem and affection of all who knew him; his students gave him and his wife some charming gifts when he set sail for his first leave. Tired with an arduous session's work he was looking forward, as we in University College were looking forward, to his return to Scotland. He left Singapore on 24th March, and died on 30th March from malaria. He was buried at sea 65 miles south-east of Colombo.

To his former colleagues and students in Dundee he leaves a memory whose fragrance will long persist in their hearts.

He was elected a Fellow of the Society in 1915.

J. E. A. S.

James Currie, M.A. (Cantab.), LL.D. (Edin.).

By the death of Dr James Currie in November 1930 this Society has lost one who for many years rendered it valuable and devoted service, and Edinburgh a noteworthy and generous citizen. In Dr Currie there was met a combination of qualities and gifts which is somewhat rare. Though much of his life was devoted to commerce, and his success in business was notable, he was at the same time a man of wide reading and high culture, a gifted linguist, and a scientist of no mean attainments. Retiring by nature and modest to a degree, he possessed a pawky humour which was delightful to those who had the privilege of his friendship, and a ripe wisdom which made him invaluable on Public Boards and Councils.

James Currie was born at Leith on 13th April 1863. He was educated at Edinburgh Academy, and later at Godesberg on the Rhine where he soon acquired a knowledge of the German language and discovered that he had an aptitude for foreign languages, many of which he mastered during subsequent years. After spending a little time at the University of Edinburgh, he went into residence at Trinity College, Cambridge, and there graduated B.A., taking the Mathematical Tripos.

Before his marriage he took a voyage round the world, and during the course of his life yachting on the West Coast of Scotland and to the Faroe Islands occupied a considerable part of his summer holidays. He married a daughter of Mr W. A. Peterkin of H.M. Board of Supervision (now Local Government Board), to whom he was devoted, and through whom much of his philanthropic work was done in a quiet and unostentatious manner.

Mr Currie's principal business was that of a shipowner. The name of Currie is well known in the shipping world and he carried on during his business career the high traditions of the family in marine affairs. He succeeded his father, the late James Currie, as Manager of the Leith, Hull, and Hamburg Steam Packet Co., Ltd., and the success of that company was largely due to his conspicuous directing ability. In addition he was interested in many other undertakings. He was a Director for many years in the North British and Mercantile Insurance Company. Educational projects found in him a willing supporter. For many years he was one of the Governors of George Heriot's Trust, and subsequently a Governor of the Heriot-Watt College. He also served as Governor in the Leith Nautical College. During the War he acted as Chairman of the

Edinburgh Chamber of Commerce and took an active part in the institution of a Degree in Commerce at the University of Edinburgh.

He was a generous benefactor to the University, and as a recognition of his public services he was awarded the Honorary Degree of LL.D. in 1919.

His services to the Scientific Societies of Edinburgh were especially notable. His father was one of the original founders of the Royal Scottish Geographical Society. Dr James Currie himself served on the Council of that Society for many years, and for over a quarter of a century he acted as Honorary Treasurer. He was also one of the Trustees of that Society.

He took a prominent part in the affairs of the Edinburgh Geological Society and served as Member of the Council of that Society, acted as Secretary from 1898–1903, and filled the Presidential Chair (1904–1906).

His scientific interests were wide. He was a keen botanist and archæologist, but his special field of work was in mineralogy, especially on its crystallographic side. He published several important papers in the *Transactions of the Edinburgh Geological Society*, the most notable of which was the Presidential Address given in 1905 on "The Mineralogy of the Faroes," in which he dealt especially with the zeolites, a group of minerals in which he took a particular interest. Of the other papers, mention may be made of one in particular, namely, that on "The Minerals of the Tertiary Eruptive Rocks of Ben More, Mull," published in 1909. He was also a frequent contributor to the *Scottish Geographical Magazine*. The section on "Pseudomorphs" in Heddle's *Mineralogy of Scotland* came from his pen. He was an enthusiastic collector of minerals and travelled widely in search of rare specimens. For some time he was closely associated in this work with the late Professor Heddle of St Andrews.

In the course of years he gathered together a large and valuable collection, which was housed in a museum at Larkfield, his residence in Edinburgh, and overflowed to other portions of the house to the embarrassment of his good lady, who already found it difficult to find place for valuable collections of china and other rarities. He made a speciality of zeolites from the Faroe Islands and the Inner Hebrides. On his death this splendid collection was most generously presented to the Geology Department of the University of Edinburgh by Mrs Currie and her family as a permanent memorial to him.

After acquiring the fine mansion and estate of Inverawe, near Loch Etive, he devoted much of his time to sylviculture, for which he found scope in the beautiful woodlands surrounding the house. He took a great interest in the place-names of Argyllshire and in that way acquired some knowledge of the Gaelic language.

He was elected a Fellow of the Royal Society of Edinburgh in 1898, became Treasurer of the Society in 1906, and held that office until 1926, a period of twenty years. He became Vice-President in 1926, which office he held until his retirement by rotation in 1929. During these twenty-three years as Member of the Council, the Society had the benefit of his experience and wisdom in every question that came before it, particularly regarding the finances of the Society. During the difficult War and post-War periods his services were especially valuable in piloting the Society's finances, which, on his retirement, were left in a very satisfactory state.

For some years he served as Honorary Treasurer of the Royal Society Club, and his geniality and wit will long be remembered by those who sat around that festive board.

It is but fitting that we should pay a warm tribute to one who gave such unstinted and willing aid to the Royal Society in difficult times. Dr Currie passed away on 3rd November 1930, and he will be greatly missed, not only on public bodies, but also by a wide circle of friends.

T. J. J.

J. D. Hamilton Dickson, M.A.

By the death of Mr J. D. Hamilton Dickson, Cambridge has lost a man whose mental alertness and youthfulness at an advanced age was a source of envy to many of his physically younger acquaintances.

The son of a Glasgow physician, he was born there on 1st May 1849, and received his education at Glasgow University, where he distinguished himself especially on the mathematical and physical side. Working in the laboratory, as was the practice then of only a few of the Natural Philosophy students, he had the privilege of helping the late Lord Kelvin in his determination of the relation between the electromagnetic and electrostatic units of electricity, and in some of his work connected with deep-sea cables.

In 1870 he proceeded to Cambridge, forming a connection with that University that was to last over sixty years, and it is not to be wondered at that he chose to follow Kelvin's footsteps and enter his name as an undergraduate of Peterhouse, which at that time had the double reputation of being both very mathematical and Scottish in bias, a tradition which he himself helped to keep up by becoming Fifth Wrangler in 1874. In 1877 he was appointed Tutor of the College, a post he held for twenty years, and when Lord Kelvin died in 1907 became Senior Fellow. The present writer first met him when he entered Peterhouse, and was immediately struck by the great pains that Dickson took to make all arrangements clear and impossible to be misunderstood by the raw undergraduate. This carefulness was characteristic of all his work. He had very considerable teaching abilities, too, for while almost any man of average ability can instruct clever pupils, he did the more difficult thing and laid himself out to teach the less clever, in which effort he did very useful work. While very successful as a tutor, he was not so successful as his merits deserved, perhaps owing to this very capacity for detail, dislike of loose ends, and love of order: things too often unappreciated by the ordinary undergraduate, recently released from the bonds of school and feeling his freedom. His mathematical mind may have been too much inclined to arrange every detail of the puzzle, and to be too much upset when every last little piece did not fit in quietly and exactly into its place. But he was much liked, and the College owed a great deal to his firm and energetic supervision. This supervision was not confined to mere discipline, for

among the other interests of a versatile man, he took a keen interest in music and was a shining light of the Peterhouse Musical Society.

When the War broke out both his patriotism and his energy were shown by his taking up mathematical work on the depleted staffs of Fettes and the Edinburgh Academy, where his physical activity occasioned remark even at that time of resuscitation of "dugouts." A rumour that he had been rebuked by those in authority for being caught sliding down a banister was probably untrue, but gave a truer picture of his vitality than the plain truth would have done. Even quite near his end he was taking a keen interest in the approaching Centenary of Clerk Maxwell, who had been a Peterhouse man before he migrated to Trinity.

While most of his life was spent in Cambridge, he had an Edinburgh connection, for in addition to his election to this Society in 1876 he, like his brother Lord Scott Dickson, and his brother-in-law, Sir James Dewar, married an Edinburgh lady, one of the three Misses Banks, by whom he is survived. He died on 6th February 1931 at the age of 81.

M. M'C. F.

John Dunstan, M.R.C.V.S.

John Dunstan, M.R.C.V.S., died on 9th September 1930. He belonged to a well-known Cornish family of veterinary surgeons and obtained the Diploma of Membership of the Royal College of Veterinary Surgeons in 1895. In 1900 he was appointed Professor of Surgery in the Royal (Dick) Veterinary College, Edinburgh, a position he held for five years. On resigning this appointment, he returned to Liskeard, Cornwall, where he practised until ill-health compelled him to retire a year or two before his death. He was held in high esteem by his fellow-members of the profession, who elected him to a seat on the Council of the Royal College of Veterinary Surgeons. On this body he served from 1910 to 1919, and occupied the Vice-Presidential chair in 1911.

Though John Dunstan made no notable contribution to veterinary literature, he was able, from his rich store of clinical experience, to afford help on many occasions to the Ministry of Agriculture in elucidating problems presented by diseases of animals. He was a man of tremendous energy and had many interests outside his professional work.

He was elected a Fellow of the Society in 1903.

O. C. B.

Richard Taunton Francis, F.Z.S., M.B.O.U.

RICHARD TAUNTON FRANCIS, younger son of Dr William Francis, of the firm of Taylor and Francis, printers and publishers, was born in 1883. He was educated at St Paul's School, and on leaving school spent some months in his father's firm, becoming practically conversant with the work of the composing-room and reading department. He then resumed his studies at Göttingen University, but his university career was cut short by a very serious bicycle accident, by which it would seem that his liver was ruptured, and which necessitated immediate operation. He used to say that if the accident had happened in England he would probably have died, owing to the time which would have been spent in formalities, such as getting consent for operation. He was always in after years liable to pain in the region of the liver. Soon after returning from Germany he joined the firm, and on the death of Dr Francis in 1904 he became a partner with his elder brother William; after the retirement of the latter in 1917 he carried on the business of the firm alone.

The house of Taylor and Francis is well known in scientific circles, and specialises in scientific, and especially in zoological, publications. Three important journals, *The Philosophical Magazine*, *The Annals and Magazine of Natural History*, and *The Journal of Botany*, are the property of the firm; and though Francis disclaimed any pretensions to scientific knowledge, his name appeared on the *Annals* as one of the editors. In this journal he took, perhaps, a specially keen interest—an interest which was certainly not due to any financial success which it attained, for it was carried on for years at a loss. It would have been a calamity if this old and famous journal had been discontinued; and Francis deserves the thanks of zoologists and palæontologists for having seen it through its period of depression. The firm has, under contract with the India Office, published, since the inception of the series in 1888, *The Fauna of British India*, which now comprises some 40 volumes; and has also produced the stately tomes of Godman and Salvin's *Biologia Centrali-Americana*. The printing by the firm, partly in the days before he became a partner, of a number of sumptuous ornithological works was probably the cause of Francis's interest in ornithology. He became a Member of the British Ornithologists' Union in 1921; *Ibis*, the journal of the Union, is published by the firm.

After his marriage in 1908 Francis lived at Purley. On his holidays he often took up sea-fishing; his accident always prevented his doing anything which required much exertion. His health broke down two or three years ago, and for a long time he was unable to come up to town, though he continued to manage the business from his home; he appeared latterly to be regaining strength, and had begun to go to office again, but an attack of influenza led to pneumonia, from which he died on 13th February, at the age of 47. He leaves a widow and one son; a young daughter predeceased him.

He was elected a Fellow of the Society in 1926.

J. S.

David Hepburn, C.M.G., M.D.

David Hepburn was one of the numerous demonstrators trained in Edinburgh University under Sir William Turner who subsequently became Professors of Anatomy in other Schools of Medicine.

He was long associated with the Anatomy Department of the University of Edinburgh, where he filled the honourable post of Senior Demonstrator from 1885 till 1903, and in that capacity his was a figure well known to and respected by many generations of medical students.

He excelled as a teacher of Anatomy, systematic and orderly in the presentation of facts, with clear expression and pleasing voice, and the ability to illustrate rapidly and graphically on the blackboard facts displayed in the dissections.

The number of students in the practical classes in the University ran into large figures, and the task of organising the work was in itself no light one, but Hepburn took it on his shoulders and relieved his chief of all anxiety and work in connection with it.

Research in Anatomy at that time was largely directed to problems of Comparative Anatomy, and Hepburn carried out some useful work in this field. He published a valuable paper on "Comparative Anatomy of the Muscles and Nerves in the Limbs of Man and the Anthropoid Apes."

He was an accurate observer, and the facts contained in it form a storehouse of reliable information. The discovery of the remains of the Java *Pithecanthropus* directed attention to the features characteristic of the human Femur, and in this connection Hepburn published an important paper on the racial characters of the Femora in the University of Edinburgh Anatomy Museum.

He saw the beginning of that enormous widening of the scope of Anatomy which took place as new methods in Histology and Embryology were introduced, and new material especially in Human Embryology and in Comparative Anatomy became available, and Anatomy became more associated with the study of the functions of the human body.

In 1903 he was elected Professor of Anatomy in the University College, Cardiff, and there found full scope for his abilities as a teacher, and afterwards as Dean of the Faculty of Medicine as organiser.

Although his resignation from the professorship fell due in 1924, he was

invited to retain it, and retired only in 1927 amid expressions of general regret.

No mention of Hepburn would be complete without reference to the devoted service he gave in the Volunteer and subsequently the Territorial Forces. He was for many years in command of the Edinburgh Medical Unit of the Officers' Training Corps, and was an efficient and greatly liked C.O. During the War he rose to the rank of Surgeon-Colonel, and held the responsible post of command of the Third Western General Hospital at Cardiff.

Honours came to him towards the close of his career. He was made a C.M.G. in 1917, and was also a Knight of Grace of the Order of St John of Jerusalem. His numerous friends knew how well deserved these honours were by one who had given loyal and devoted service to the universities in which he worked, and to the medical forces in which he served.

He was elected a Fellow in 1890, and died on 9th March 1931.

D. W.

David Thomas Jones, C.B.E.

David Thomas Jones was born at Gilfachgoch, Glamorgan, on 10th June 1866, and early determined to make the Civil Service his career in life. At the age of twenty-one he was posted as a junior clerk to the staff of the Fishery Board for Scotland, and only five years later he became the Chief Clerk. In 1909 he was promoted to be Secretary, which appointment he held all through the troubled years of the War when the Board's cruisers were taken into Admiralty service and he became a Paymaster-Lieutenant-Commander in the Royal Naval Reserve. It was during the War that Mr Jones consolidated his position in the Civil Service and fishery world, and brought to bear on the many problems which then arose the experience which he had gained and the judgment which he had matured during his earlier service. Only by an intimate knowledge of the fishing industry and population and, as he himself most readily admitted, by the willing co-operation of those under his command, was he able to advise on the urgent and sometimes far-reaching problems which then arose. From Mr Jones came the advice on which the Fishery Board decided the fishermen who might be taken for the country's war service and those who might be left to augment in such necessary fashion the war food supplies. He had to adjust as far as might be possible the insistent needs of war action with those other almost equally insistent needs of continuing to obtain fish supplies for an ever-ready market. No less after the War were his services in demand on various committees of reorganisation, and later he was deputed to visit and report on the suitability of the coast of British Columbia for the settlement of crofter fishermen from the Western Highlands and Islands. For his war services he was awarded the honour of C.B.E.

In 1920 Mr Jones was made Chairman of the Fishery Board and thus achieved the then most unusual distinction of passing through all the administrative grades in one department from the most junior to the senior post. Perhaps, however, the achievement of which Mr Jones was most proud was that he held Commissions from three different sovereigns in the three services—the Navy, the Army, and the Civil Service. An enthusiastic Volunteer, he served with the Royal Scots as long as age and his other duties permitted, and retired with the rank of Captain.

Keenly interested as he was in the practical side of sea-fishery

matters, Mr Jones was by no means blind to the necessity of theory and investigation. It was mainly under his chairmanship that the development of the Marine Research staff and the laboratory of the Board at Aberdeen took place, and he was one of the British Delegates to the International Council for the Exploration of the Sea. It was owing to his encouragement and advice on the administrative side, and due to the manner in which he sponsored the applications for the necessary funds, that the scientists under the charge of Dr Bowman have been able to take the place in international deliberations which is now accorded them, and to receive the consideration which their achievements merit. The author of a number of papers on fishery subjects, he was elected a Fellow of the Society in 1924, and was also a Fellow of the Royal Statistical Society.

At first a stranger to Scotland, Mr Jones, by his personality and accomplishments, quickly made himself at home and his presence appreciated wherever he went. An accomplished musician, with a deep affection more particularly for the older and more simple melodies, he was for long a member, and for a time the President, of the Edinburgh Harmonists Society, of which a predecessor in the fishery connection, Gilbert Innes of Stow, was the founder. A Freemason, a golfer, a bowler, and of a genial social disposition, Mr Jones had a very wide circle of friends and a still larger number of acquaintances who mourn the loss of one keenly interested in all that went on around, and who was always willing to help others in need with assistance and advice. His death on 4th February 1931 is the more tragic, in that it occurred within a very few months of the time when he was looking forward to some rest and recreation free from the cares of his long official life. His wife predeceased him by a few years and three daughters survive.

W. J. M. M.

Sir Francis Grant Ogilvie, C.B., LL.D., M.A. (Aberdeen), B.Sc. (Edinburgh).

Sir Francis Grant Ogilvie was a scion of an Aberdeen family that produced a number of men well known and distinguished in the educational world of Scotland. His father was Headmaster of Gordon's College; one of his uncles was Headmaster of George Watson's College, Edinburgh. The tradition continues, for his sister is a distinguished geologist, Mrs Ogilvie Gordon; and his only son is Professor of Geography in Edinburgh University.

Sir Francis Grant Ogilvie (born 1858) studied Arts in Aberdeen University. His special subject was Physics, but after graduating M.A. he came to Edinburgh, where in addition to Engineering he devoted much attention to Natural Science. He was a favourite pupil of Sir Archibald Geikie and a friend of Sir John Murray of *Challenger* fame, and throughout his life he was an ardent student of Physical Geology, and in particular of the relation of geology to scenery and the economic applications of geological investigations. In 1880 he became Assistant Professor of Natural Philosophy in Aberdeen University, in 1882 Science Master at Gordon's College, and in 1886 he was appointed Principal of the Heriot-Watt College, Edinburgh, and also Professor of Applied Physics in that institution. In that work he spent several busy years, and was well known as a member of committees and learned societies in Edinburgh, serving on the Council of the Royal Society of Edinburgh during the years 1901 to 1903. In 1900 he became Director of the Museum of Science and Art, Edinburgh, and in 1903 he was appointed Principal Assistant Secretary (Technology and Higher Education in Science and Art), Board of Education (London). In 1910 he was nominated Secretary of the Board of Education for the Science Museum and the Geological Survey, and in 1920 he passed over to the newly created Department of Scientific and Industrial Research as Principal Assistant Secretary. From this post he retired on pension in 1922, but continued voluntarily as Chairman of the Geological Survey Board till 1930.

It is sufficient to say of Sir Francis Ogilvie that he left his mark on every one of the numerous official charges that were entrusted to him. Ardent, sincere, and painstaking, he had a very wide knowledge of men and affairs, and was a very prudent and sagacious counsellor, inclined to caution but never despondent or pessimistic.

The high positions to which he attained showed how much his administrative ability was esteemed; and those who were brought into contact with him in the course of official work never failed to appreciate the breadth of his views, his mastery of detail, and his sympathy with all educational advances. In Edinburgh his work for the Heriot-Watt College gave a great stimulus to technological education, and many men who subsequently filled important posts were trained in that school. He knew most of his students personally, and in after life followed their careers with great interest. He took a part in organising displays at various exhibitions at Paris and in America, and the experience thus obtained was the foundation of his subsequent work in the Royal Scottish Museum and the Science Museum (London). At the invitation of the War Office he raised, and for many years commanded, the Forth Division (Submarine Mining) of the Royal Engineers (Volunteers). During the War he was an Assistant Controller of the Trench Warfare Department, and served also in the Chemical Warfare Department. Throughout his life he was essentially an "open-air" man, intensely fond of a tramp on the Braemar Hills or the woodlands of Surrey, and though his opportunities were limited, his geological studies continued till the close of his life; in fact he was engaged in mapping the Cretaceous rocks around Shere in Surrey, where he lived, during his years of retirement after active service in official posts. Although he has left few printed contributions to scientific literature, his influence on education and research was very great. As Chairman of the Geological Survey Board he was responsible for extensive programmes of investigation, and did much to encourage the progress of geological science in Great Britain.

Sir Francis Ogilvie was elected a Fellow in 1888, and died on 14th December 1930.

J. S. F.

Sir David Paulin.

SIR DAVID PAULIN, who passed away on 20th December 1930 at the age of eighty-three, was a son of Mr George Paulin, Rector of Irvine Academy. He commenced his business life as a Banker, but early developed a great interest in Life Assurance, which eventually led to his adopting the insurance profession for his career. In 1881, jointly with Mr James Sorley, also a Fellow of the Royal Society of Edinburgh, he founded the Scottish Life Assurance Company, of which he was Manager for the long period of thirty-seven years, retiring in 1918. The success of that Company is the best tribute to the energy and judgment of himself and of the colleagues that he gathered round him. Many honours in his profession came to him; he was President of the Actuarial Society and also of the Insurance Society of Edinburgh, and he filled for two years the position of Chairman of the Associated Scottish Life Offices. Knighthood was conferred on him in 1909, this being the first occasion in Scotland on which Royal recognition was given to insurance.

In the world of Science his interests lay mainly in Economics; from his earliest days he was given to the study of Statistics, and quite a number of papers came from his pen for the benefit of various Insurance, Economic, and Statistical Societies. At one time he was a forceful and attractive lecturer on Thrift, and his influence in pressing his teaching was widespread. All his life he took a very great interest in ecclesiastical affairs. He was prominent as a Churchman, having long been a leading layman of the United Free Church of Scotland. The cause of union with the Church of Scotland had in him a warm supporter, and it was a matter of great satisfaction to him when the cause of Union finally triumphed and the two churches became one. He was a strong promoter of temperance reform, and had a life-long connection with the Y.M.C.A., being at one time Treasurer of the National Council. A keen and successful man of business, he was generous to a degree in the support which he gave to every movement which in his opinion made for the well-being of his fellow-men.

In the days when the Royal and Ancient Game was not so popular as it is now, Sir David was an enthusiastic golfer, and many a one owed to him his initiation into the game. His country dwelling was at Machrihanish in Argyllshire, where he kept open house to such an extent as almost to make it a Clubhouse for the district.

Interesting as were some of his contributions to various learned Societies, it is not for these that he will be remembered, but for his chief characteristic—a genius for friendship. Full of years and honour, he has been gathered to his rest, leaving behind him troops of friends who will long cherish the memory of a very gracious personality.

He was elected a Fellow of the Society in 1892.

L. P. O.

Peter Pinkerton, M.A., D.Sc.

PETER PINKERTON died on 22nd November 1930.

Born in 1870, he received his early education in Kilmarnock Academy. He left this school, medallist in Classics and Mathematics, and proceeded to Glasgow University in 1886. Four years later he graduated M.A. with first-class honours in Mathematics and Natural Philosophy. He was awarded at the same time the Breadalbane and John Clark Scholarships in those subjects.

Having decided to prosecute still further his studies in these favourite subjects, he attended for two years the Royal College of Science, Dublin, where an intimate association with Professor Orr greatly influenced his intellectual outlook.

He was appointed a Mathematical Master in Allan Glen's School, Glasgow, in 1893, and during six years' splendid work in this school he laid the foundation of a great teaching career.

From 1899 to 1902 he was Headmaster of the Mathematical Department of the Royal Academical Institution, Belfast; from 1903 to 1913 he was head of the Mathematical Department in George Watson's Boys' College, Edinburgh, acting also as Deputy-Headmaster in this school during his last two years of service. The extraordinary success of his pupils at Edinburgh University and elsewhere bears eloquent testimony to his brilliant mathematical teaching.

In 1909 Glasgow University conferred upon him the degree of Doctor of Science, and in 1930 that of Doctor of Laws.

From January 1914 to his death he was Rector of the High School of Glasgow, and his occupancy of this distinguished post still further enhanced his great reputation as an educationist and administrator.

He always took a leading interest in the affairs of the Edinburgh Mathematical Society. As an author, the publication by which his name is most widely known is the book on the *Elements of Analytical Geometry* which he wrote in collaboration with the late Professor Gibson of Glasgow University.

He was elected a Fellow of the Society in 1905.

W. A.

James Lorrain Smith, M.A., M.D., LL.D., D.Sc., F.R.C.P.Ed., F.R.S.

JAMES LORRAIN SMITH died in Edinburgh on 18th April 1931. He was in his sixty-ninth year, having been born on 21st August 1862, at Half Morton, Dumfriesshire, the fourth son of the late Rev. Walter Smith.

He was educated at George Watson's College, Edinburgh, and afterwards at Edinburgh University, where, in 1884, he graduated M.A. with first-class honours in Philosophy. He then commenced the study of medicine, and in 1889 graduated M.B., C.M., with first-class honours. In 1893 he obtained his M.D. degree, with a gold medal for his thesis on "Thyroidectomy and Respiratory Exchange—a Contribution to the Pathology of Myxœdema."

Following his graduation in medicine in 1889, he had a term as resident in the Edinburgh Royal Infirmary under Dr Affleck (later Sir James Affleck), and then he went to Oxford University, where he worked under Professor Burdon-Sanderson. While there he collaborated with Dr J. S. Haldane in a brilliant series of investigations on the respiratory function and oxygen capacity of the blood. One outcome of this work was the now well-known method of estimating the blood volume by means of carbon monoxide.

In 1892 he went to Cambridge as John Lucas Walker Scholar under Professor Roy and continued his physiological researches.

In 1895 he was appointed Lecturer in Pathology in Queen's College, Belfast (now Queen's University), and soon his lectureship was made a Chair. He was also Honorary Pathologist to the Royal Victoria Hospital in that city.

In 1904 he became Professor of Pathology in Manchester, and in 1912 he was called to the Chair of Pathology in Edinburgh to succeed his former teacher, Professor Greenfield.

Such is the bald recital of his progress from one area of work to another.

Beginning as a student on the Arts side, he gained distinction in pure philosophy. For many men this would have determined their direction of life work. For him it served but as a rich background for his later more practical studies. In medicine he had a like brilliant course, and naturally to his type of mind the scientific side of the

profession offered ripe opportunities for probing the unknown and elucidating the obscure. At first it seemed as if he were destined for a career in physiology, but for him this was but a means towards the study of abnormal function, and his appointment to Belfast was the definite departure on his life work.

The earlier training, philosophical and physiological, gave him a wonderful breadth of view and width of resource in his subsequent pathological work—indeed it explains the trend of his observations and methods of teaching.

His researches were in many fields, but whatever he touched he illuminated. The basic problems interested him, as is seen first in his respiratory and blood work. Later, in Manchester and in Edinburgh, the primary changes in degeneration, especially fat and lipoid changes, occupied his attention. During the War his laboratory became the scene of activity on some of the very practical problems affecting the troops. The best-known result is the hypochlorite antiseptic that became known as "Eusol," but he also, in conjunction with James Ritchie and J. W. Dawson, demonstrated experimentally the pathogenesis of "Trench Foot." In addition to these matters he attacked the problem of the prevention of gassing in characteristic fashion. He showed by personal experiment how charcoal could absorb large amounts of chlorine—one of the early war gases used. Charcoal was available in the trenches for braziers: the soldiers had socks; therefore he argued and demonstrated that—as an emergency measure at least—by filling a sock with charcoal and holding it over mouth and nostrils it was possible to exist for a time in an atmosphere thick with chlorine. This work is mentioned here in some detail as it was never published and, indeed, more efficient and elaborate methods were devised for combating gas, but it shows his practical mind and how he seized upon what was to hand and put it to good use.

It was the same practical spirit which led him to devise the method of teaching pathology by concrete examples. This method, begun in Manchester and perfected in Edinburgh, depends upon the intensive study of a series of cases. The functional disturbances as shown by the clinical signs and symptoms are compared with the disordered structures as seen in the pathological lesions. In a selected series of cases the main pathological processes can be illustrated and the student has definite pictures around which he can group his knowledge: also he is initiated into the method of accurate observation of data and the correlation of these as regards cause and effect. In Edinburgh Lorrain

Smith still further extended his method by organising a course for the third-year medical students wherein the subjects of medicine and surgery are studied concurrently with those of pathology, pharmacology, and therapeutics: each one complementary to the other.

The Medical Faculty of Edinburgh University, recognising his organising ability, elected Lorrain Smith Dean in 1919, an office he held till his death. To him this office was no sinecure, for he devoted much care and thought to the multifarious matters that came before him. His clear grasp of essentials and his breadth of view and ready sympathy made his occupancy a happy one for his colleagues. Having arrived at a decision he adhered to it and saw to its accomplishment, not ruthlessly but gently and smoothly, though quite effectually.

In the wider sphere of medical administration he took his full share, as in various Government inquiries such as the Royal Commission on Irish University Education in 1901, and on the pathological effects of factory conditions in the spinning industries. He was a member of the General Medical Council in 1912–1913, representing Manchester University, and again in 1927 as the representative of Edinburgh University. He acted as Vice-President of the section of Pathology at the Annual Meeting of the British Medical Association in Oxford in 1904, and as President of the section of Pathology and Bacteriology at the annual meetings at Cambridge in 1920 and at Edinburgh in 1928.

Lorrain Smith was possessed of a strong personality. Handsome of face, tall and spare in stature, he was rather reserved in speech, weighing his words carefully before giving them forth. Though deeply versed in academic philosophy and in the practical philosophy of life his was a mind simple, direct, and eminently sympathetic. He was not easily ruffled: serene in trouble, he possessed that optimistic outlook that refused to be daunted by difficulties, which, for him, were but created to be tackled cheerfully and valiantly. Externally somewhat austere, he preserved a quiet humour close beneath the surface, ready to break forth and ease the tiresome toil or cheer the respite from work. Quietly happy at work, patently happy at play, he had the faculty of attracting people to himself and of working with them in brotherly collaboration, and he was ever generous in according credit to those co-workers for results achieved.

His friendship was a thing to value and is a treasured memory to many.

He gave of his wisdom and of his humour freely and joyfully.

He received without asking the best his friends could give in

return. In time of trouble no one could be more helpful or more gentle.

He has left an unfinished monograph on "Growth," a product of his ripe wisdom and experience. He has bequeathed to his generation and his friends a finished and beautiful product in his life work and personality, a product which will grow and bear fruit in ways none may foresee.

He was elected a Fellow in 1915, and served on the Council of the Society from 1918-1921.

A. M. D.

Spencer Campbell Thomson.

We regret to announce the death of Mr Spencer C. Thomson, which took place in Edinburgh on 11th May 1931. Mr Thomson carried on a great family connection with the Insurance world in Scotland, when he succeeded his father, the late Mr William Thomas Thomson, as Manager of the Standard Life Assurance Company in 1874.

Mr Spencer Thomson was born on 16th October 1842 at 3 George Street, Edinburgh, where his father occupied a flat over the Standard building there. He was educated at the Edinburgh Academy and Rugby, and graduated at Cambridge. In 1865 he entered the service of the Standard Life Assurance Company as Assistant Actuary, and after being Assistant Manager, succeeded his father as Manager in 1874.

Mr Thomson was Honorary President of the Actuarial Society of Edinburgh in 1877 and again in 1886; and in 1890–1892 he was President of the Faculty of Actuaries. Among his contributions to the *Transactions* of that body, the best known is the one relating to Mortality in India and other tropical countries, which for years was a standard work of reference on that subject. From 1892 to 1895 he was Chairman of the Associated Scottish Life Offices, a position which his father had held from 1867 to 1873. He was a Fellow of the Royal Society of Edinburgh (1870) and a former President of the Cockburn Association, an organisation in which he took a keen interest.

In 1869 Mr Thomson married Georgina Maria Joanna, daughter of George F. Cockburn of the Bengal Civil Service, a son of Lord Cockburn. After her death he married in 1919 Helen Gladys Walker, a granddaughter of Sir William Stewart Walker, K.C.B., of Bowland. He is survived by his widow, two sons, and one daughter.

Sir Byrom Bramwell, M.D., LL.D., D.C.L.

Byrom Bramwell, whose death occurred upon 27th April 1931, was the son and grandson of medical men. An Englishman by birth, descent, and sympathy—though when south of the border it would be difficult to find one who was more loyal to the country of his adoption—he came of a long line of yeoman farmers who hailed from the Duchy of Lancaster. Born at North Shields on 18th December 1847, the eldest of a large family, he was sent at an early age to Cheltenham College, where he made his mark both in the class-rooms and on the field. For three years he was a member of the cricket eleven, receiving his cap from R. T. Reid (afterwards Lord Loreburn), and during his last year he played football for the school. His success in after life was predicted by his master, J. Brook Smith, who, in a letter to his father, wrote: "Byrom has not a sufficiently exalted opinion of his powers. I am writing to request you to urge him to aim at the highest honours in his profession, for he has much more than average ability."

Upon leaving school it was only natural that he should proceed to Edinburgh, for it was at the Academy and University of Edinburgh that his father had been educated. His early promise was fulfilled at the University, for during his undergraduate career he gained many medals, and in the class of Medicine obtained 100 per cent. of the available marks. Nor did he confine himself to his books, for, *inter alia*, he captained the University cricket eleven in his second year. The popularity of "the Baron," as he was known to his intimates, was evinced by a remarkable ovation at the time of his graduation.

After acting for six months as house surgeon to Professor Spence, he was invited by Laycock, the then Professor of Medicine, to become his University Assistant. It was with reluctance that he decided to decline this invitation and the opportunity it afforded of staying in Edinburgh, but he felt that it was his duty—and one of Bramwell's striking characteristics was his sense of duty—to return to North Shields and assist his father, whose health was not too robust, in carrying on a large general practice. This was a turning-point in his career. It is interesting to speculate as to what the effect of Laycock's mental outlook, habit of thought, and general influence might have been upon an enthusiast of impressionable age whose subsequent reputation, determined

of necessity by the opportunities afforded in practice, was essentially characterised by exact personal observation, exceptional powers of logical inference, and unusual clarity of expression.

During the next five years he was strenuously engaged in general practice, while he also held appointments as Medical Officer of Health, Police Surgeon, and Physician to the local Dispensary, Fever Hospital, and Convalescent Home. Yet from the first, despite his busy life, he found time to keep records of his cases and post-mortems, a habit which he continued throughout his life, and to make the most of an experience which, as he often said, proved invaluable to him later. And how different general practice was in those days! One has only to read the vivid descriptions of personal experiences of cholera and smallpox in his *Atlas of Clinical Medicine* to realise this. For his father he had a deep affection and admiration. Thus he writes, "Any clinical ability and knowledge which I myself may have acquired are in great part due to his teaching and example, and to the experience I gained when I had the advantage of being associated with him. I well remember a most typical case of myxœdema to which my father drew my attention when I joined him in practice in the year 1869. (The first written description of myxœdema was published by Sir William Gull in 1873.) He pointed out to me all the essential features of the disease, the peculiar character of the œdema, the normal condition of the urine, the delicate pink flush on the cheeks, the transparent, waxy-like character of the skin of the face and eyelids, the dullness of the intellect, the persistent low temperature, and the fact that the patient was most susceptible to cold. All these—the most striking features of the disease—my father pointed out to me in detail. He further stated that he believed the condition was a new disease —a disease which had never been described. How well I remember the incredulity with which I received his statement." His father died in 1884 from the effects of a rheumatic carditis, and the book upon *Diseases of the Heart and Thoracic Aorta*, which was published in the same year, was dedicated to his memory.

Appointed in 1874, at the early age of twenty-seven, Honorary Physician and Pathologist to the Newcastle Royal Infirmary and Lecturer on Clinical Medicine in the Durham University School of Medicine (he had for three years previously held the appointment of Lecturer on Medical Jurisprudence), Bramwell left North Shields and commenced practice as a consulting physician in Newcastle-upon-Tyne. Practice was at first scarcely to be expected, and the next five years were largely devoted to clinical and pathological observation and teaching. An indefatigable

worker, with wards under his own care, he spent much of his time in hospital, revelling in the opportunity thus afforded of acquiring clinical experience, and collecting material which was to form the basis of many of his subsequent publications. The *Transactions of the Northumberland and Durham Medical Society* at this date contain numerous contributions from his pen. It was then that he had as his colleague David Drummond, with whom he formed an intimate and lifelong friendship. Sir David Drummond many years later, when Bramwell was presented with his portrait, gives a vivid description of his personality in the Newcastle days. "When I first met him," Sir David says, "I had just returned from Germany and had only recently passed from under the beneficent and dominating influence of Professor William Stokes, my revered clinical chief; and my satisfaction may be imagined when, on my appointment as physician to the Royal Infirmary, I found in the colleague who was to be my most intimate associate on the staff, and co-worker in the same wards, a man after my own heart, and one who combined the thoroughness of the German with the courtesy and qualities of the heart and scientific enthusiasm—the characteristics of my old teacher, Stokes—of the best type of British physician. I felt that I had left the spirit of Stokes in the December of his life to meet it afresh in the springtime of Bramwell's."

Ambitious for a larger sphere for his activities, Bramwell left Newcastle for Edinburgh in 1879. His ultimate success appeared to be assured, but at first he had a hard struggle. Edinburgh was very conservative in those days. The portal for the consultant had always been through general practice, and no one in Scotland had hitherto confined himself to consulting practice at such an early age, although this was done in London and in some of the English provincial towns, and is nowadays a very general custom. But, further, he had the temerity to take a house to the west of Queensferry Street in a part of Edinburgh in which no consultant had hitherto practised. The presumption of this young man from south of the Tweed incurred the resentment of some of the senior members of the profession, and even his old friends shook their heads. It may be that his critics failed to realise that he had already had a clinical experience such as no one at his age had hitherto enjoyed in Edinburgh. To those who said to him, "No one has done this in Edinburgh," his reply was "But I intend to do so, and is there any reason why I should not?" There can be no doubt that an atmosphere of antagonism was thereby engendered which persisted for years and, as he admitted, rather embittered his outlook for many a day.

But "no difficulty baffles great zeal," and throwing himself heart and soul into the work of teaching, he soon became a tower of strength in the Edinburgh Academical School. In 1879 he commenced a course of lectures upon Practical Medicine and Medical Diagnosis—another innovation—which soon became very popular; and in 1880 he began to lecture on the Principles and Practice of Medicine, a course of lectures which he continued for many years. Appointed pathologist to the Royal Infirmary in 1882, a post which afforded him the opportunity he desired for observation and research, he became an assistant physician in 1885. The would-be consultant in those days—and this applies to the assistant physician—took no part in the clinical teaching in the wards, and little attention had previously been given to out-patient teaching in the Royal Infirmary. Bramwell's out-patient cliniques became so popular that after a time he had to transfer his teaching to one of the large clinical theatres. When, in 1897, he was appointed a full physician, his Wednesday cliniques on Clinical Medicine became one of the most popular classes in the Edinburgh Medical School. In 1886 he was elected a Fellow of the Royal Society of Edinburgh, and during the years 1891, 1892, and 1893 he was a member of the Council. His pen was never idle during those strenuous years, his reputation as a clinician, teacher, and investigator was world-wide, and for many years he enjoyed one of the largest consulting practices in Scotland.

His appointment to the Chair of Medicine in Edinburgh in 1900 would have met with general acclamation both in this country and abroad, but this was not to be. "We thought," writes Professor Lovell Gulland, his old friend, former assistant physician, and colleague, "that his opportunity and that of the School had come, when at the death of Grainger Stewart the Chair of Medicine became vacant. Bramwell presented a marvellous collection of testimonials from every part of the world, a wonderful evidence of his fame and reputation, but to our—and his—bitter disappointment he was not elected." He continued his teaching, however, up to 1912, when his term of office as physician to the Royal Infirmary expired, and for the next five years, as physician to Chalmers Hospital, he was still in charge of wards. When his time at the Royal Infirmary came to an end, his colleagues and friends entertained him to dinner in the Royal College of Physicians, an honour which has not been extended to any other clinician in Edinburgh.

"As a clinical teacher," Professor Lovell Gulland writes, "he was superlatively good, clear, concise; dogmatic when it was possible to be so, cautious when it was not, bringing to the illustration and elucidation of a doubtful case the whole of his vast experience. He *was* the Edinburgh method of teaching at its best."

Of his teaching Dr Robert Hutchison writes, "He was pre-eminently a clinical teacher and was always happiest in the out-patient room or at the bedside. Many of his old students must have vivid recollections of his Saturday morning out-patient cliniques. He was seen in these at his very best, and the keen and accurate observation which he showed, and his brilliant and sometimes dramatic demonstration of cases, were a lasting inspiration to his audience."

"No one," writes Dr S. A. Kinnier Wilson, "who in other years was associated closely with Dr Byrom Bramwell, as he then was, as senior student, house-physician, or assistant is ever likely to forget the immensely stimulating influence of contact with him. At a time when the Edinburgh school was adorned with a number of distinguished men on the medical side, when its reputation was supreme and its classes thronged, Bramwell easily took his place in the front rank of students' favourites."

"As a consulting physician," writes Professor W. T. Ritchie, "Sir Byrom Bramwell stood in the very forefront, not only because of his intellectual attainments, but also inasmuch as his professional colleagues knew that in him implicit trust and confidence could be reposed. A clinician of vast experience, most impressive manner, and extraordinary sound judgment, he had the true clinical instinct of recognising and interpreting the essential while discarding the immaterial. In an incredibly short time he would solve the problem of a difficult case; his opinion regarding diagnosis, prognosis, and treatment was authoritative; he was the final court of appeal."

"Byrom Bramwell," writes Dr Robert Hutchison, "was a type of physician now becoming rare, for although his special leanings were towards neurology, yet he touched medicine at every point, and wherever he touched he adorned."

Sir Humphry Rolleston, the President of the Royal College of Physicians of London, in presenting a replica of his portrait to the sister college in Edinburgh, said of him: "To recite what he has done for medicine would be a task like reading an address on the modern advances of the last forty years."

There were indeed few departments of medicine upon which he did not leave his mark. During thirty years, from the time when he commenced practice in Newcastle to the end of the century, several books and more than a hundred and sixty communications issued from his pen. His work on *Diseases of the Spinal Cord*, published in 1881, was richly illustrated by sections and drawings, chiefly made by his own hand. It was translated into German, French, and Russian, while several editions were reproduced

in America. His *Diseases of the Heart and Thoracic Aorta*, published in 1884, was largely a record of personal experience, both clinical and pathological, and again its numerous illustrations were mainly derived from his own work. This book attained a great success in its time, and was often referred to in his lectures by one of the most distinguished German clinical teachers of the day; but being produced immediately before what may be called the instrumental epoch of cardiology, it is not now consulted as much as its usefulness deserves. *Intracranial Tumours* was published in 1888. Dr Harvey Cushing of Boston, in referring to this work in his 1930 Lister Memorial Lecture, remarked: "It has been said that if some unusual clinical condition turns up concerning which one seeks information, an account of it is likely to be found in Jonathan Hutchinson's *Archives*. To this I would like to add, particularly for the benefit of neurologists, that if Hutchinson fails, then try Byrom Bramwell." In this book too Bramwell was the first to draw attention to the connection of tumours of the pituitary body with an excessive development of fat and the presence of diabetes. His monumental *Atlas of Clinical Medicine* in three large volumes, illustrated by many coloured plates, some of them perfect examples of the lithographer's art, was issued between 1892 and 1896. The work on *Anæmia and Diseases of the Blood-forming Organs and Ductless Glands*, published in 1899, a record of his personal experience, was a comprehensive account of what was then known in this department of medicine. But of all his publications his *Clinical Studies*, which appeared in eight volumes from 1903 to 1910, were most widely read by the profession. Embodying as they did his bedside teaching and cliniques, they had a large circulation in this country and in America.

Among Bramwell's particular contributions to the advance of medicine, it may be recalled that he was the first, in 1875, to introduce the arsenical treatment of pernicious anæmia, which was the method employed all the world over prior to the recent introduction of liver therapy. Again, in 1872, in a paper read to the Northumberland and Durham Medical Society, he described for the first time the transmission of scarlet fever by milk supply. He was a strong advocate of the compulsory notification of phthisis long before its introduction, and in 1893 he introduced the thyroid treatment of psoriasis and other skin diseases. Then in 1902 he first described a condition of infantilism due to pancreatic disease.

Honours naturally came to him, though he was not one of those who sought them. The Universities of Edinburgh, St Andrews, and Birmingham conferred on him their honorary LL.D. degrees, and from the University of Durham he received the degree of D.C.L. He was chosen

in 1899 to deliver the Morison Lectures in the Royal College of Physicians of Edinburgh, and in 1910 he was elected President of that College. In 1885 he presided over the section of Medicine, and in 1898 over that of Neurology, at the Annual Meeting of the British Medical Association. He became President of the Medico-Chirurgical Society of Edinburgh in 1909, and President of the Association of Physicians of Great Britain and Ireland in 1923, when his admirable summing-up of several of the discussions was the subject of general comment. In the same year he was elected under By-law XXXVIII (*b*) a Fellow of the Royal College of Physicians of London, the first clinician to receive that distinction, and at this time his friends and former pupils took occasion to present him with his portrait. At this time too he was elected the first Honorary President of the Royal Medical Society of Edinburgh, a distinction which he greatly prized. In 1924 he received the honour of knighthood "for services to medicine." He was a corresponding member of the Neurological Societies of Paris and Philadelphia and of the German Neurological Society, and an honorary member of the Société des Médecins Russes de St Petersburg, of the American Neurological Association, and other medical societies in this country.

A man of particularly robust physique, of boundless energy and power of concentration, a strenuous, untiring, and very thorough worker, every moment of his day from early morning to bedtime was, in his earlier years, fully occupied. His activities were at their height at a time when attention was focussed upon the application of morbid anatomy to symptomatology. A profuse note-taker, he was able to review his personal experience upon the topic of the moment. Possessed to a quite exceptional degree of the faculties of observation, clarity of thought, logical inference, perspective, and judgment, he regarded the methods of precision and of the laboratory from the standpoint of the clinician as adjuncts which were to be valued in direct relation to the information they afforded and their practical utility. He did not suffer fools gladly; he had no use for the "text-book clinician," nor for the man whose thoughts were always "in the air." But for a great scientific thinker (and here Hughlings Jackson was his prototype) he had an admiration almost amounting to reverence. He took little interest in medical politics and procedure, and committee work did not appeal to him. On the other hand, his mental attributes were such as would have insured his success at the Bar or on the Bench.

A good athlete as a youth, he continued to take a keen interest in cricket and football up to the end. Even a few years before his death he

liked nothing better than to spend a whole day watching county cricket at "Lords" or at the "Oval." A keen angler from boyhood, he boarded as a student with William Stewart of *The Practical Angler*; this no doubt accounted for his skill with the "upstream worm." Many a pleasant day he spent on Loch Leven in an annual competition between the staff of his wards and that of the late Dr George Gibson, each side captained by its Chief, and many a happy day too he had on Threipmuir, in the Pentlands, during the years 1896 to 1919, when Baberton House was his country residence.

The last few years of his life, after the death of his beloved wife, who had done so much to encourage him and in his earlier days to help him with his work, were spent at 10 Heriot Row, where his wife's cousin and lifelong friend, Miss Meggison, tended him with every care. His time was largely occupied in reading, and he kept himself abreast of all that was going on through the columns of the *Times*. He took a special interest in astronomy, and would read any popular books upon which he could lay his hands, more than once saying that if he had to begin life over again he would adopt astronomy as the profession of his choice. He lived for his children and his grandchildren, whose health, happiness, and success were ever in his thoughts. All the old antagonisms and struggles were memories of the distant past. He lived, indeed, at peace and happiness with all men. The visits of his old friends Sir David Drummond, Professor Lovell Gulland, Sir Edward Sharpey-Schafer, the late Dr John Macdougall of Cannes, Dr G. Sandison Brock, Sir Ashley Mackintosh, Dr F. N. Kay Menzies, Dr R. M. Ronaldson, and many others, were to him a special pleasure; while he often spoke of the visit paid him by his former house-physicians Dr George Gibson, Mr J. M. Graham, and Dr Hugh Davidson on his seventy-fifth birthday. Clear in his mind to the very end, he greeted Dr Fergus Hewat, who called to see him the day before his death, though speaking with difficulty, by telling him that he was glad to hear that he had been elected a member of the Threipmuir Angling Club, though the fishing had never been so good there after the old plantation was cut down.

There are many who will feel of Bramwell as did Sir David Drummond when he wrote: "He is long to be remembered as one of the greatest clinicians of our time, but to me his reputation as a physician is overborne by the memory of a loyal and warm friend whom to know was to love and to admire." E. B.

WILLIAM ELDER, M.D., F.R.C.P.E., was educated at the Knox Institute, Haddington, and Edinburgh University. He became a Resident Physician in Edinburgh Royal Infirmary, and in 1893, after assisting in general practice in Kirkcaldy, Manchester, and Leith, he was appointed Assistant Physician to Leith Hospital. From 1894 until 1909 he was Physician with full charge of wards, and from 1909 was Consulting Physician to the Hospital. He was the author of several important medical works and a frequent contributor to medical journals.

Dr Elder was elected a Fellow of the Society in 1904, and died in Edinburgh on 2nd July 1931, aged 66.

JOHN EDWARD GEMMELL, M.B., C.M. (Edin.), became House Surgeon at the Royal Infirmary, Liverpool, and Demonstrator in Pathology and Holt Scholar in Pathology and Physiology at University College. Later he held the appointments of Honorary Consulting Surgeon to the Hospital for Women, and to the Maternity Hospital, Liverpool. Dr Gemmell was a past President of the Liverpool Medical Institute, and was twice President of the North of England Obstetrical and Gynæcological Society. He was the author of papers in the *Brit. Journ. of Obstetrics and Gynæcology*, 1905, 1908, with A. L. Robinson in the *Lancet*, 1921, and in the *Med. Pres. Circ.*, 1926, etc.

He was elected a Fellow of the Society in 1914, and died on 24th January 1931, aged 68.

CHARLES ROBERT GIBSON, LL.D. (Glas.), who was born in 1870, was a well-known writer of popular science books. From 1922 until 1925 he was President of the Royal Philosophical Society of Glasgow. Dr Gibson did most of his literary work whilst actively engaged in business as head of the firm of Gibson Bros., curtain manufacturers, Pollokshaws.

He was elected a Fellow of the Society in 1910, and died on 6th January 1931. (See also notice in *Proc. Roy. Phil. Soc. Glas.*, vol. lix, 1930–31, p. 59.)

JOHN ANGUS MACDONALD, M.A., B.Sc., lately Science Master, King Edward VII College, Johannesburg, died in Edinburgh on 28th June 1931.

He graduated at Edinburgh University in Arts in 1895 and in Science in 1896. In the course of his University career, in addition to graduating with honours, he obtained the Neil Arnott Prize in Physics and the John Edward Baxter Scholarship in Mathematics. Mr Macdonald was elected a Fellow of the Society in 1904.

ERNEST ROMNEY MATTHEWS, M.Inst.C.E., was born on 16th January 1873, and educated at St Michael's School, Hastings. His father, Chief Officer of the Coast Guards at Hastings, was one of the few survivors of the troopship *Birkenhead*. He held the appointment of Chief Drainage Engineer to H.M. Office of Works, and was formerly Chadwick Professor of Civil Engineering in the University of London. He addressed the British Association on Coast Erosion, gave evidence before the Royal Commission, and also supplied the Royal Commission on Canals and Waterways with data on the silting of estuaries. He was Bessemer and Nursery Prizeman of the Society of Engineers, a medallist of the Royal Society of Arts, and a former member of the Councils of the Institution of Municipal and County Engineers and the Royal Sanitary Institute. Professor Matthews rendered distinguished service as Commanding Officer, during the War, of the 34th and 57th Sanitary Sections, B.E.F., was twice mentioned in dispatches and received the 1914 star. His publications include *Coast Erosion and Protection*, *Reinforced-Concrete Construction* (joint-author), *Refuse Disposal*, *Logarithms and Trigonometry* (joint-author), *Dam Construction*, and many papers to the technical journals.

He was elected a Fellow in 1902, and died on 6th November 1930, aged 57.

ALBERT ABRAHAM MICHELSON, Nobel Laureate, Physics, 1907, was born in Strelno, Poland, on 19th December 1852. He was Professor of Physics in the University of Chicago from 1892 until 1929, and was probably best known for his wonderful experimental work to detect any effect of the earth's rotation on the velocity of light.

He was elected a Foreign Honorary Fellow of the Society in 1910, and died on 9th May 1931. (For details of his life and work see notice in *Proc. Roy. Soc. Lond.* (not yet published), *Nature*, 16th May 1931, and other notices.)

HON. ALFRED GEORGE NASH, B.Sc. (Edin.), A.M.I.C.E., was born in Mandeville, Jamaica, and was educated at St George's College, Kingston, and Edinburgh University. After holding engineering posts in this country, he returned to Jamaica in 1882, and for many years was engaged in various engineering enterprises. He was a member of the Legislative Council of Jamaica.

He was elected a Fellow of the Society in 1897, and died on 23rd December 1930.

THOMAS P. WATSON, M.A., B.Sc., was from 1899 to 1902 Head Science Master in George Watson's College, Edinburgh. Thereafter he was appointed Principal of the Trade and Grammar School, Keighley. He was elected a Fellow in 1900, and died on 25th September 1930.

APPENDIX.

CONTENTS.

PROCEEDINGS OF THE STATUTORY GENERAL MEETING
Beginning the 148th Session, 1930–1931.

At the Statutory General Meeting of the Royal Society of Edinburgh, held in the Society's Rooms, 24 George Street, on Monday, October 27, 1930, at 4.30 P.M.,

Sir E. A. Sharpey-Schafer, F.R.S., President, in the Chair,

the Minutes of the Statutory Meeting held on October 28, 1929, were read, approved, and signed.

The GENERAL SECRETARY submitted the following Report:—

SECRETARY'S REPORT, OCTOBER 27, 1930.

By request of the Council addresses were delivered on "The Early Colonisation of Northern Scotland as Illustrated by the Recent Discoveries in Orkney," by Professor V. G. CHILDE, on December 2, 1929; "Do the Wireless Echoes of Long Delay come from Space Outside the Moon's Orbit?" by Professor CARL STÖRMER, on February 17, 1930; "Base Exchange," by Professor GEORG WIEGNER, on May 16, 1930; "Philosophical Aspects of Atomic Theory," by Professor NIELS BOHR, on May 26, 1930; and on "Climate during the Pleistocene Period," by Dr G. C. SIMPSON, on June 16, 1930. Professor CHILDE'S, Professor STÖRMER'S, and Dr SIMPSON'S addresses have been published in the *Proceedings* of this session, and it is expected that Professor BOHR'S address will be published at an early date. 35 papers were read as compared with 42 in the previous session. The papers were divided among subjects as follows: Mathematics, 6; physics, 4; chemistry, 1; geology, 4; palæontology, 2; botany, 4; zoology, 4; animal genetics, 8; physiology, 1; engineering, 1. 10 papers have been published in the *Transactions* and 22 in the *Proceedings*. 2 papers were read but have not yet been submitted in final form, 6 have been declined for publication, and 1 is at present under revision.

The Society has lost by death 16 Ordinary Fellows and 1 Honorary Fellow, and 4 Ordinary Fellows have resigned. 27 Ordinary Fellows and 11 Honorary Fellows (6 British and 5 Foreign) were elected.

Invitations were received, and the Society was represented as follows on the occasions mentioned:—

1. Meeting to consider formation of Freshwater Biological Association in London, on February 21, 1930. Professor J. H. ASHWORTH.
2. Second International Congress for Sex Research, London, August 3 to 9, 1930. Professor F. A. E. CREW and Dr B. P. WIESNER.
3. International Botanical Congress (Discussions on Nomenclature), Cambridge, 16 to 23 August. Professor W. WRIGHT SMITH.
4. Centenary of the Royal Geographical Society, London, October 21, 1930, and two following days. Professor D'ARCY W. THOMPSON.

The undernoted gentlemen were appointed to represent the Society on the National Committee for the Biological Sciences—Professor J. GRAHAM KERR (until December 31, 1930), Professor J. H. ASHWORTH (until December 31, 1933).

Correspondence was received from the Foreign Office in regard to International Co-operation in the Scientific Study of the Polar Regions during a year to be called the "Polar Year." The GENERAL SECRETARY, Dr A. CRICHTON MITCHELL, and Mr J. M. WORDIE were appointed to represent the Society on a Joint Committee of the Royal Society of London and the Royal Society of Edinburgh, which had been formed to advise as to the feasibility of British collaboration in the project. The Report of this Committee has now been forwarded to the Foreign Office.

Professor T. J. MACKIE, University, Edinburgh, was appointed as the Joint Representative of the Royal Society of Edinburgh and the University of Edinburgh on a consultative Committee consisting of representatives of the India Office, Medical Research Council, Royal Society of London, London School of Hygiene and Tropical Medicine, University of Edinburgh, and the Royal Society of Edinburgh, the object of the Committee being to secure closer liaison between Medical Research Organisations in India and in this country.

The Association of Special Libraries and Information Bureaux has informed the Society that a panel of expert translators has been formed under its auspices.

In response to an appeal from the International Institute of Intellectual Co-operation, the Council decided to allow the name of the Royal Society of Edinburgh to be placed upon a list of offices, collaborating to assist investigators, on the understanding that the Society's position is that of an independent body which has volunteered its assistance.

The BRUCE-PRELLER Bequest of £100 has been used for binding books in the Library, in addition to £100 from the General Income, with a view to overtaking arrears.

£1000 was received under the will of the late Dr DAVID ANDERSON-BERRY, to be used "for the purposes hereinafter appearing; that is to say: I DIRECT the Society, through its proper officers, to invest the said sum of One Thousand pounds, and apply the accumulated income thereof every third year, reckoned from the date of the payment over of the said legacy, in the first place in the presentation of a gold medal, and in the second place in the payment of a sum of money to the winner for the year of such gold medal, the winner being the person who, in the opinion of the Society, shall be the producer for the year of the best essay on the nature of X-rays and their therapeutical effect on human diseases, on which subject I have worked for twenty years, this bequest to be in remembrance of myself."

New lighting has been installed in the Front Hall and Back Saloon (street floor) and in the Back Saloon, Geological Room, and Colonial Room in the basement.

During the session wooden shelving, costing £27, 17s. 6d., was erected in the basement, and two cases of steel shelving, by Roneo, Ltd., costing £105, in the Back Saloon (street floor), the cost of both being met from the proceeds of the sale of a spare set of *Transactions*, certain duplicates of the *Philosophical Transactions* of the Royal Society of London, and of the *Journal für Praktische Chemie*, together with certain other volumes examined by the Curator and General Secretary, and marked for disposal.

The following prizes were awarded during the session:—

KEITH PRIZE (1927–1929), to Dr CHRISTINA C. MILLER.
NEILL PRIZE (1927–1929), to Professor E. B. BAILEY, F.R.S.
JAMES SCOTT PRIZE (1927–1930), to Professor NIELS BOHR.
BRUCE PRIZE (1930), to Mr N. A. MACKINTOSH, M.Sc., A.R.C.S.

The thanks of the Society are due to the Carnegie Trust for the Universities of Scotland for grants to authors towards the cost of illustrations of papers published by the Society, amounting to £53, 4s. 2d.; and to the Royal Society of London, who dispense the Government Publication Grant, for £250 towards the printing of the *Transactions* and *Proceedings*; also to Professor W. C. M'INTOSH, F.R.S., for a donation of £30 towards the cost of printing his paper in the *Transactions*.

TREASURER'S Report:—

The TREASURER in presenting his accounts for the session compared the various items of income and expenditure with those of the previous session, and drew attention to the satisfactory position in regard to arrears of subscriptions, the amount outstanding being lower than it had been for many years. He mentioned that a balance of £100 on account of shelving purchased in 1927 had now been written off.

The PRESIDENT then made some remarks, mentioning that a former President of the Society—Professor F. O. BOWER, F.R.S.—had been President of the British Association Meeting at Bristol this year.

The PRESIDENT nominated as Scrutineers of the Ballot, Mr D. T. JONES and Professor F. W. SHARPLEY.

The Ballot for the Election of Council and Office-Bearers was then taken.

Mr J. BARTHOLOMEW moved the adoption of the Reports and the reappointment of Messrs LINDSAY, JAMIESON & HALDANE, C.A., as auditors for the ensuing Session. These motions were seconded by Mr J. MATHIESON, and approved.

The Scrutineers reported that the Ballot Papers were in order, and the PRESIDENT declared that the following Office-Bearers and Members of Council had been duly elected:—

Professor Sir E. A. SHARPEY-SCHAFER, M.D., D.Sc., LL.D., F.R.S., President.

Professor J. GRAHAM KERR, M.A., F.R.S., F.L.S. Professor W. WRIGHT SMITH, M.A. Professor F. G. BAILY, M.A., M.Inst.E.E. Professor T. J. JEHU, M.A., M.D., F.G.S. Professor J. H. ASHWORTH, D.Sc., F.R.S. ARTHUR LOGAN TURNER, M.D., LL.D., F.R.C.S.E.	Vice-Presidents.

Professor R. A. SAMPSON, M.A., D.Sc., LL.D., F.R.S., General Secretary.

Professor C. G. DARWIN, M.A., F.R.S. Professor JAMES RITCHIE, M.A., D.Sc.	Secretaries to Ordinary Meetings.

JAMES WATT, W.S., F.F.A., LL.D., Treasurer.
Professor D'ARCY W. THOMPSON, C.B., D.Litt., F.R.S., Curator of Library and Museum.

ORDINARY MEMBERS OF COUNCIL.

J. B. Clark, M.A., LL.D., J.P.
Professor F. A. E. Crew, M.D., Ch.B., D.Sc., Ph.D.
Professor J. Montagu F. Drummond, M.A.
David Alan Stevenson, B.Sc., M.Inst.C.E.
Professor H. W. Turnbull, M.A.
Sir James Walker, Kt., D.Sc., LL.D., F.R.S.
James Drever, M.A., B.Sc., D.Phil.
A. H. R. Goldie, M.A., B.A.
Robert Alex. Houstoun, M.A., Ph.D., D.Sc.
The Hon. Lord Sands, Kt., K.C., LL.D., D.D.
Murray Macgregor, M.A., B.Sc.
A. Crichton Mitchell, D.Sc.

The President, before closing the meeting, thanked the Scrutineers for their services.

PROCEEDINGS OF THE ORDINARY MEETINGS, Session 1930-1931.

FIRST ORDINARY MEETING.

Monday, November 3, 1930.

Professor Sir E. A. Sharpey-Schafer, F.R.S., President, in the Chair.

The Minutes of the last Meeting were taken as read.

By request of the Council, PHILIP EGGLETON, M.Sc., Lecturer on Bio-chemistry in the Physiology Department of the University of Edinburgh, delivered an address entitled "Recent Work in the Bio-chemistry of Muscle."

A short discussion followed, in which Professors CLARK and BARGER, and Dr KERMACK took part.

SECOND ORDINARY MEETING.

Monday, December 1, 1930.

Professor J. H. Ashworth, D.Sc., F.R.S., Vice-President, in the Chair.

The Minutes of the last Meeting were taken as read.

Dr J. W. Low signed the Roll, and was formally admitted a Fellow.

The following Communications were submitted:—

1. The Stem-endodermis in the Genus Piper. By GEORGE BOND, B.Sc., Ph.D. Communicated by Professor MONTAGU F. DRUMMOND, M.A. *Trans.*, vol. 56, pp. 695–724.
2. On the Morphology, Feeding Mechanisms, and Digestion of *Ensis siliqua*. By ALASTAIR GRAHAM, M.A., B.Sc.(Edin.). Communicated by Professor J. H. ASHWORTH, F.R.S. *Trans.*, vol. 56, pp. 725–751.
3. On the Pregnancy Rate in the Lactating Mouse and the Effect of Suckling on the Duration of Pregnancy. By L. MIRSKAIA, Ph.D., and F. A. E. CREW, M.D., D.Sc. *Proc.*, vol. 51, pp. 1–7.
4. Further Observations on the Mechanism of the Diphasic Sex Cycle. By B. P. WIESNER, Ph.D.

THIRD ORDINARY MEETING.

Monday, January 12, 1931.

Professor Sir E. A. Sharpey-Schafer, F.R.S., President, in the Chair.

The Minutes of the last Meeting were taken as read.

The following Communications were submitted:—

1. The Dalradian Rocks of Scotland and the Structure of the Southern Highlands. By Em.-Professor J. W. GREGORY, F.R.S.
2. The British and Belgian Carboniferous *Bellerophontidæ*. By JOHN WEIR, Ph.D., F.G.S. Communicated by Professor E. B. BAILEY, F.R.S. *Trans.*, vol. 56, pp. 767–861.
3. Life History of *Didymium nigripes*. By ELSIE J. CADMAN, M.A., B.Sc., Ph.D. Communicated by Dr MALCOLM WILSON. *Trans.*, vol. 57, pp. 93–142.
4. The Genus *Lyginorachis* Kidston. By R. CROOKALL, Ph.D. Communicated by MURRAY MACGREGOR, M.A., B.Sc. *Proc.*, vol. 51, pp. 27–34.
5. On Some Problems involving the Persymmetric Determinants. By Professor J. GERONIMUS. Communicated by the General Secretary. *Proc.*, vol. 51, pp. 14–18. (Read by title.)

FOURTH ORDINARY MEETING.

Monday, February 2, 1931.

Professor Sir E. A. Sharpey-Schafer, F.R.S., President, in the Chair.

The Minutes of the last Meeting were taken as read.

The following Communications were submitted :—

1. A Note on the Secular Changes of Rock Temperature on the Calton Hill. By F. J. W. WHIPPLE, Sc.D. Communicated by A. H. R. GOLDIE, M.A., B.A. *Proc.*, vol. 51, pp. 19–24.
2. Observations on the Relative Rate of Growth of the Nails of the Right and Left Hands respectively : on Seasonal Variations in the Rate, and on the Influence of Nerve Section upon it. By Sir E. A. SHARPEY-SCHAFER, F.R.S., President. *Proc.*, vol. 51, pp. 8–13.
3. Zeros and Turning-Points of the Elliptic Cylinders. By Professor E. L. INCE, M.A., D.Sc. (Read by title.)

FIFTH ORDINARY MEETING.

Monday, March 2, 1931.

Professor Sir E. A. Sharpey-Schafer, F.R.S., President, in the Chair.

The Minutes of the last Meeting were taken as read.

The Society proceeded to the election of Fellows, and the undernoted were elected, Dr R. CAMPBELL and Mr J. W. BUTTERS acting as Scrutineers :—

WILLIAM ALEXANDER BAIN, WILLIAM MACDONALD BAIRD, THOMAS PURVES BLACK, JOHN ANTHONY CARROLL, JOHN MACQUEEN COWAN, JOHN CRICHTON, SHEPHERD DAWSON, PHILIP EGGLETON, WILLIAM RONALD DODDS FAIRBAIRN, ROBERT GRANT, JOHN RUSSELL GREIG, JOHN HENDERSON, THOMAS JOHNSON, JOHN DU PLESSIS LANGRISHE, NICHOLAS MORPETH HUTCHINSON LIGHTFOOT, WILLIAM JOHN MCCALLIEN, WILLIAM HUNTER MCCREA, JOHN BOWES M'DOUGALL, JOHN HUXLEY MASON, FRANK CHARLES MEARS, ALEXANDER NELSON, JAMES PREMISTER, WILLIAM ROBB, HAROLD STANLEY RUSE, JOHN JAMES M'INTOSH SHAW, JAMES FLEMING SHEARER, GEORGE ALEXANDER STEVEN, CORBET PAGE STEWART, DAVID CLEGHORN THOMSON, WILLIAM JAMES WALKER, JOHN WALTON, THOMAS BARNBY WHITSON, JOHN WISHART.

The Secretary read a communication regarding an International Geographical Congress to be held in Paris from September 16–25, 1931.

The following Communications were submitted :—

1. Some Noteworthy Examples of Parallel Evolution in the Molluscan Faunas of South-eastern Asia and South America. By BAINI PRASHAD, D.Sc., F.A.S.B. *Proc.*, vol. 51, pp. 42–53.
2. The Classification and Development of Carbonaceous Minerals. By Professor HENRY BRIGGS, D.Sc., Ph.D. *Proc.*, vol. 51, pp. 54–63.

Read by title :

3. Properties of the Function Ein (x). By H. V. LOWRY, M.A. Communicated by O. F. T. ROBERTS, M.C., M.A.
4. On Charlier's New Form of the Frequency Function. By A. C. AITKEN, M.A., D.Sc., and A. OPPENHEIM, M.A., Ph.D. *Proc.*, vol. 51, pp. 35–41.

SIXTH ORDINARY MEETING.

Monday, May 4, 1931.

Professor Sir E. A. Sharpey-Schafer, F.R.S., President, in the Chair.

The Minutes of the last Meeting were taken as read.

The undernoted Gentlemen signed the Roll, and were formally admitted as Fellows :— Mr W. A. BAIN, Mr WM. MACDONALD BAIRD, Dr T. P. BLACK, Dr W. R. D. FAIRBAIRN, Mr ROBERT GRANT, Mr JOHN HENDERSON, Professor T. JOHNSON, Dr J. DU PLESSIS LANGRISHE, Mr N. M. H. LIGHTFOOT, Dr W. J. MCCALLIEN, Mr F. C. MEARS, Mr G. A. STEVEN, Dr C. P. STEWART, Mr D. CLEGHORN THOMSON, Dr W. J. WALKER, Professor J. WALTON, Rt. Hon. T. B. WHITSON (Lord Provost of Edinburgh).

The President made the following announcements :—

The Council has awarded the MAKDOUGALL-BRISBANE PRIZE for the period 1926–1930 to Miss NELLIE B. EALES, D.Sc., University of Reading, for her papers on the Anatomy of a Fœtal African Elephant, published in the Society's *Transactions*.

The BRUCE-PRELLER LECTURE on July 6, 1931 will be devoted to a Commemoration of the Centenary of the Birth of JAMES CLERK MAXWELL. The Council has invited Professor HORACE LAMB, F.R.S., Hon. F.R.S.E., to deliver the lecture.

On June 15, 1931, Professor A. H. R. BULLER, F.R.S., will address the Society "On Recent Advances in our Knowledge of the Higher Fungi."

The following Communications were submitted :—

1. A Contribution to the Molluscan Fauna of the Laki and Basal Khirthar Groups of the Indian Eocene. By L. R. Cox, M.A., British Museum (Natural History). Communicated by Lt.-Col. L. M. DAVIES, R.A., F.G.S. *Trans.*, vol. 57, pp. 25–92.

2. On the Identity of *Sacculina triangularis* and *Sacculina inflata*. By Dr H. BOSCHMA (Zoological Laboratory of the University of Leiden). Communicated by Professor J. H. ASHWORTH, F.R.S. *Proc.*, vol. 51, pp. 64–70.

3. An Analysis of the Vegetative Organs of *Selaginella grandis*, Moore, together with some Observations on Abnormalities and Experimental Results. By S. WILLIAMS, M.Sc., Ph.D. *Trans.*, vol. 57, pp. 1–24.

The undernoted were read by title :—

4. On the Operational Solution of the Homogeneous Linear Equation of Finite Differences by Generalised Continued Fractions. By Professor L. M. MILNE-THOMSON. Communicated by Professor B. B. BAKER. *Proc.*, vol. 51, pp. 91–96.

5. Further Numerical Studies in Algebraic Equations and Matrices. By A. C. AITKEN, M.A., D.Sc. *Proc.*, vol. 51, pp. 80–90.

6. Electromagnetic Phenomena in a Uniform Gravitational Field. By D. MEKSYN, Ph.D. Communicated by Professor E. T. WHITTAKER, F.R.S. *Proc.*, vol. 51, pp. 71–79.

SEVENTH ORDINARY MEETING.

Tuesday, May 19, 1931.

Professor J. H. Ashworth, F.R.S., Vice-President, in the Chair.

The Minutes of the last Meeting were taken as read.

The undernoted Gentlemen signed the Roll, and were formally admitted as Fellows :—JOHN CRICHTON and Dr JOHN RUSSELL GREIG.

The following Communications were submitted :—

1. Male Haploidy and Female Diploidy in *Sirex cyaneus*, F (Hymen). By Professor A. D. PEACOCK, D.Sc., and Dr R. A. R. GRESSON. *Proc.*, vol. 51, pp. 97–103.

2. Some New Facts about the Structure of the Cuticles in the Russian Paper-Coal and their Bearing on the Systematic Position of some Fossil Lycopodiales. By Miss J. A. R. WILSON, B.Sc. *Proc.*, vol. 51, pp. 104–115.

With a Note on the Absence of Eligulate Heterosporous Lycopodiales in the Fossil-Record. By Professor J. WALTON. *Proc.*, vol. 51, pp. 114–115.

3. The Electric Field in Terrestrial Magnetic Storms. By A. H. R. GOLDIE, M.A. *Trans.*, vol. 57, pp. 143–177.

4. Fourier Integrals. By Professor T. M. MACROBERT. *Proc.*, vol. 51, pp. 116–126. (Read by title.)

EIGHTH ORDINARY MEETING.

Monday, June 1, 1931.

Professor Sir E. A. Sharpey-Schafer, F.R.S., President, in the Chair.

The Minutes of last Meeting were taken as read.

The undernoted Gentlemen signed the Roll, and were formally admitted as Fellows of the Society :—Dr JOHN MACQUEEN COWAN, Dr PHILIP EGGLETON, Dr SUNDER LAL HORA, and Mr JOHN HUXLEY MASON.

The following Communications were submitted :—

1. The Relation between the Yield of Crude Oil and the Composition of Retortable Carbonaceous Minerals. By Professor HENRY BRIGGS, D.Sc. *Proc.*, vol. 51, pp. 142–149.

2. Abnormalities of the Vascular System of the Anura. By C. H. O'DONOGHUE, D.Sc. *Trans.*, vol. 57, pp. 179–224.

3. The Early Stages of Development of the Vertebrates. By G. L. PURSER, M.A.

4. Studies in the Scottish Marine Fauna.—The Crustacea of the Sandy and Muddy Areas of the Tidal Zone. By R. ELMHIRST, F.L.S., Superintendent, Millport Marine Biological Station. Communicated by A. C. STEPHEN, B.Sc. *Proc.*, vol. 51, pp. 169–175.

5. Note on Cayley's Elimination-problem involving Superfluous Data. By Sir THOMAS MUIR, C.M.G., F.R.S. *Proc.*, vol. 51, pp. 162–168. (Read by title.)

NINTH ORDINARY MEETING.

Monday, June 15, 1931.

Professor W. Wright Smith, M.A., Vice-President, in the Chair.

The Minutes of the last Meeting were taken as read.

The Chairman intimated the title of the BRUCE-PRELLER LECTURE, to be delivered by Sir HORACE LAMB, F.R.S., on July 6—CLERK MAXWELL CENTENARY—"The Dynamical Age in Physical Science."

By request of the Council, Professor A. H. R. BULLER, D.Sc., LL.D., F.R.S., Professor of Botany in the University of Manitoba, delivered an address entitled "Recent Advances in Our Knowledge of the Higher Fungi."

The Ionizing Efficiency of Electronic Impacts in Air. By JOHN THOMSON, M.A., B.Sc., Ph.D. Communicated by Professor E. TAYLOR JONES. *Proc.*, vol. 51, pp. 127–141. (Read by title.)

TENTH AND LAST ORDINARY MEETING.

Monday, July 6, 1931.

Professor Sir E. A. Sharpey-Schafer, F.R.S., President, in the Chair.

JAMES CLERK MAXWELL CENTENARY (1831–1879).

The Minutes of last Meeting were taken as read.

The undernoted Gentlemen signed the Roll, and were formally admitted as Fellows :—Dr W. H. McCREA, Dr SHEPHERD DAWSON, Mr H. S. RUSE, Mr J. J. M. SHAW, and Professor T. A. BROWN.

On presenting the MAKDOUGALL-BRISBANE PRIZE (1928–1930) to Dr NELLIE B. EALES, the President read the following statement :—

Owing to the great difficulty of investigating their gigantic bodies the elephants had not been thoroughly studied, nor had their relationships to other mammals been satisfactorily determined, despite the able researches of palæontologists and their brilliant interpretations of the fossil-record. Elephants have been placed among the Ungulates, but this proves not to be justified by a study of the soft parts of the animal which Dr EALES has carried out with great patience and skill on a fœtal example. Only twelve fœtal elephants have so far been recorded in the literature, and none of them had been examined anatomically. The specimen investigated by Dr EALES was in a good state of preservation and it was therefore imperative to make the most of it. The dissection was carried out in great detail under a binocular dissecting microscope, and the results testify to Dr EALES'S meticulous care in technique and to her judgment in interpretation. The systems of organs are described and compared with those of other mammals, and the author concludes that the Proboscidea, exhibiting as they do a mixture of primitive and specialised characters, sprang from a generalised stock having affinities with the ancestors of the Sirenia, the Rodents, and the Primates, but not directly with the Ungulates.

The palæontological researches of ANDREWS, OSBORN, and others, showed that the mesozoic ancestors of the Elephants were long-jawed and that their jaws turned downwards at the chin. The study of the fœtal specimen shows that its jaw is similar to that described for the mesozoic forms—long and deflected downwards at the chin—but later it shortens and loses the downwardly deflected part, thus furnishing an example, rare among the mammals, of the presence of an ancestral character during fœtal life. Altogether the investigation is valuable for its interest and for its importance as affording for the first time the basis for a detailed anatomical comparison of the Proboscidea with other mammals.

DAVID HUME STATUETTE.

The President made the following statement regarding the gift to the Society by Dr PITTENDRIGH MACGILLIVRAY, R.S.A., of a Statuette of DAVID HUME :—

Fellows will have noticed in the outer Reading-Room an interesting statuette. It is a gift to the Society by Dr PITTENDRIGH MACGILLIVRAY, R.S.A., and is the original of his full-size statue of DAVID HUME, which stands at the north-west corner of the National Gallery of Scotland. Though DAVID HUME the elder died before the Society received its first Charter, our position with regard to him is unique and personal, because his nephew gave the Society the manuscripts of many of his writings, and also the priceless series of letters which passed between HUME and his contemporaries, both here and in France. These manuscripts have proved a mine for scholars, and are still not exhausted. The statuette by Dr PITTENDRIGH MACGILLIVRAY will be a reminder to Fellows and visitors using our Rooms of the connection of the Society with DAVID HUME, and we thank the sculptor most warmly for his gift.

CLERK MAXWELL CENTENARY (1831–1879).

On the platform were Lady CLERK, a representative of the CLERK MAXWELL family, and Dr W. P. D. WIGHTMAN, representing Edinburgh Academy.

The President said :—

This year is the Centenary of the birth of JAMES CLERK MAXWELL, who was born at 14 India Street, Edinburgh, on June 13, 1831, and who for many reasons is intimately connected with the Society. It was the intention of the Council to devote the first BRUCE-PRELLER Lecture—a foundation that we owe to the benefaction of our well-known Fellow, the late Dr DU RICHE PRELLER—to the purpose of commemorating MAXWELL'S work, and they had secured for that purpose the services of our Honorary Fellow, Sir HORACE LAMB, F.R.S. Unfortunately Sir HORACE LAMB has been overtaken by illness—happily not of a serious kind—and is obliged by his doctor's orders to disappoint us. Under these circumstances, the Council is greatly indebted to Professor E. T. WHITTAKER, F.R.S., who at the briefest notice has consented to fill the gap and address the Society.

Professor E. T. WHITTAKER, F.R.S., then gave his address on "James Clerk Maxwell and Mechanical Descriptions of the Universe."

The following Paper was read by title :—

Photochemical Measurements of Light-Intensity in Two Common Vegetation Types in Tropical Africa, by means of an Improved Eder-Hecht Photometer. By J. F. V. PHILLIPS, D.Sc., J. D. SCOTT, B.Sc., and J. Y. MOGGRIDGE, Department of Tsetse Research, Kondoa, Irangi. *Proc.*, vol. 51, pp. 150–161.

PROCEEDINGS OF THE STATUTORY GENERAL MEETING
Ending the 148th Session, 1930–1931.

At the Statutory General Meeting of the Royal Society of Edinburgh, held in the Society's Rooms, 24 George Street, on Monday, October 26, 1931, at 4.30 P.M.,

Sir E. A. Sharpey-Schafer, F.R.S., President, in the Chair,

the Minutes of the Statutory Meeting held on October 27, 1930, were read, approved, and signed.

Dr J. D. SUTHERLAND, Dr ALEX. NELSON, and Mr R. A. ROBB signed the Roll, and were formally admitted Fellows.

The GENERAL SECRETARY submitted the following Report :—

SECRETARY'S REPORT, OCTOBER 26, 1931.

By request of the Council addresses were delivered on "Recent Work in the Bio-chemistry of Muscle," by PHILIP EGGLETON, M.Sc., on November 3, 1930; "Recent Advances in Our Knowledge of the Higher Fungi," by Professor A. H. R. BULLER, F.R.S., on June 15, 1931; and on "JAMES CLERK MAXWELL (1831–1879) and Mechanical Descriptions of the Universe," by Professor E. T. WHITTAKER, F.R.S., on July 6, 1931, being the BRUCE-PRELLER lecture. 34 papers were read, as compared with 35 in the previous session. The papers were divided among subjects as follows: Mathematics, 8; physics, 5; geology and mineralogy, 3; palæontology, 5; botany, 3; zoology, 6; animal genetics, 3; and physiology, 1. 8 papers have been published or are being published in the *Transactions*, and 21 in *Proceedings*. 4 papers were read, but have not yet been submitted in final form, and 3 were declined for publication.

The Society has lost by death 22 Ordinary Fellows and 2 Foreign Honorary Fellows, and 3 Ordinary Fellows have resigned. 33 Ordinary Fellows were elected.

Invitations were received, and the Society was represented as follows on the occasions mentioned :—

1. Memorial Service in St Giles, loss of Airship R 101, October 10, 1930. Sir E. A. SHARPEY-SCHAFER, President; Dr A. CRICHTON MITCHELL, and Mr A. H. R. GOLDIE.
2. Collège de France. 4th Centenary. Paris, June 18–20, 1931. Sir E. A. SHARPEY-SCHAFER, President.
3. Royal Dublin Society. Bi-Centenary. June 23–26, 1931. Sir E. A. SHARPEY-SCHAFER, President.
4. British Association. Centenary Meeting, London, September 23–30, 1931. Professor R. A. SAMPSON, General Secretary.
5. Faraday Centenary Celebrations, London, September 21–25, 1931. Professor R. A. SAMPSON, General Secretary.
6. British Museum (Natural History). 50th Anniversary. End of September 1931. Professor J. H. ASHWORTH.
7. Clerk Maxwell Centenary Celebrations, Cambridge, October 1 and 2, 1931. Professor C. G. DARWIN and Professor R. A. SAMPSON, General Secretary.
8. Memorial Service of Dr JAMES CURRIE, November 7, 1930. Professor T. J. JEHU and the GENERAL SECRETARY.
9. Memorial Service of Sir F. G. OGILVIE. Principal J. C. SMAIL.

Dr A. CRICHTON MITCHELL was re-elected to represent the Society on the Governing Body of Heriot-Watt College.

The undernoted Gentlemen were nominated as representatives of the Society on National Committees:—Astronomy, Sir A. S. EDDINGTON; Geography, Professor A. G. OGILVIE; Geodesy and Geophysics, Dr A. CRICHTON MITCHELL; Mathematics, Professor E. T. WHITTAKER; Physics, Professor E. TAYLOR JONES; Radio-telegraphy, Professor F. G. BAILY; Biology, Professor J. GRAHAM KERR.

Professor F. G. BAILY and the GENERAL SECRETARY, representing the Council, attended meetings convened by the Royal Scottish Society of Arts, to discuss a project for a group of buildings dedicated to the furtherance of Edinburgh's intellectual interests. The chief part of the project is the housing, under one roof, of the principal scientific societies of the City.

The Rooms of the Society were opened to the delegates attending the Congress of the

Universities of the Empire, held in July 1931, so that they could read the reviews, consult journals, write letters, etc.

The Feu Charter conveying to the Society the ground on which the Peach and Horne Memorial stands at Inchnadamph has been received from Major-General STEWART of Assynt and has been recorded.

Attention is called to the List of Periodicals, received by exchange and purchase, published in the *Proceedings*, vol. 50, part 4.

The issue of *Proceedings* has been increased from 1460 to 1500 owing to the increase in membership, etc.

The Hume MSS. in the possession of the Society have been collated and calendared by Dr J. Y. T. GREIG, Armstrong College, Newcastle-on-Tyne. It is hoped to print the Calendar in the *Proceedings*. The collection of 13 volumes, which includes—

7 volumes of correspondence,
3 volumes, miscellaneous,
2 volumes, History of England,
1 volume, Dialogues on Natural Religion,

have been bound by Messrs HENDERSON & BISSET, Edinburgh. The thanks of the Society are due to Dr J. Y. T. GREIG for his valuable services in rearranging the letters and in preparing the Calendar.

A statuette of DAVID HUME has been presented to the Society by Dr PITTENDRIGH MACGILLIVRAY, R.S.A. It has been cast in bronze, and will be set up in the Rooms. The statuette is the original model of the figure of HUME in stone on the North-west corner of the National Portrait Gallery in Queen Street.

During the session, at the request of the Air Ministry, the Society has continued its collaboration with the Royal Society of London in regard to the arrangements of the Second Polar Year (1932-1933). A proposal was considered to have observations made at Fort Rae in Canada, and to conduct a high-level observing station on Ben Nevis. The Air Ministry intends to allot £10,000 as a Grant in Aid to the British part of the programme. The Ben Nevis station was found to be impracticable at short notice, but it is intended to proceed with the other portions.

The MAKDOUGALL-BRISBANE PRIZE for the period 1928 to 1930 was awarded to Dr NELLIE B. EALES of the University of Reading.

It was agreed to devote the BRUCE-PRELLER LECTURE for 1931 to a commemoration of the Centenary of the birth of JAMES CLERK MAXWELL, and Professor Sir HORACE LAMB, F.R.S., was invited to deliver the lecture, but through illness was unable to do so. Professor E. T. WHITTAKER, F.R.S., at the briefest notice, gave an address on July 6, on "James Clerk Maxwell (1831 to 1879) and Mechanical Descriptions of the Universe," as recorded above, and was appointed BRUCE-PRELLER lecturer in the place of Sir HORACE LAMB.

The acknowledgment of the Society is due to the Carnegie Trust for the Universities of Scotland for grants to authors towards the cost of illustrations of papers published by the Society, amounting to £126, 12s. 9d., and to the Royal Society of London for a sum of £250 from the Government Publication Grant, in aid of the cost of printing the Society's *Transactions* and *Proceedings* during the session 1930-1931.

A photographic enlargement of a portrait of Lord Kelvin has been secured by the Society. It has been hung in the Back Saloon (Street Floor).

During the Session a supply of hot water has been provided in the Lavatory (Street Floor).

During the year, painting, etc. has been renewed in the Back Saloon, the Council Room, Secretary's Room, and the upper flat.

TREASURER'S REPORT :—

Dr JAMES WATT, Treasurer, in submitting the Annual Financial Statement, entered into a comparison of the detailed figures of the past two years.

The PRESIDENT in the course of his remarks gave particulars of the Fellows, Honorary and Ordinary, who died during the session, and made reference to his visits, as the delegate of the Society, to the Fourth Centenary Celebration of the Collège de France, held in Paris from June 18-20, 1931, and to the Bi-Centenary Celebration of the Royal Dublin Society, held in Dublin from June 23-26, 1931.

The PRESIDENT nominated as Scrutineers of the Ballot, Mr J. A. INGLIS and Principal J. C. SMAIL.

The Ballot for the Election of Council and Office-Bearers was then taken.

Mr J. GALL INGLIS moved the adoption of the Reports and the reappointment of Messrs LINDSAY, JAMIESON, and HALDANE, C.A., as auditors for the ensuing Session. These motions were seconded and approved.

The Scrutineers reported that the Ballot Papers were in order, and the PRESIDENT declared that the following Office-Bearers and Members of Council had been duly elected :—

Professor Sir E. A. SHARPEY-SCHAFER, M.D., D.Sc., LL.D., F.R.S., President.
Professor F. G. BAILY, M.A., M.Inst.E.E.
Professor T. J. JEHU, M.A., M.D., F.G.S.
Professor J. H. ASHWORTH, D.Sc., F.R.S.
ARTHUR LOGAN TURNER, M.D., LL.D., F.R.C.S.E.
J. B. CLARK, M.A., LL.D., J.P.
Professor JAMES RITCHIE, M.A., D.Sc.
} Vice-Presidents.
Professor R. A. SAMPSON, M.A., D.Sc., LL.D., F.R.S., General Secretary.
Professor C. G. DARWIN, M.A., F.R.S.
Professor F. A. E. CREW, M.D., D.Sc., Ph.D.
} Secretaries to Ordinary Meetings.
JAMES WATT, W.S., F.F.A., LL.D., Treasurer.
Professor D'ARCY W. THOMPSON, C.B., D.Sc., D.Litt., F.R.S., Curator of Library and Museum.

ORDINARY MEMBERS OF COUNCIL.

Professor JAMES DREVER, M.A., B.Sc., D.Phil.
A. H. R. GOLDIE, M.A., B.A.
ROBERT ALEX. HOUSTOUN, M.A., Ph.D., D.Sc.
The Hon. LORD SANDS, Kt., K.C., LL.D., D.D.
MURRAY MACGREGOR, M.A., D.Sc.
A. CRICHTON MITCHELL, D.Sc.
Professor P. T. HERRING, M.D., F.R.C.P.E.
Principal Sir THOMAS H. HOLLAND, K.C.S.I., K.C.I.E., Hon. D.Sc., LL.D., F.R.S.
Professor JAMES KENDALL, M.A., D.Sc., F.R.S.
Professor THOMAS M. MACROBERT, M.A., D.Sc.
Professor GODFREY H. THOMSON, D.Sc., Ph.D.
MALCOLM WILSON, D.Sc., A.R.C.Sc., F.L.S.

The PRESIDENT, before closing the Meeting, thanked the Scrutineers for their services.

THE KEITH, MAKDOUGALL-BRISBANE, NEILL, GUNNING VICTORIA JUBILEE, JAMES SCOTT, BRUCE, AND DAVID ANDERSON-BERRY PRIZES, AND THE BRUCE-PRELLER LECTURE FUND.

The above Prizes will be awarded by the Council in the following manner:—

I. KEITH PRIZE.

The KEITH PRIZE, consisting of a Gold Medal and from £40 to £50 in Money, will be awarded in the Session 1933–1934 for the "best communication on a scientific subject, communicated,* in the first instance, to the Royal Society of Edinburgh during the Sessions 1931–1932 and 1932–1933." Preference will be given to a paper containing a discovery. (See also Council's resolutions at the end of these regulations.)

II. MAKDOUGALL-BRISBANE PRIZE

(*Amended June* 7, 1926.)

This Prize is to be awarded biennially by the Council of the Royal Society of Edinburgh to such person, for such purposes, for such objects, and in such manner as shall appear to them the most conducive to the promotion of the interests of science; with the *proviso* that the Council shall not be compelled to award the Prize unless there shall be some individual engaged in scientific pursuit, or some paper written on a scientific subject, or some discovery in science made during the biennial period, of sufficient merit or importance in the opinion of the Council to be entitled to the Prize.

1. The Prize, consisting of a Gold Medal and a sum of Money, will be awarded before the close of the Session 1932–1933, for an Essay, Paper, or other work having reference to any branch of scientific inquiry, either material or mental.
2. It is open to all men of science.
3. The specific subjects taken into consideration in the current award are governed by the resolutions of the Council as stated at the end of these regulations.
4. For the current period the Committee is representative of Group A.
5. The Committee will consider papers presented to the Society within the Sessions 1930–1931 and 1931–1932, and will make a recommendation.

It is empowered to recommend either:—

(*a*) An award to the Author of an Essay or Paper considered as above, or

(*b*) That no award be made on the ground that, within its group, no paper of sufficient merit has been presented, or

(*c*) That the Prize be awarded to some distinguished man of learning, who may not have presented a paper to the Society within the period considered, but who is willing to deliver an address.

* For the purposes of this award the word "communicated" shall be understood to mean the date on which the manuscript of a paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

III. NEILL PRIZE.

The Council of the Royal Society of Edinburgh having received the bequest of the late Dr Patrick Neill of the sum of £500, for the purpose of "the interest thereof being applied in furnishing a Medal or other reward every second or third year to any distinguished Scottish Naturalist, according as such Medal or reward shall be voted by the Council of the said Society," hereby intimate:

1. The Neill Prize, consisting of a Gold Medal and a sum of Money, will be awarded during the Session 1933–1934.

2. The Prize will be given for a Paper of distinguished merit, on a subject of Natural History, by a Scottish Naturalist, which shall have been presented * to the Society during the two years preceding the fourth Monday in October 1933,—or failing presentation of a paper sufficiently meritorious, it will be awarded for a work or publication by some distinguished Scottish Naturalist, on some branch of Natural History, bearing date within five years of the time of award. (See also Council's resolutions at the end of these regulations.)

IV. GUNNING VICTORIA JUBILEE PRIZE.

This Prize, founded in the year 1887 by Dr R. H. Gunning, is to be awarded quadrennially by the Council of the Royal Society of Edinburgh, in recognition of original work in Physics, Chemistry, or Pure or Applied Mathematics.

Evidence of such work may be afforded either by a Paper presented * to the Society, or by a Paper on one of the above subjects, or some discovery in them elsewhere communicated or made, which the Council may consider to be deserving of the Prize.

The Prize consists of a sum of money, and is open to men of science resident in or connected with Scotland. The first award was made in the year 1887. The next award will be made in Session 1932–1933.

In accordance with the wish of the Donor, the Council of the Society may on fit occasions award the Prize for work of a definite kind to be undertaken during the three succeeding years by a scientific man of recognised ability.

V. JAMES SCOTT PRIZE.

This Prize, founded in the year 1918 by the Trustees of the James Scott Bequest, is to be awarded triennially, or at such intervals as the Council of the Royal Society of Edinburgh may decide, "for a lecture or essay on the fundamental concepts of Natural Philosophy."

VI. BRUCE PRIZE.

The Royal Society is trustee of a fund, instituted in 1923, to commemorate the work of Dr W. S. Bruce as an explorer and scientific investigator in polar regions.

The Committee of Award is appointed jointly by the Royal Society, the Royal Physical Society, and the Royal Scottish Geographical Society.

* For the purposes of this award the word "presented" shall be understood to mean the date on which the manuscript of a paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

The Prize consists of a Bronze Medal and sum of money. It is open to workers of all nationalities, with a preference, *ceteris paribus*, for those of Scottish birth or origin, and is to be awarded biennially for some notable contribution to Natural Sciences, such as Zoology, Botany, Geology, Meteorology, Oceanography, and Geography; the contribution to be in the nature of new knowledge, the outcome of a personal visit to polar regions on the part of the recipient. The recipient shall preferably be at the outset of his career as an investigator.

The next award will be made in 1934. Papers for the consideration of the Committee should be in the hands of the General Secretary of the Royal Society, 22 George Street, Edinburgh, not later than March 31 of that year.

VII. BRUCE-PRELLER LECTURE FUND.

The Council of the Royal Society of Edinburgh having received in 1929 the bequest of the late Dr CHARLES DU RICHE PRELLER of the sum of £500, decided that the income thereof be applied by the Council biennially as an honorarium for a special BRUCE-PRELLER Lecture or Address by an outstanding man of science, its subject to be Geology or Electrical or Physical Science, or in the discretion of the Council some other branch of science. The next award will be made in session 1932–1933.

VIII. DAVID ANDERSON-BERRY FUND.

The Council of the Royal Society of Edinburgh having received in the year 1930, free of duty, the capital sum of one thousand pounds (£1000), to be used in terms of the will of the late Dr DAVID ANDERSON-BERRY, dated 23rd April 1926, decided that the income thereof be applied triennially, "in the first place in the presentation of a gold medal, and in the second place in the payment of a sum of money to the winner for the year of such gold medal, the winner being the person who, in the opinion of the Society, shall be the producer for the year of the best essay on the nature of X-rays and their therapeutical effect on human diseases."

RESOLUTIONS OF COUNCIL IN REGARD TO THE MODE OF AWARDING PRIZES.

(*See Minutes of Meeting of January* 18, 1915.)

I. With regard to the Keith and Makdougall-Brisbane Prizes, which are open to all Sciences, the mode of award will be as follows:—

1. Papers or essays to be considered shall be arranged in two groups, A and B, —Group A to include Astronomy, Chemistry, Mathematics, Metallurgy, Meteorology, and Physics; Group B to include Anatomy, Anthropology, Botany, Geology, Pathology, Physiology, and Zoology.
2. These two Prizes shall be awarded to each group in alternate biennial periods, provided papers worthy of recommendation have been communicated to the Society.

3. Prior to the adjudication the Council shall appoint, in the first instance, a Committee composed of representatives of the group of Sciences which did not receive the award in the immediately preceding period. The Committee shall consider the Papers which come within their group of Sciences, and report in due course to the Council.

4. In the event of the aforesaid Committee reporting that within their group of subjects there is, in their opinion, no paper worthy of being recommended for the award, the Council, on accepting this report, shall appoint a Committee representative of the alternate group to consider papers coming within their group and to report accordingly.

5. Papers to be considered by the Committees shall fall within the period dating from the last award in groups A and B respectively.

II. With regard to the Neill Prize, the term "Naturalist" shall be understood to include any student in the Sciences composing group B, namely, Anatomy, Anthropology, Botany, Geology, Pathology, Physiology, Zoology.

AWARDS OF THE KEITH, MAKDOUGALL-BRISBANE, NEILL, GUNNING, JAMES SCOTT, BRUCE, AND DAVID ANDERSON-BERRY PRIZES, AND THE BRUCE-PRELLER LECTURE FUND.

I. KEITH PRIZE.

1ST BIENNIAL PERIOD, 1827–29.—Dr BREWSTER, for his papers "on his Discovery of Two New Immiscible Fluids in the Cavities of certain Minerals," published in the Transactions of the Society.

2ND BIENNIAL PERIOD, 1829–31.—Dr BREWSTER, for his paper "on a New Analysis of Solar Light," published in the Transactions of the Society.

3RD BIENNIAL PERIOD, 1831–33.—THOMAS GRAHAM, Esq., for his paper "on the Law of the Diffusion of Gases," published in the Transactions of the Society.

4TH BIENNIAL PERIOD, 1833–35.—Professor J. D. FORBES, for his paper "on the Refraction and Polarization of Heat," published in the Transactions of the Society.

5TH BIENNIAL PERIOD, 1835–37.—JOHN SCOTT RUSSELL, Esq., for his researches "on Hydrodynamics," published in the Transactions of the Society.

6TH BIENNIAL PERIOD, 1837–39.—Mr JOHN SHAW, for his experiments "on the Development and Growth of the Salmon," published in the Transactions of the Society.

7TH BIENNIAL PERIOD, 1839–41.—Not awarded.

8TH BIENNIAL PERIOD, 1841–43.—Professor JAMES DAVID FORBES, for his papers "on Glaciers," published in the Proceedings of the Society.

9TH BIENNIAL PERIOD, 1843–45.—Not awarded.

10TH BIENNIAL PERIOD, 1845–47.—General Sir THOMAS BRISBANE, Bart., for the Makerstoun Observations on Magnetic Phenomena, made at his expense, and published in the Transactions of the Society.

11TH BIENNIAL PERIOD, 1847–49.—Not awarded.

12TH BIENNIAL PERIOD, 1849–51.—Professor KELLAND, for his papers "on General Differentiation, including his more recent Communication on a process of the Differential Calculus, and its application to the solution of certain Differential Equations," published in the Transactions of the Society.

13TH BIENNIAL PERIOD, 1851–53.—W. J. MACQUORN RANKINE, Esq., for his series of papers "on the Mechanical Action of Heat," published in the Transactions of the Society.

14TH BIENNIAL PERIOD, 1853–55.—Dr THOMAS ANDERSON, for his papers "on the Crystalline Constituents of Opium, and on the Products of the Destructive Distillation of Animal Substances," published in the Transactions of the Society.

15TH BIENNIAL PERIOD, 1855–57.—Professor BOOLE, for his Memoir "on the Application of the Theory of Probabilities to Questions of the Combination of Testimonies and Judgments," published in the Transactions of the Society.

16TH BIENNIAL PERIOD, 1857–59.—Not awarded.

17TH BIENNIAL PERIOD, 1859–61.—JOHN ALLAN BROUN, Esq., F.R.S., Director of the Trevandrum Observatory, for his papers "on the Horizontal Force of the Earth's Magnetism, on the Correction of the Bifilar Magnetometer, and on Terrestrial Magnetism generally," published in the Transactions of the Society.

18TH BIENNIAL PERIOD, 1861–63.—Professor WILLIAM THOMSON, of the University of Glasgow, for his Communication "on some Kinematical and Dynamical Theorems."

19TH BIENNIAL PERIOD, 1863–65.—Principal FORBES, St Andrews, for his "Experimental Inquiry into the Laws of Conduction of Heat in Iron Bars," published in the Transactions of the Society.

20TH BIENNIAL PERIOD, 1865–67.—Professor C. PIAZZI SMYTH, for his paper "on Recent Measures at the Great Pyramid," published in the Transactions of the Society.

21ST BIENNIAL PERIOD, 1867–69.—Professor P. G. TAIT, for his paper "on the Rotation of a Rigid Body about a Fixed Point," published in the Transactions of the Society.

22ND BIENNIAL PERIOD, 1869-71.—Professor CLERK MAXWELL, for his paper "on Figures, Frames, and Diagrams of Forces," published in the Transactions of the Society.

23RD BIENNIAL PERIOD, 1871-73.—Professor P. G. TAIT, for his paper entitled "First Approximation to a Thermo-electric Diagram," published in the Transactions of the Society.

24TH BIENNIAL PERIOD, 1873-1875.—Professor CRUM BROWN, for his Researches "on the Sense of Rotation, and on the Anatomical Relations of the Semicircular Canals of the Internal Ear."

25TH BIENNIAL PERIOD, 1875-77.—Professor M. FORSTER HEDDLE, for his papers "on the Rhombohedral Carbonates," and "on the Felspars of Scotland," published in the Transactions of the Society.

26TH BIENNIAL PERIOD, 1877-79.—Professor H. C. FLEEMING JENKIN, for his paper "on the Application of Graphic Methods to the Determination of the Efficiency of Machinery," published in the Transactions of the Society; Part II having appeared in the volume for 1877-78.

27TH BIENNIAL PERIOD, 1879-81.—Professor GEORGE CHRYSTAL, for his paper "on the Differential Telephone," published in the Transactions of the Society.

28TH BIENNIAL PERIOD, 1881-83.—THOMAS MUIR, Esq., LL.D., for his "Researches into the Theory of Determinants and Continued Fractions," published in the Proceedings of the Society.

29TH BIENNIAL PERIOD, 1883-85.—JOHN AITKEN, Esq., for his paper "on the Formation of Small Clear Spaces in Dusty Air," and for previous papers on Atmospheric Phenomena, published in the Transactions of the Society.

30TH BIENNIAL PERIOD, 1885-87.—JOHN YOUNG BUCHANAN, Esq., for a series of communications, extending over several years, on subjects connected with Ocean Circulation, Compressibility of Glass, etc.; two of which, viz., "On Ice and Brines," and "On the Distribution of Temperature in the Antarctic Ocean," have been published in the Proceedings of the Society.

31ST BIENNIAL PERIOD, 1887-89.—Professor E. A. LETTS, for his papers on the Organic Compounds of Phosphorus, published in the Transactions of the Society.

32ND BIENNIAL PERIOD, 1889-91.—R. T. OMOND, Esq., for his contributions to Meteorological Science, many of which are contained in vol. xxxiv of the Society's Transactions.

33RD BIENNIAL PERIOD, 1891-93.—Professor THOMAS R. FRASER, F.R.S., for his papers on *Strophanthus hispidus*, Strophanthin, and Strophanthidin, read to the Society in February and June 1889 and in December 1891, and printed in vols. xxxv, xxxvi, and xxxvii of the Society's Transactions.

34TH BIENNIAL PERIOD, 1893-95.—Dr CARGILL G. KNOTT, for his papers on the Strains produced by Magnetism in Iron and in Nickel, which have appeared in the Transactions and Proceedings of the Society.

35TH BIENNIAL PERIOD, 1895-97.—Dr THOMAS MUIR, for his continued communications on Determinants and Allied Questions.

36TH BIENNIAL PERIOD, 1897-99.—Dr JAMES BURGESS, for his paper "on the Definite Integral $\frac{2}{\sqrt{\pi}}\int_0^t e^{-t^2}dt$, with extended Tables of Values," printed in vol. xxxix of the Transactions of the Society.

37TH BIENNIAL PERIOD, 1899-1901.—Dr HUGH MARSHALL, for his discovery of the Persulphates, and for his Communications on the Properties and Reactions of these Salts, published in the Proceedings of the Society.

38TH BIENNIAL PERIOD, 1901-03.—Sir WILLIAM TURNER, K.C.B., LL.D., F.R.S., etc., for his memoirs entitled "A Contribution to the Craniology of the People of Scotland," published in the Transactions of the Society, and for his "Contributions to the Craniology of the People of the Empire of India," Parts I, II, likewise published in the Transactions of the Society.

39TH BIENNIAL PERIOD, 1903-05.—THOMAS H. BRYCE, M.A., M.D., for his two papers on "The Histology of the Blood of the Larva of *Lepidosiren paradoxa*," published in the Transactions of the Society within the period.

40TH BIENNIAL PERIOD, 1905-07.—ALEXANDER BRUCE, M.A., M.D., F.R.C.P.E., for his paper entitled "Distribution of the Cells in the Intermedio-Lateral Tract of the Spinal Cord," published in the Transactions of the Society within the period.

41ST BIENNIAL PERIOD, 1907-09.—WHEELTON HIND, M.D., B.S., F.R.C.S., F.G.S., for a paper published in the Transactions of the Society, "On the Lamellibranch and Gasteropod Fauna found in the Millstone Grit of Scotland."

42ND BIENNIAL PERIOD, 1909–11.—Professor ALEXANDER SMITH, B.Sc., Ph.D., of New York, for his researches upon "Sulphur" and upon "Vapour Pressure," appearing in the Proceedings of the Society.

43RD BIENNIAL PERIOD, 1911–1913.—JAMES RUSSELL, Esq., for his series of investigations relating to magnetic phenomena in metals and the molecular theory of magnetism, the results of which have been published in the Proceedings and Transactions of the Society, the last paper having been issued within the period.

44TH BIENNIAL PERIOD, 1913–15.—JAMES HARTLEY ASHWORTH, D.Sc., for his papers on "Larvæ of Lingula and Pelagodiscus," and on "Sclerocheilus," published in the Transactions of the Society, and for other papers on the Morphology and Histology of Polychæta.

45TH BIENNIAL PERIOD, 1915–17.—ROBERT C. MOSSMAN, for his work on the Meteorology of the Antarctic Regions, which originated with the important series of observations made by him during the voyage of the "Scotia" (1902–1904), and includes his paper "On a Sea-Saw of Barometric Pressure, Temperature, and Wind Velocity between the Weddell Sea and the Ross Sea," published in the Proceedings of the Society.

46TH BIENNIAL PERIOD, 1917–19.—JOHN STEPHENSON, Lt.-Col. I.M.S., for his series of papers on the Oligochæta and other Annelida, several of which have been published in the Transactions of the Society.

47TH BIENNIAL PERIOD, 1919–21.—RALPH ALLEN SAMPSON, F.R.S., for his Astronomical Researches, including the papers "Studies in Clocks and Time-keeping: No. 1, Theory of the Maintenance of Motion; No. 2, Tables of the Circular Equation," published in the Proceedings of the Society within the period of the award.

48TH BIENNIAL PERIOD, 1921–23.—JOHN WALTER GREGORY, F.R.S., for his papers published in the Transactions of the Society, and in recognition of his numerous contributions to Geology, extending over a period of thirty-six years.

49TH BIENNIAL PERIOD, 1923–25.—HERBERT WESTREN TURNBULL, M.A., for the papers on "Hyper-Algebra," "Invariant Theory," and "Algebraic Geometry," three of which have been published in the Proceedings within the period of award.

50TH BIENNIAL PERIOD, 1925–27.—THOMAS JOHN JEHU, M.A., M.D., F.G.S., and ROBERT MELDRUM CRAIG, M.A., B.Sc., F.G.S., for the joint series of papers which have recently appeared in the Transactions of the Society on the "Geology of the Outer Hebrides."

51ST BIENNIAL PERIOD, 1927–29.—Dr CHRISTINA C. MILLER, B.Sc., for her papers on the "Slow Oxidation of Phosphorus Trioxide," published in the Proceedings within the period of the award, and in consideration of subsequent developments on "Slow Oxidation of Phosphorus," published elsewhere.

II. MAKDOUGALL-BRISBANE PRIZE.

1ST BIENNIAL PERIOD, 1859.—Sir RODERICK IMPEY MURCHISON, on account of his Contributions to the Geology of Scotland.

2ND BIENNIAL PERIOD, 1860–62.—WILLIAM SELLER, M.D., F.R.C.P.E., for his "Memoir of the Life and Writings of Dr Robert Whytt," published in the Transactions of the Society.

3RD BIENNIAL PERIOD, 1862–64.—JOHN DENIS MACDONALD, Esq., R.N., F.R.S., Surgeon of H.M.S. "Icarus," for his paper "on the Representative Relationships of the Fixed and Free Tunicata, regarded as Two Sub-classes of equivalent value; with some General Remarks on their Morphology," published in the Transactions of the Society.

4TH BIENNIAL PERIOD, 1864–66.—Not awarded.

5TH BIENNIAL PERIOD, 1866–68.—Dr ALEXANDER CRUM BROWN and Dr THOMAS RICHARD FRASER, for their conjoint paper "on the Connection between Chemical Constitution and Physiological Action," published in the Transactions of the Society.

6TH BIENNIAL PERIOD, 1868–70.—Not awarded.

7TH BIENNIAL PERIOD, 1870–72.—GEORGE JAMES ALLMAN, M.D., F.R.S., Emeritus Professor of Natural History, for his paper "on the Homological Relations of the Cœlenterata," published in the Transactions, which forms a leading chapter of his Monograph of Gymnoblastic or Tubularian Hydroids—since published.

8TH BIENNIAL PERIOD, 1872–74.—Professor LISTER, for his paper "on the Germ Theory of Putrefaction and the Fermentive Changes," communicated to the Society, 7th April 1873.

9TH BIENNIAL PERIOD, 1874–76.—ALEXANDER BUCHAN, A.M., for his paper "on the Diurnal Oscillation of the Barometer," published in the Transactions of the Society.

10TH BIENNIAL PERIOD, 1876-78.—Professor ARCHIBALD GEIKIE, for his paper "on the Old Red Sandstone of Western Europe," published in the Transactions of the Society.

11TH BIENNIAL PERIOD, 1878-80.—Professor PIAZZI SMYTH, Astronomer-Royal for Scotland, for his paper "on the Solar Spectrum in 1877-78, with some Practical Idea of its probable Temperature of Origination," published in the Transactions of the Society.

12TH BIENNIAL PERIOD, 1880-82.—Professor JAMES GEIKIE, for his "Contributions to the Geology of the North-West of Europe," including his paper "on the Geology of the Faroes," published in the Transactions of the Society.

13TH BIENNIAL PERIOD, 1882-84.—EDWARD SANG, Esq., LL.D., for his paper "on the Need of Decimal Subdivisions in Astronomy and Navigation, and on Tables requisite therefor," and generally for his Recalculations of Logarithms both of Numbers and Trigonometrical Ratios,—the former communication being published in the Proceedings of the Society.

14TH BIENNIAL PERIOD, 1884-86.—JOHN MURRAY, Esq., LL.D., for his papers "On the Drainage Areas of Continents, and Ocean Deposits," "The Rainfall of the Globe, and Discharge of Rivers," "The Height of the Land and Depth of the Ocean," and "The Distribution of Temperature in the Scottish Lochs as affected by the Wind."

15TH BIENNIAL PERIOD, 1886-88.—ARCHIBALD GEIKIE, Esq., LL.D., for numerous Communications, especially that entitled "History of Volcanic Action during the Tertiary Period in the British Isles," published in the Transactions of the Society.

16TH BIENNIAL PERIOD, 1888-90.—Dr LUDWIG BECKER, for his paper on "The Solar Spectrum at Medium and Low Altitudes," printed in vol. xxxvi, Part I, of the Society's Transactions.

17TH BIENNIAL PERIOD, 1890-92.—HUGH ROBERT MILL, Esq., D.Sc., for his papers on "The Physical Conditions of the Clyde Sea Area," Part I being already published in vol. xxxvi of the Society's Transactions.

18TH BIENNIAL PERIOD, 1892-94.—Professor JAMES WALKER, D.Sc., Ph.D., for his work on Physical Chemistry, part of which has been published in the Proceedings of the Society, vol. xx, pp. 255-263. In making this award, the Council took into consideration the work done by Professor Walker along with Professor Crum Brown on the Electrolytic Synthesis of Dibasic Acids, published in the Transactions of the Society.

19TH BIENNIAL PERIOD, 1894-96.—Professor JOHN G. M'KENDRICK, for numerous Physiological papers, especially in connection with Sound, many of which have appeared in the Society's publications.

20TH BIENNIAL PERIOD, 1896-98.—Dr WILLIAM PEDDIE, for his papers on the Torsional Rigidity of Wires.

21ST BIENNIAL PERIOD, 1898-1900.—Dr RAMSAY H. TRAQUAIR, for his paper entitled "Report on Fossil Fishes collected by the Geological Survey in the Upper Silurian Rocks of Scotland," printed in vol. xxxix of the Transactions of the Society.

22ND BIENNIAL PERIOD, 1900-02.—Dr ARTHUR T. MASTERMAN, for his paper entitled "The Early Development of *Cribrella oculata* (Forbes), with remarks on Echinoderm Development," printed in vol. xl of the Transactions of the Society.

23RD BIENNIAL PERIOD, 1902-04.—Mr JOHN DOUGALL, M.A., for his paper on "An Analytical Theory of the Equilibrium of an Isotropic Elastic Plate," published in vol. xli of the Transactions of the Society.

24TH BIENNIAL PERIOD, 1904-06.—JACOB E. HALM, Ph.D., for his two papers entitled "Spectroscopic Observations of the Rotation of the Sun," and "Some Further Results obtained with the Spectroheliometer," and for other astronomical and mathematical papers published in the Transactions and Proceedings of the Society within the period.

25TH BIENNIAL PERIOD, 1906-08.—D. T. GWYNNE-VAUGHAN, M.A., F.L.S., for his papers, 1st, "On the Fossil Osmundaceæ," and 2nd, "On the Origin of the Adaxially-curved Leaf-trace in the Filicales," communicated by him conjointly with Dr R. Kidston.

26TH BIENNIAL PERIOD, 1908-10.—ERNEST MACLAGAN WEDDERBURN, M.A., LL.B., for his series of papers bearing upon "The Temperature Distribution in Fresh-water Lochs," and especially upon "The Temperature Seiche."

27TH BIENNIAL PERIOD, 1910-12.—JOHN BROWNLEE, M.A., M.D., D.Sc., for his contributions to the Theory of Mendelian Distributions and cognate subjects, published in the Proceedings of the Society within and prior to the prescribed period.

28TH BIENNIAL PERIOD, 1912-14.—Professor C. R. MARSHALL, M.D., M.A., for his studies "On the Pharmacological Action of Tetra-alkyl-ammonium Compounds."

29TH BIENNIAL PERIOD, 1914-16.—ROBERT ALEXANDER HOUSTOUN, Ph.D., D.Sc., for his series of papers on "The Absorption of Light by Inorganic Salts," published in the Proceedings of the Society.

30TH BIENNIAL PERIOD, 1916-18.—Professor A. ANSTRUTHER LAWSON, for his Memoirs on "The Prothalli of *Tmesipteris Tannensis* and of *Psilotum*," published in the Transactions of the Society, together with previous papers on Cytology and on The Gametophytes of various Gymnosperms.

31ST BIENNIAL PERIOD, 1918-20.—Professor J. H. MACLAGAN WEDDERBURN of Princeton University for his Memoirs in Universal Algebra, etc., published in the Transactions and Proceedings of the Society, and elsewhere.

32ND BIENNIAL PERIOD, 1920-22.—Professor W. T. GORDON, M.A., D.Sc., for his paper on "Cambrian Organic Remains from a Dredging in the Weddell Sea," published in the Transactions of the Society within the period, and for his investigations on the Fossil Flora of the Pettycur Limestone, previously published in the Transactions.

33RD BIENNIAL PERIOD, 1922-24.—Professor H. STANLEY ALLEN, D.Sc., for his papers on the "Quantum and Atomic Theory," published in the Society's Proceedings within the period.

34TH BIENNIAL PERIOD, 1924-26.—Dr CHARLES MORLEY WENYON, C.M.G., C.B.E., F.R.S. for his distinguished work in Protozoology extending over a period of twenty-one years.

35TH BIENNIAL PERIOD, 1926-28.—Dr W. O. KERMACK, M.A., for his contributions to Chemistry, published in the Society's Proceedings and elsewhere.

36TH BIENNIAL PERIOD, 1928-30.—Dr NELLIE B. EALES, for her papers in the Society's Transactions on "The Anatomy of a Fœtal African Elephant."

III. THE NEILL PRIZE.

1ST TRIENNIAL PERIOD, 1856-59.—Dr W. LAUDER LINDSAY, for his paper "on the Spermogones and Pycnides of Filamentous, Fruticulose, and Foliaceous Lichens," published in the Transactions of the Society.

2ND TRIENNIAL PERIOD, 1859-62.—ROBERT KAYE GREVILLE, LL.D., for his contributions to Scottish Natural History, more especially in the department of Cryptogamic Botany, including his recent papers on Diatomaceæ.

3RD TRIENNIAL PERIOD, 1862-65.—ANDREW CROMBIE RAMSAY, F.R.S., Professor of Geology in the Government School of Mines, and Local Director of the Geological Survey of Great Britain, for his various works and memoirs published during the last five years, in which he has applied the large experience acquired by him in the Direction of the arduous work of the Geological Survey of Great Britain to the elucidation of important questions bearing on Geological Science.

4TH TRIENNIAL PERIOD, 1865-68.—Dr WILLIAM CARMICHAEL M'INTOSH, for his paper "on the Structure of the British Nemerteans, and on some New British Annelids," published in the Transactions of the Society.

5TH TRIENNIAL PERIOD, 1868-71.—Professor WILLIAM TURNER, for his papers "on the Great Finner Whale; and on the Gravid Uterus, and the Arrangement of the Fœtal Membranes in the Cetacea," published in the Transactions of the Society.

6TH TRIENNIAL PERIOD, 1871-74.—CHARLES WILLIAM PEACH, Esq., for his Contributions to Scottish Zoology and Geology, and for his recent contributions to Fossil Botany.

7TH TRIENNIAL PERIOD, 1874-77.—Dr RAMSAY H. TRAQUAIR, for his paper "on the Structure and Affinities of *Tristichopterus alatus* (Egerton)," published in the Transactions of the Society, and also for his contributions to the Knowledge of the Structure of Recent and Fossil Fishes.

8TH TRIENNIAL PERIOD, 1877-80.—JOHN MURRAY, Esq., for his paper "on the Structure and Origin of Coral Reefs and Islands," published (in abstract) in the Proceedings of the Society.

9TH TRIENNIAL PERIOD, 1880-83.—Professor HERDMAN, for his papers "on the Tunicata," published in the Proceedings and Transactions of the Society.

10TH TRIENNIAL PERIOD, 1883-86.—B. N. PEACH, Esq., for his Contributions to the Geology and Palæontology of Scotland, published in the Transactions of the Society.

11TH TRIENNIAL PERIOD, 1886-89.—ROBERT KIDSTON, Esq., for his Researches in Fossil Botany, published in the Transactions of the Society.

12TH TRIENNIAL PERIOD, 1889-92.—JOHN HORNE, Esq., F.G.S., for his Investigations into the Geological Structure and Petrology of the North-West Highlands.

13TH TRIENNIAL PERIOD, 1892-95.—ROBERT IRVINE, Esq., for his papers on the Action of Organisms in the Secretion of Carbonate of Lime and Silica, and on the solution of these substances in Organic Juices. These are printed in the Society's Transactions and Proceedings.

14TH TRIENNIAL PERIOD, 1895-98.—Professor COSSAR EWART, for his recent Investigations connected with Telegony.

15TH TRIENNIAL PERIOD, 1898-1901.—Dr JOHN S. FLETT, for his papers entitled "The Old Red Sandstone of the Orkneys" and "The Trap Dykes of the Orkneys," printed in vol. xxxix of the Transactions of the Society.

16TH TRIENNIAL PERIOD, 1901-04.—Professor J. GRAHAM KERR, M.A., for his Researches on *Lepidosiren paradoxa*, published in the Philosophical Transactions of the Royal Society, London.

17TH TRIENNIAL PERIOD, 1904-07.—FRANK J. COLE, B.Sc., for his paper entitled "A Monograph on the General Morphology of the Myxinoid Fishes, based on a Study of Myxine," published in the Transactions of the Society, regard being also paid to Mr Cole's other valuable contributions to the Anatomy and Morphology of Fishes.

1ST BIENNIAL PERIOD, 1907-09.—FRANCIS J. LEWIS, M.Sc., F.L.S., for his papers in the Society's Transactions "On the Plant Remains of the Scottish Peat Mosses."

2ND BIENNIAL PERIOD, 1909-11.—JAMES MURRAY, Esq., for his paper on "Scottish Rotifers collected by the Lake Survey (Supplement)," and other papers on the "Rotifera" and "Tardigrada," which appeared in the Transactions of the Society—(this Prize was awarded after consideration of the papers received within the five years prior to the time of award: see Neill Prize Regulations).

3RD BIENNIAL PERIOD, 1911-13.—Dr W. S. BRUCE, in recognition of the scientific results of his Arctic and Antarctic explorations.

4TH BIENNIAL PERIOD, 1913-15.—ROBERT CAMPBELL, D.Sc., for his paper on "The Upper Cambrian Rocks at Craigeven Bay, Stonehaven," and "Downtonian and Old Red Sandstone Rocks of Kincardineshire," published in the Transactions of the Society.

5TH BIENNIAL PERIOD, 1915-17.—W. H. LANG, F.R.S., M.B., D.Sc., for his paper in conjunction with Dr R. KIDSTON, F.R.S., on *Rhynia Gwynne-Vaughani*, Kidston and Lang, published in the Transactions of the Society, and for his previous investigations on Pteridophytes and Cycads.

6TH BIENNIAL PERIOD, 1917-19.—JOHN TAIT, D.Sc., M.D, for his work on Crustacea, published in the Proceedings of the Society, and for his papers on the blood of Crustacea.

7TH BIENNIAL PERIOD, 1919-21.—Sir EDWARD A. SHARPEY-SCHAFER, F.R.S., for his recent contributions to our knowledge of Physiology, and in recognition of his published work extending over a period of fifty years.

8TH BIENNIAL PERIOD, 1921-23.—JOHN M'LEAN THOMPSON, M.A., D.Sc., University of Liverpool, for his series of Memoirs on Staminal Zygomorphy, and on the Anatomy of the Filicales.

9TH BIENNIAL PERIOD, 1923-25.—FREDERICK ORPEN BOWER, F.R.S., for his recent contributions to Botanical knowledge and in recognition of his published work extending over a period of forty-five years.

10TH BIENNIAL PERIOD, 1925-27.—ARTHUR ROBINSON, M.D., M.R.C.S., for his contributions to Comparative Anatomy and Embryology.

11TH BIENNIAL PERIOD, 1927-29.—Professor ED. BATTERSBY BAILEY, M.C., F.R.S., in recognition of his valuable contributions to the Geology of Scotland, two of which have recently appeared in the Transactions of the Society.

IV. GUNNING VICTORIA JUBILEE PRIZE.

1ST TRIENNIAL PERIOD, 1884-87.—Sir WILLIAM THOMSON, Pres. R.S.E., F.R.S., for a remarkable series of papers "on Hydrokinetics," especially on Waves and Vortices which have been communicated to the Society.

2ND TRIENNIAL PERIOD, 1887-90.—Professor P. G. TAIT, Sec. R.S.E., for his work in connection with the "Challenger" Expedition, and his other Researches in Physical Science.

3RD TRIENNIAL PERIOD, 1890-93.—ALEXANDER BUCHAN, Esq., LL.D., for his varied, extensive, and extremely important Contributions to Meteorology, many of which have appeared in the Society's publications.

4TH TRIENNIAL PERIOD, 1893-96.—JOHN AITKEN, Esq., for his brilliant Investigations in Physics, especially in connection with the Formation and Condensation of Aqueous Vapour.

1ST QUADRENNIAL PERIOD, 1896-1900.—Dr T. D. ANDERSON, for his discoveries of New and Variable Stars.

2ND QUADRENNIAL PERIOD, 1900-04.—Sir JAMES DEWAR, LL.D., D.C.L., F.R.S., etc., for his researches on the Liquefaction of Gases, extending over the last quarter of a century, and on the Chemical and Physical Properties of Substances at Low Temperatures: his earliest papers being published in the Transactions and Proceedings of the Society.

3RD QUADRENNIAL PERIOD, 1904-08.—Professor GEORGE CHRYSTAL, M.A., LL.D., for a series of papers on "Seiches," including "The Hydrodynamical Theory and Experimental Investigations of the Seiche Phenomena of Certain Scottish Lakes."

4TH QUADRENNIAL PERIOD, 1908-12.—Professor J. NORMAN COLLIE, Ph.D., F.R.S., for his distinguished contributions to Chemistry, Organic and Inorganic, during twenty-seven years, including his work upon Neon and other rare gases. Professor Collie's early papers were contributed to the Transactions of the Society.

5TH QUADRENNIAL PERIOD, 1912-16.—Sir THOS. MUIR, C.M.G., LL.D., F.R.S., for his series of *Memoirs upon "The Theory and History of Determinants and Allied Forms,"* published in the Transactions and Proceedings of the Society between the years 1872 and 1915.

6TH QUADRENNIAL PERIOD, 1916-20.—C. T. R. WILSON, Esq., F.R.S., in recognition of his important discoveries in relation to Condensation Nuclei, Ionisation of Gases, and Atmospheric Electricity.

7TH QUADRENNIAL PERIOD, 1920-24.—Sir J. J. THOMSON, O.M., F.R.S., in recognition of his great discoveries in Physical Science.

8TH QUADRENNIAL PERIOD, 1924-28.—Professor E. T. WHITTAKER, F.R.S., in recognition of his distinguished contributions to Mathematical Science, and of his promotion of Mathematical Research in Scotland.

V. JAMES SCOTT PRIZE.

1ST AWARD, 1918-22.—Professor A. N. WHITEHEAD, F.R.S., for his lecture delivered on June 5, 1922, "*On the Relatedness of Nature.*"

2ND AWARD, 1922-27.—Sir JOSEPH LARMOR, M.A., D.Sc., LL.D., F.R.S., for his lecture delivered on July 4, 1927, on "The Grasp of Mind on Nature."

3RD AWARD, 1927-30.—Professor NIELS BOHR, for his lecture delivered on May 26, 1930, on "Philosophical Aspects of Atomic Theory."

VI. BRUCE PRIZE.

1ST AWARD 1926.—JAMES MANN WORDIE, M.A., for his Oceanographical and Geological work in both Polar Regions.

2ND AWARD, 1928.—H. U. SVERDRUP, for his contributions to the knowledge of the Meteorology, Magnetism, and Tides of the Arctic, as an outcome of his travels with the Norwegian Expedition in the "Maud" from 1918 to 1925.

3RD AWARD, 1930.—N. A. MACKINTOSH, M.Sc., A.R.C.S., for his researches into the Biology of Whales in the Waters of the Falkland Islands Dependencies.

VII. BRUCE-PRELLER LECTURE FUND.

1ST AWARD, 1931.—Professor E. T. WHITTAKER, F.R.S., for his lecture, "*James Clerk Maxwell and Mechanical Descriptions of the Universe.*"

VIII. DAVID ANDERSON-BERRY FUND.

ABSTRACT

OF

THE ACCOUNTS

OF

THE ROYAL SOCIETY OF EDINBURGH,

SESSION — 1st October 1930 to 30th September 1931.

JAMES WATT, LL.D., W.S.,
Treasurer.

I. GENERAL FUND

CHARGE.

1. Arrears of Contributions at 30th September 1930			£72 9 0
2. Contributions for present Session :—			
1. 388 Fellows at £3, 3s. each		£1222 4 0	
2. Fees of Admission and Contributions of thirty-five new Fellows at £6, 6s. each (including two outstanding at 30th September 1930)		220 10 0	
			1442 14 0
3. Extra Contributions for 1930–31 under Amended Law, No. 6 :—			
1. Voluntary Contributions		£37 13 0	
2. Commutations		31 10 0	
			69 3 0
4. Contributions for next Session :—			
2 Fellows at £3, 3s. each			6 6 0
5. Interest received—			
a. Interest on £445, 10s. 5% War Loan, 1929–47 (R. M. Smith Legacy), Untaxed		£22 5 6	
b. Interest on £751, 16s. 5% War Loan, 1929–47 (Special Subscription Fund), Untaxed		37 11 10	
c. General—			
Interest on £7830 5% War Loan, 1929–47, Untaxed	£391 10 0		
Interest on £2100 2½% Consolidated Stock, Untaxed	52 10 0		
Interest on Deposit Receipts	9 16 7		
		453 16 7	
			513 13 11
6. Transactions and Proceedings sold			155 3 7
7. Grants—Annual Grant from Government		£600 0 0	
Grant from Royal Society, Government Publication Grant		250 0 0	
			850 0 0
Amount of the Charge			**£3109 9 6**

DISCHARGE.

1. Taxes, Insurance, Coal and Lighting:—				
Insurance				£25 19 1
Coal, etc., to 11th June 1931				42 14 0
Gas to 9th July 1931				4 2 3
Electric Light to 6th April 1931				10 6 4
Water, 1930–31				4 4 0
Equitable Life Assurance Co., Ltd., for Superannuation Scheme			£45 0 0	
Less—Received from Assistant Secretary			15 0 0	
				30 0 0
				£117 5 8
2. Salaries:—				
General Secretary			£100 0 0	
Librarian and Assistant Secretary			320 0 0	
Assistant Librarian			145 0 0	
Office Keeper			151 12 0	
Treasurer's Clerk			35 0 0	
				751 12 0
3. Expenses of Transactions and Proceedings:—				
a. Transactions:—				
Neill & Co., Ltd., Printers			£504 5 0	
Hislop & Day, Ltd., Engravers			53 6 0	
The Zinco-Collotype Co.			149 5 0	
			£706 16 0	
Less Receipts—				
Grants by Carnegie Trustees towards Papers, etc., of the Misses Frances Ballantyne, Nicol, and E. J. Cadman; Messrs D. G. Catcheside, T. M. Finlay, A. Graham, and McCallien and Anderson			122 12 9	
			£584 3 3	
b. Proceedings:—				
Neill & Co., Ltd., Printers		£547 2 9		
Hislop & Day, Ltd., Engravers		17 17 6		
The Zinco-Collotype Co.		22 2 6		
		£587 2 9		
Less Receipts:—				
Grant by Carnegie Trustees towards Illustration of S. Williams' paper	£4 0 0			
Revenue of Publication Fund	104 1 5			
		108 1 5		
			479 1 4	
				1063 4
4. Books, Periodicals, Newspapers, etc.:—				
James Thin, Bookseller			£300 18 3	
R. Grant & Son, Booksellers			1 19 8	
Robertson & Scott, News Agents			7 7 8	
Ray Society, Subscription			1 1 0	
Berwickshire Naturalists' Club, Do.			1 0 0	
Palæontographical Society, Do. for years 1929–31			3 3 0	
History of Science Society, Do.			1 1 0	
Edinburgh and Leith Post-Office Directory, Ltd.			0 18 6	
Institution of Civil Engineers, for Abstracts			0 10 0	
Bombay Natural History Society, for *Journal*			2 1 0	
Scientific and Learned Societies' Year Book 1931			0 8 6	
				319 18 7
Carry forward				£2252 0 10

	Brought forward		£2252 0 10
5. Other Payments:—			
Neill & Co., Ltd., Printers		£119 18 6	
A. Cowan & Sons, Ltd.		19 14 3	
S. Duncan & Sons, Tailors (uniform)		9 9 0	
Macvitties, Guest & Co., Ltd.		38 11 9	
Andrew H. Baird		9 9 2	
Lindsay, Jamieson & Haldane, C.A., Auditors		10 10 0	
The Window Cleaning Co., Ltd.		16 4 0	
Orrock & Son, Ltd., Bookbinders		200 1 9	
Federated Superannuation System for Universities		5 0 0	
G. Waterston & Sons, Ltd.		14 14 1	
Telephone Accounts		12 15 8	
G. Cairns, Plumber		11 2 2	
W. S. Brown & Sons, Upholsterers, etc.		22 3 6	
Doig, Wilson, & Wheatley, Picture Restorers		7 13 6	
Edward & Co., Electrical Repairs		21 13 2	
Miscellaneous Accounts under £5		33 1 11	
Charwoman		63 14 0	
Petty Expenses, Postages, Carriage, etc.		39 16 6½	
T. & A. Constable, Printers		5 19 6	
"Polar Year" Expenses		66 4 6	
Underwood Typewriter		21 9 10	
Henderson & Bisset—Binding Hume Manuscripts		70 0 0	
			819 6 9½
6. Arrears of Contributions outstanding at 30th September 1931:—			
Present Session		£78 15 0	
Previous Sessions		22 1 0	
			100 16 0
	Amount of the Discharge		**£3172 3 7½**
Amount of the Charge			**£3109 9 6**
Amount of the Discharge			**3172 3 7½**
Excess of Discharge transferred to Special Subscription Fund			**£62 14 1½**

SPECIAL SUBSCRIPTION FUND

To 30th September 1931.

Charge.

Total Subscriptions towards Fund		£1128 17 9

Discharge.

1. Sum required to write War Loan Investment down to par		£7 12 0
2. Transfers to General Fund to meet Deficits—		
Up to 30th September 1930	£331 11 7	
Deficit for year to 30th September 1931	62 14 1½	
		394 5 8½
3. Balance of Fund—		
£751, 16s. 5% War Loan, 1929–47, at par	£751 16 0	
Due by Treasurer	18 17 10½	
	£770 13 10½	
Less—Due to Union Bank of Scotland, Ltd., on Current Account	43 13 10	
		727 0 0½
		£1128 17 9

II. KEITH FUND

To 30th September 1931.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1930		£43 14 9
2. Interest Received:—		
On £650 5% War Loan, 1929–47, Untaxed	£32 10 0	
On Deposit Receipts	0 19 3	
		33 9 3
		£77 4 0

DISCHARGE.

Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1931	£77 4 0

III. NEILL FUND

To 30th September 1931.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1930		£19 3 10
2. Interest Received:—		
On £300 5% War Loan, 1929–47, Untaxed	£15 0 0	
On Deposit Receipts	0 8 2	
		15 8 2
		£34 12 0

DISCHARGE.

Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1931	£34 12 0

IV. MAKDOUGALL-BRISBANE FUND

To 30th September 1931.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1930		£70 4 0
2. Interest Received:—		
On £400 5% War Loan, 1929–47, Untaxed	£20 0 0	
On Deposit Receipts	1 1 4	
		21 1 4
		£91 5 4

DISCHARGE.

1. Alex. Kirkwood & Son, for Gold Medal	£17 10 0
Miss Nellie B. Eales, 1928–30 award	24 5 8
	£41 15 8
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1931	49 9 8
	£91 5 4

V. MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND

To 30th September 1931.

CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1930		£64 12 4
2. INTEREST RECEIVED:—		
On £250 5% War Loan, 1929-47, Untaxed	£12 10 0	
On Deposit Receipts	1 2 4	
		13 12 4
		£78 4 8

DISCHARGE.

BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1931	£78 4 8

VI. GUNNING VICTORIA JUBILEE PRIZE FUND

To 30th September 1931.

CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1930		£100 18 5
2. INTEREST RECEIVED:—		
On £599, 14s. 5% War Loan, 1929-47, Untaxed	£29 19 8	
On Deposit Receipts	1 17 0	
		31 16 8
		£132 15 1

DISCHARGE.

BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1931	£132 15 1

VII. JAMES SCOTT PRIZE FUND

To 30th September 1931.

CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1930		£2 17 10
2. INTEREST RECEIVED:—		
On £247, 10s. 5% War Loan, 1929-47, Untaxed	£12 7 6	
On Deposit Receipts	0 2 4	
		12 9 10
		£15 7 8

DISCHARGE.

BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1931	£15 7 8

VIII. PUBLICATION FUND

(Comprising Peter Guthrie Tait Memorial Fund and Dr John Aitken Fund)

To 30th September 1931.

CHARGE.

1. Peter Guthrie Tait Memorial Fund :— Year's Interest on £1550 5% War Loan, 1929-47, Untaxed		£77 10 0
2. Dr John Aitken Fund :—		
Year's Interest on £445, 10s. 5% War Loan, 1929-47, Untaxed	£22 5 6	
Interest on Deposit Receipts	1 6 10	
Sale of Volumes	2 19 1	
		26 11 5
		£104 1 5

DISCHARGE.

Transferred to General Fund to meet Cost of Publications (see General Fund Discharge, No. 3)	£104 1 5

IX. DR W. S. BRUCE MEMORIAL FUND

To 30th September 1931.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1930		£7 15 2
2. Interest Received :—		
On £233 3½% Conversion Loan, 1961	£8 3 0	
On Deposit Receipts	0 2 7	
		8 5 7
		£16 0 9

DISCHARGE.

Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1931	£16 0 9

X. BRUCE-PRELLER LECTURE FUND

To 30th September 1931.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1930		£33 9 7
2. Dividend and Interest Received :—		
On £140, 9s. Royal Bank of Scotland Stock, less Tax £5, 7s. 6d.	£18 10 0	
On Deposit Receipts	0 9 0	
		18 19 0
		£52 8 7

DISCHARGE.

1. Professor E. T. Whittaker, Fee as Lecturer, 1931	£40 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1931	12 8 7
	£52 8 7

XI. DR DAVID ANDERSON-BERRY FUND

To 30th September 1931.

CHARGE.

1. BALANCE of Revenue due by Union Bank of Scotland, Ltd., on Deposit Receipt, at 30th September 1930		£5 1 8
2. INTEREST RECEIVED :—		
On £1528, 0s. 4d. Local Loans 3% Stock from 5th October 1930 to 5th July 1931, *less* tax £7, 14s. 6d.	£26 13 0	
On Deposit Receipts	0 3 1	
		26 16 1
		£31 17 9

DISCHARGE.

BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt, at 30th September 1931	£31 17 9

STATE OF THE FUNDS BELONGING TO THE ROYAL SOCIETY OF EDINBURGH

As at 30th *September* 1931.

1. GENERAL FUND—

1. £7830 5% War Loan, 1929–47		£7830 0 0
2. £2100 2½% Consolidated Stock at 53%		1113 0 0
3. £445, 10s. 5% War Loan, 1929–47. Mr Robert Mackay Smith, Legacy		445 10 0
4. Arrears of Contributions, as per preceding Abstract of Accounts		100 16 0
5. Balance of Special Subscription Fund—		
£751, 16s. 5% War Loan, 1929–47	£751 16 0	
Cash due by Treasurer	18 17 10½	
	£770 13 10½	
Less—Cash due to Union Bank of Scotland, Ltd., on Current Account	43 13 10	
		727 0 0½
AMOUNT		£10,216 6 0½

Exclusive of Library, Museum, Pictures, etc., and Furniture in the Society's Rooms at George Street, Edinburgh.

2. KEITH FUND—

1. £650 5% War Loan, 1929–47	£650 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	77 4 0
AMOUNT	£727 4 0

3. NEILL FUND—

1. £300 5% War Loan, 1929–47	£300 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	34 12 0
AMOUNT	£334 12 0

4. MAKDOUGALL-BRISBANE FUND—

1. £400 5% War Loan, 1929–47	£400 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	49 9 8
AMOUNT	£449 9 8

5. MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND—

1. £250 5% War Loan, 1929–47	£250	0	0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	78	4	8
AMOUNT	£328	4	8

6. GUNNING VICTORIA JUBILEE PRIZE FUND—Instituted by Dr Gunning of Edinburgh and Rio de Janeiro—

1. £599, 14s. 5% War Loan, 1929–47	£599	14	0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	132	15	1
AMOUNT	£732	9	1

7. JAMES SCOTT PRIZE FUND—

1. £247, 10s. 5% War Loan, 1929–47	£247	10	0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	15	7	8
AMOUNT	£262	17	8

8. PUBLICATION FUND—

(COMPRISING PETER GUTHRIE TAIT MEMORIAL FUND AND DR JOHN AITKEN FUND)

1. PETER GUTHRIE TAIT MEMORIAL FUND:— £1550 5% War Loan, 1929–47				£1550	0	0
2. DR JOHN AITKEN FUND:— £445, 10s. 5% War Loan, 1929–47	£445	10	0			
Deposit Receipt with Union Bank of Scotland, Ltd.	71	6	1			
				516	16	1
AMOUNT				£2066	16	1

9. DR W. S. BRUCE MEMORIAL FUND—

1. £233 $3\frac{1}{2}$% Conversion Loan, at $72\frac{7}{8}$%	£169	15	11
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	16	0	9
AMOUNT	£185	16	8

10. BRUCE-PRELLER LECTURE FUND—

1. £140, 9s. Royal Bank of Scotland Stock, taken over at 350%	£491	11	6
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	12	8	7
AMOUNT	£504	0	1

11. DR DAVID ANDERSON-BERRY FUND—

1. £1528, 6s. 4d. Local Loans 3% Stock at $65\frac{7}{16}$%	£1000	0	0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	31	17	9
AMOUNT	£1031	17	9

Note.—As previously, 5% War Stock, 1929–47, has been uniformly valued at par in the above State of Funds.

EDINBURGH, 16*th October* 1931.—We have examined the preceding Accounts of the Treasurer of the Royal Society of Edinburgh for the Session 1930–1931, and have found them to be correct. The securities for the various Investments, as noted in the foregoing Statement of Funds, have been verified by us as at 30th September 1931.

LINDSAY, JAMIESON & HALDANE, C.A.,

Auditors.

LIST OF VOLUNTARY CONTRIBUTORS who have made a Single Payment under Law VI (end of para. 3), up to 30th September 1931.

Dr Alexander Scott, F.R.S.,	£21 0 0
Professor J. W. Gregory, F.R.S.,	10 10 0
Total,	£31 10 0

LIST OF VOLUNTARY CONTRIBUTORS under Law VI (end of para. 3), up to 30th September 1931.

Col. A. F. Appleton,	£1 1 0
Sir James Barr,	1 1 0
Professor F. O. Bower, F.R.S.,	1 1 0
Professor Sir T. H. Beare,	1 1 0
Judge F. E. Bradley,	1 1 0
Principal O. C. Bradley,	1 1 0
Dr G. S. Brock,	1 1 0
Mr J. W. Butters,	1 1 0
Professor E. W. Carlier,	1 1 0
Mr. David Carnegie,	1 1 0
Professor E. G. Coker,	1 1 0
Dr T. W. Dewar,	2 0 0
Dr J. Haig Ferguson,	1 1 0
Dr R. A. Fleming,	1 1 0
Mr J. S. Ford,	1 1 0
Dr E. M. Horsburgh,	1 1 0
Dr W. F. Hume,	1 1 0
Professor T. J. Jehu,	1 1 0
Total,	£19 17 0
Carried forward,	£19 17 0
Mr F. H. Lightbody,	1 1 0
Dr A. Veitch Lothian,	1 1 0
Dr Peter M'Bride,	1 1 0
Mr James A. Macdonald,	1 1 0
Dr George M'Gowan,	1 1 0
Sir W. Leslie Mackenzie,	1 1 0
Dr Hugh R. Mill,	1 1 0
Sir Thomas Muir, F.R.S.,	1 1 0
Professor W. Peddie,	1 0 0
Em. Professor A. G. Perkin, F.R.S.,	1 1 0
Mr A. G. Ramage,	1 1 0
Mr Ralph Richardson,	1 1 0
Professor R. A. Robertson,	1 1 0
Professor H. F. Stockdale,	1 1 0
Mr Gilbert Thomson,	1 1 0
Sir James Walker, F.R.S.,	1 1 0
Mr J. C. Wright,	1 1 0
Total,	£37 13 0

Single payments,	£31 10 0
Other payments,	37 13 0
Total,	£69 3 0

THE COUNCIL OF THE SOCIETY.

26th October 1931.

PRESIDENT.

PROFESSOR SIR E. A. SHARPEY-SCHAFER, M.D., D.Sc., LL.D., F.R.S.

VICE-PRESIDENTS.

PROFESSOR FRANCIS G. BAILY, M.A., M.Inst.E.E.
PROFESSOR T. J. JEHU, M.A., M.D., F.G.S.
PROFESSOR J. H. ASHWORTH, D.Sc., F.R.S.
ARTHUR LOGAN TURNER, M.D., LL.D., F.R.C.S.E.
J. B. CLARK, M.A., LL.D., J.P.
PROFESSOR JAMES RITCHIE, M.A., D.Sc.

GENERAL SECRETARY.

PROFESSOR R. A. SAMPSON, M.A., D.Sc., LL.D., F.R.S.

SECRETARIES TO ORDINARY MEETINGS.

PROFESSOR C. G. DARWIN, M.A., F.R.S.
PROFESSOR F. A. E. CREW, M.D., D.Sc., Ph.D.

TREASURER.

JAMES WATT, W.S., LL.D.

CURATOR OF LIBRARY AND MUSEUM.

PROFESSOR D'ARCY W. THOMPSON, C.B., F.R.S.

COUNCILLORS.

PROFESSOR JAMES DREVER, M.A., B.Sc., D.Phil.
A. H. R. GOLDIE, M.A., B.A.
ROBERT ALEX. HOUSTOUN, M.A., Ph.D., D.Sc.
THE HON. LORD SANDS, Kt., K.C., LL.D., D.D.
MURRAY MACGREGOR, M.A., D.Sc.
A. CRICHTON MITCHELL, D.Sc.
PROFESSOR P. T. HERRING, M.D., F.R.C.P.Ed.
PRINCIPAL SIR THOMAS H. HOLLAND, K.C.S.I., K.C.I.E., Hon. D.Sc., LL.D., F.R.S.
PROFESSOR JAMES KENDALL, M.A., D.Sc., F.R.S.
PROFESSOR THOMAS M. MACROBERT, M.A., D.Sc.
PROFESSOR GODFREY H. THOMSON, D.Sc., Ph.D.
MALCOLM WILSON, D.Sc., A.R.C.Sc., F.L.S.

OFFICE STAFF.

Assistant Secretary and Librarian, G. A. STEWART.
Assistant Librarian, R. J. B. MUNRO.
Housekeeper, SAMUEL HEDDLE.

ALPHABETICAL LIST OF THE ORDINARY FELLOWS OF THE SOCIETY,

Corrected to 26th October, 1931.

N.B.—*Those marked* * *are Annual Contributors.*
,, ,, † *have commuted Voluntary Contribution* (*see* 3rd *Paragraph, Law VI*).

M.B. prefixed to a name indicates that the Fellow has received a Makdougall-Brisbane Medal.
K. ,, ,, ,, ,, Keith Medal.
N. ,, ,, ,, ,, Neill Medal.
V. J. ,, ,, ,, ,, the Gunning Victoria Jubilee Prize.
B. ,, ,, ,, ,, Bruce Medal.
B-P. ,, ,, ,, ,, Bruce-Preller Lectureship.
C. ,, ,, ,, has contributed one or more Communications to the Society's TRANSACTIONS OF PROCEEDINGS.

Date of Election.			Service on Council, etc.
1922		* Abernethy, Charles Lawrence, M.A. (Hons.), B.Sc., Research Physicist, "Exnaböe," Craiglockhart Avenue, Slateford, Edinburgh	
1925	C.	* Aitken, Alexander Craig, M.A., D.Sc., Lecturer in Mathematics in the University of Edinburgh. Mathematical Institute, 16 Chambers Street, Edinburgh	
1889		† Alison, John, M.A., LL.D., formerly Head Master, George Watson's College. 126 Craiglea Drive, Edinburgh	
1927	C.	* Allan, Douglas Alexander, Ph.D. (Edin.), D.Sc., Director, City of Liverpool Public Museums, William Brown Street, Liverpool	
1894		† Allan, Francis John, M.D., C.M. (Edin.), M.O.H. City of Westminster. Lincluden, 33 Cromwell Road, Teddington, Middlesex	
1920	C. M-B.	* Allen, Herbert Stanley, M.A. (Cambridge), D.Sc. (London), F.R.S., Professor of Natural Philosophy in the University of St Andrews	1921–24.
1920	C.	* Anderson, Ernest Masson, M.A., B.Sc., F.G.S., 50 Greenbank Crescent, Edinburgh	
1905		Anderson, William, M.A., Head Science Master, George Watson's College, Edinburgh. 6 Lockharton Crescent, Edinburgh	
1905		Andrew, George, M.A., B.A., H.M.I.S., Royal Technical College, George Street, Glasgow. Hamewith, Kilmacolm, Renfrewshire	
1881	C.	Anglin, Arthur H., M.A., LL.D., M.R.I.A., formerly Professor of Mathematics, Queen's College, Cork	
1930		* Annan, William, M.A., C.A., F.C.W.A., Professor of Accounting and Business Method in the University of Edinburgh. Tofthill, Ferry Road West, Edinburgh	
1915		Anthony, Charles, M.Inst.C.E., M.Am.Soc.C.E., F.R.San.I., F.R.Met.S., F.R.A.S., F.C.S., c/o Royal Society, Edinburgh	
1906		Appleton, Colonel Arthur Frederick, F.R.C.V.S., Nutwell, 34 Shortlands Road, Shortlands, Kent	
1910	C.	Archibald, E. H., B.Sc., Professor of Chemistry, University of British Columbia, Vancouver, Canada	
1921		* Arthur, William, M.A., Lecturer in Mathematics in the University of Glasgow. 148 Carmunnock Road, Cathcart, Glasgow	
1911	C. K.	* Ashworth, James Hartley, D.Sc., F.R.S. (VICE-PRESIDENT), Professor of Natural History, University of Edinburgh. "Hillbank," Grange Loan, Edinburgh	1912–14, 1915–18, 1927–30. Sec. 1918–23. V-P 1923–26, 1930–
1920		* Bagnall, Richard Siddoway, Hon. D.Sc., Member of the Entomological and other Scientific Societies. C/o Macdonald, Brown & Co., 9 Charles Street, London, S.W. 1	
1920	C. N.	* Bailey, Edward Battersby, M.C., M.A., F.R.S., F.G.S., Professor of Geology in the University of Glasgow	

Date of Election.			Service on Council, etc.
1896	C.	† Baily, Francis Gibson, M.A., M.Inst.E.E. (VICE-PRESIDENT), Professor of Electrical Engineering, Heriot-Watt College, Edinburgh. Newbury, Colinton, Midlothian	1909-12, 1920-23. V.P 1929-
1931		* Bain, William Alexander, B.Sc., Lecturer in Biophysics, Department of Physiology, University of Edinburgh. 9 Falcon Gardens, Edinburgh	
1931		* Baird, William Macdonald, Fellow and Past President of the Faculty of Surveyors of Scotland, F.S.A.Scot., J.P. Dalveen, Barnton Avenue, Davidson's Mains, Edinburgh	
1921		* Baker, Bevan Braithwaite, M.A., D.Sc., Professor of Mathematics, Royal Holloway College, Englefield Green, London	
1928	C.	* Baker, Edwin Arthur, D.Sc. (Edin.), Assistant at the Royal Observatory, Edinburgh. 17 Ladysmith Road, Edinburgh	
1905	C.	Balfour-Browne, William Alexander Francis, M.A., F.Z.S., F.L.S., F.E.S., Barrister-at-Law, late Professor of Entomology at the Imperial College of Science and Technology, South Kensington, London, S.W. 7. Winscombe Court, Winscombe, Somerset	
1923		* Bannerman, David Armitage, M.A. (Cantab.), M.B.E., M.A., F.R.G.S., Special Assistant (Ornithology) in the Department of Zoology, British Museum (Natural History), London. 7 Pembroke Gardens, Kensington, London, W. 8	
1928		* Barbour, George Brown, M.A. (Edin.), M.A. (Camb.), Ph.D., F.G.S., c/o Dr R. L Dickinson, 438 West 116th Street, New York City, N.Y., U.S.A.	
1886		Barclay, A. J. Gunion, M.A., 3 Chandos Avenue, Oakleigh Park, London, N.	
1903		Bardswell, Noël Dean, M.V.O., M.D., M.R.C.P. (Ed. and Lond.), New County Hall, Westminster Bridge Road, London, S.E. 1	
1922		* Barger, George, M.A., D.Sc., Dr *h. c.* (Padua), Hon. D.Sc. (Liverp.), F.R.S., *Hon. Mem. Nederl. Chem. Vereen ; Corr. Mem. Bayerische Akad. d. Wissensch. München and Ges. d. Wissensch. Göttingen*, Professor of Chemistry (Medical) in the University of Edinburgh. 48 St Alban's Road, Edinburgh	1925-28.
1929		* Barker, Sydney George, Ph.D., D.I.C., F.Inst.P., Director of Research, British Research Association for the Woollen and Worsted Industries. Torridon, Headingley, Leeds	
1914	C.	* Barkla, Charles Glover, M.A., D.Sc., F.R.S., Professor of Natural Philosophy in the University of Edinburgh, Nobel Laureate, Physics, 1917. The Hermitage of Braid, Edinburgh	1915-18, 1924-27.
1925		* Barlow, Thomas William Naylor, O.B.E., M.R.C.S., D.P.H., Barrister-at-Law, Past President of the Incorporated Society of Medical Officers of Health. 23 North Drive, New Brighton, Cheshire	
1927		* Barnett, John, F.F.A., C.A., Scottish Widows' Fund Life Assurance Society, 9 St Andrew Square, Edinburgh	
1904		Barr, Sir James, C.B.E., M.D., LL.D., F.R.C.P. (Lond.), Hindhead Brae, Hindhead, Surrey	
1921		* Bartholomew, John, M.C., M.A., F.R.G.S., Geographical Institute, Duncan Street, Edinburgh. Nairne Lodge, Duddingston	1925-28.
1927		* Bastow, Stephen Everard, M.Inst.E.E., M.Inst.Min.E., Managing Director, Bruce Peebles & Co., Ltd., Edinburgh. Northwood, Russell Place, Trinity, Edinburgh	
1929		* Bath, Frederick, B.Sc., Ph.D., Lecturer in Mathematics, University of St Andrews, Assistant to the Professor of Mathematics, University College, Dundee	
1913		† Beard, Joseph, F.R.C.S. (Edin.), M.R.C.S. (Eng.), L.R.C.P. (Lond.), D.P.H. (Camb.), formerly Medical Officer of Health and School Medical Officer, City of Carlisle. 8 Carlton Gardens, Carlisle	
1888		Beare, Sir Thomas Hudson, B.A., B.Sc., M.Inst.C.E., J.P., D.L., Professor of Engineering in the University of Edinburgh	1907-09. V.P 1909-15, 1923-26.
1897	C.	Beattie, Sir John Carruthers, K.B., D.Sc., LL.D., Vice-Chancellor and Principal, The University, Cape Town	
1893	C. M-B.	Becker, Ludwig, Ph.D., *Regius Professor of Astronomy in the University of Glasgow.* The Observatory, Dowanhill, Glasgow	
1916		* Bell, Robert John Tainsh, M.A., D.Sc., Professor of Mathematics in the University of Otago, Dunedin, New Zealand	
1915		Bell, Walter Leonard, M.D. (Edin.), F.S.A.Scot., Langarth, Brisco, Carlisle	
1929		* Bennet, George, B.Sc., A.M.I.Mech.E., Lecturer in Mechanical Engineering, 68 Arden Street, Edinburgh	
1893	C.	Berry, Sir George A., M.B., C.M., LL.D., F.R.C.S.E., King's Knoll, North Berwick	1916-19. V.P 1919-22.

Date of Election.			Service on Council, etc.
1897	C.	Berry, Richard J. A., M.D., F.R.C.S.E., Director of Medical Services, Stoke Park Colony, Stapleton, Bristol. Rufford, Canford Lane, Westbury-on-Trym, Bristol	
1880	C.	Birch, de Burgh, C.B., M.D., Em. Professor of Physiology in the University of Leeds	
1907		Black, Frederick Alexander, T.D., Solicitor, 57 Academy Street, Inverness	
1931	C.	* Black, Thomas Purves, M.A. (Edin.), B.Sc. (Lond.), Ph.D. (Edin.), Head of Department of Mathematics, Trinity Academy, Leith. 32 Eastfield, Joppa	
1918		* Blight, Francis James, formerly Chairman and Managing Director of Charles Griffin & Co., Ltd., Publishers, Belstone Tor, Uphill Road, Mill Hill, London, N.W. 7	
1894		Bolton, Herbert, D.Sc., F.G.S., F.Z.S., formerly Director of the Bristol Museum and Art Gallery, Bristol. 318 Tilehurst Road, Reading, Berks	
1915		* Boon, Alfred Archibald, D.Sc., B.A., F.I.C., Em. Professor of Chemistry, Heriot-Watt College, Edinburgh	
1925		* Borthwick, Albert William, O.B.E., D.Sc., Professor of Forestry in the University of Aberdeen	
1925		* Bose, Sahay Ram, M.A., D.Sc., F.L.S., Professor of Botany in the Carmichael Medical College, Belgachia, Calcutta, India	
1886	C. N.	Bower, Frederick Orpen, M.A., D.Sc., LL.D., F.R.S., F.L.S., Em. Regius Professor of Botany in the University of Glasgow. 2 The Crescent, Ripon, Yorks	1887–90, 1893–96, 1907–09, 1917–19. V-P 1910–16. P 1919–24.
1924		* Bowman, Alexander, D.Sc., Scientific Superintendent, Marine Laboratory, Fishery Board for Scotland, Torry, Aberdeen	
1916		Bradley, His Honour Judge (Francis Ernest), M.A., M.Com., LL.D., Member of the Court of Governors of Manchester University. 8 Balmoral Road, St Annes-on-the-Sea	
1903	C.	Bradley, O. Charnock, M.D., D.Sc., Principal, Royal (Dick) Veterinary College, Edinburgh	1907–10, 1915–17.
1926		* Braid, Kenneth William, B.A. (Cantab.), B.Sc., Professor of Botany, West of Scotland Agricultural College, 6 Blythswood Square, Glasgow	
1907		Bramwell, Edwin, Professor of Clinical Medicine in the University of Edinburgh, M.D., F.R.C.P.E., F.R.C.P. (Lond.). 23 Drumsheugh Gardens, Edinburgh	
1918		* Bremner, Alexander, M.A., D.Sc., Headmaster, Demonstration School, Training Centre, Aberdeen. 13 Belgrave Terrace, Aberdeen	
1916	C.	* Briggs, Henry, O.B.E., D.Sc., Ph.D., A.R.S.M., James A. Hood Professor of Mining in the University of Edinburgh. 12 Gordon Terrace, Edinburgh	1923–26.
1895		Bright, Sir Charles, M.Inst.C.E., M.Inst.E.E., F.R.Aë.S., F.Inst.Radio.E., F.R.A.S., F.R.G.S., Little Brewers', Hatfield Heath, Harlow, Essex, and Athenæum Club, Pall Mall, London, S.W.	
1893		Brock, G. Sandison, M.D., F.R.C.P.E., 53 Chemiston Gardens, Kensington, London, W. 8	
1901	C.	Brodie, W. Brodie, M.D., Camden House, Bletchingley, Surrey	
1907		Brown, Alexander, M.A., B.Sc., Professor of Applied Mathematics, The University, Cape Town	
1928		* Brown, Hugh Wylie, F.I.A., F.F.A., 1 Cobden Crescent, Edinburgh	
1885	C.	Brown, J. Macdonald, M.D., F.R.C.S., 10 Bryanston Street, London, W. 1	
1924		* Brown, Thomas Arnold, M.A., B.Sc., Professor of Mathematics in University College, Exeter	
1923		* Brown, Walter, M.A., B.Sc., Professor of Mathematics, The University, Hong Kong, China	
1921		* Bruce, Alexander, B.Sc. (Edin.), Government Agricultural Chemist and City Analyst, The Laboratory, Turret Road S., Colombo, Ceylon	
1912		* Bruce, Alexander Ninian, D.Sc., M.D., 8 Ainslie Place, Edinburgh	
1927		* Bryce, David Lawrence. Vice-President (1918–1925), Quekett Microscopical Club. Salfords Parsonage, Redhill, Surrey	
1898	C. K.	† Bryce, Thomas Hastie, M.A., M.D. (Edin.), F.R.S., Professor of Anatomy in the University of Glasgow. 2 The College, Glasgow	1911–14, 1922–25. V-P 1925–28.
1902		Burgess, A. G., M.A., D.Sc., Rector of The Academy, Rothesay. Midmar, Crichton Road, Rothesay	

Date of Election.			Service on Council, etc.
1887		† Burnet, Sir John James, R.A., R.S.A., LL.D., Architect, Killermont, Rowledge, Farnham, Surrey	
1888		Burns, Rev. Thomas, C.B.E., D.D., J.P., F.S.A.Scot., Minister of Lady Glenorchy's Parish Church, Croston Lodge, Chalmers Crescent, Edinburgh	
1917		* Burnside, George Barnhill, M.I.Mech.E., 104 Beechwood Drive, Glasgow, W.	
1930	C.	* Burt, David Raitt Robertson, B.Sc. (St Andrews), F.L.S., Lecturer in Zoology, Ceylon University College, Colombo. Fernbank, Kirkcaldy	
1896		Butters, John W., M.A., B.Sc., formerly Rector of Ardrossan Academy. 116 Comiston Drive, Edinburgh	
1887	C.	Cadell, Henry Moubray, of Grange, B.Sc., M.Inst.M.E., D.L., Linlithgow	1919-22.
1929	C.	* Calder, Alexander, B.Sc., Ph.D., Assistant in the Institute of Animal Genetics, The University, Edinburgh	
1910		* Calderwood, Rev. Robert Sibbald, D.D., Minister of Cambuslang, The Old Manse, Cambuslang, Lanarkshire	
1893	C.	Calderwood, W. L., I.S.O., formerly Inspector of Salmon Fisheries of Scotland, Canaan Grove, 82 Newbattle Terrace, Edinburgh	1923-26.
1926	C.	* Cameron, Alfred E. Henderson, M.A., D.Sc. (Aberd.), M.Sc. (Vict.), Lecturer in Entomology, Zoology Department, The University, Edinburgh. 8 West Savile Road, Edinburgh	
1905	C.	Cameron, John, M.D., D.Sc., M.R.C.S.Eng., formerly Professor of Anatomy, Dalhousie University, Halifax, Nova Scotia. Wingfield, Grosvenor Gardens, Golders Green, London, N.W. 11	
1921		* Campbell, Andrew, Advisory Chemist, Burmah Oil Co., Ltd., and Anglo-Persian Oil Co., Ltd. 36 Abbey Lodge, Hanover Gate, Regent's Park, London, N.W.	
1918		* Campbell, John Menzies, D.D.S. (Toronto), L.D.S. (Glas.), L.D.S. (Ontario), F.I.C.D., 14 Buckingham Terrace, Glasgow, W.	
1915	C. N.	* Campbell, Robert, M.A., D.Sc., F.G.S., Lecturer in Petrology, University of Edinburgh. Maryton, Colinton	1920-23.
1927	C.	* Cannon, Herbert Graham, M.A., Sc.D. (Cantab.), D.Sc. (Lond.), F.L.S., Beyer Professor of Zoology in the University of Manchester	
1899	C.	Carlier, Edmund W. W., B. ès Sc. (France), M.Sc., M.D., F.E.S., Em. Professor of Physiology, University, Birmingham. Morningside, Dorridge, near Birmingham	
1910		Carnegie, Col. David, C.B.E., M.Inst.C.E., J.P., Haven, Seasalter, Whitstable	
1931		* Carroll, John Anthony, M.A., Ph.D. (Camb.), Professor of Natural Philosophy, University of Aberdeen. Marischal College, Aberdeen	
1920	C.	* Carruthers, R. G., F.G.S., District Geologist, H.M. Geological Survey, High Barn, Stocksfield-on-Tyne	
1905	C.	Carse, George Alexander, M.A., D.Sc., Reader in Natural Philosophy, University of Edinburgh. 3 Middleby Street, Edinburgh	
1901		Carslaw, Horatio Scott, M.A., Sc.D. (Camb.), D.Sc., LL.D. (Glasg.), Fellow of Emmanuel College, Cambridge, Professor of Mathematics in the University of Sydney, New South Wales	
1925		* Carter, George Stuart, M.A., Ph.D., Corpus Christi College, Cambridge	
1898		Carus-Wilson, Cecil, J.P., F.R.G.S., F.G.S., Mayor of Twickenham, 16 Waldegrave Park, Strawberry Hill, Middlesex, and Sandacres Lodge, Parkstone-on-Sea, Dorset	
1899		Chatham, James, Actuary, c/o Robert Murrie, Esq., 48 Morningside Park, Edinburgh	
1912		Chaudhuri, Banawari Lal, B.A. (Cal.), D.Sc. (Edin.), formerly Superintendent, Natural History Section, Indian Museum, Sherpurtown P.O., India	
1925	C.	* Chumley, James, M.A., Ph.D., Lecturer on Oceanography, Department of Zoology, University of Glasgow. Thalassa, Thorn Drive, Bearsden, Dumbartonshire	
1928	C.	* Clark, Alfred Joseph, B.A., M.D., F.R.S., Professor of Materia Medica in the University of Edinburgh. 67 Braid Avenue, Edinburgh	
1891		Clark, John Brown, M.A., LL.D., J.P. (Vice-President), formerly Head Master of George Heriot's School. Garleffin, 146 Craiglea Drive, Edinburgh	1928-1931 V.P 1931-
1911		* Clark, William Inglis, D.Sc., 1 Belgrave Crescent, Edinburgh	
1903		Clarke, William Eagle, I.S.O., LL.D., F.L.S., Honorary Supervisor of the Bird Collection and formerly Keeper of the Natural History Collections in the Royal Scottish Museum, Edinburgh. 8 Grosvenor Street, Edinburgh	
1909		Clayton, Thomas Morrison, M.D., D.Hy., B.Sc., D.P.H., Medical Officer of Health, Gateshead, Health Department, Greenesfield House, Gateshead-on-Tyne	
1922		* Clerk, Sir Dugald, K.B.E., D.Sc., LL.D., F.R.S., M.Inst.C.E., etc., Lukyns, Ewhurst, Surrey	

Date of Election.			Service on Council, etc.
1904	C.	Coker, Ernest George, M.A. (Cantab.), D.Sc. (Edin.), Hon. D.Sc. (Sydney and Louvain), M.Sc. (M'Gill), F.R.S., M.Inst.C.E., M.I.Mech.E., Kennedy Professor of Civil and Mechanical Engineering, and Director of the Engineering Laboratories, University of London, University College, Gower Street, London, W.C. 1	
1904		Coles, Alfred Charles, M.D., D.Sc., York House, Poole Road, Bournemouth, W.	
1886	V. J. C.	Collie, John Norman, Ph.D., D.Sc., LL.D., F.R.S., F.C.S., F.I.C., F.R.G.S., Em. Professor of Organic Chemistry in the University College, Gower Street, London	
1909	C.	* Comrie, Peter, M.A., B.Sc., LL.D., Rector, Leith Academy. 19 Craighouse Terrace, Edinburgh	
1924	C.	* Copson, Edward Thomas, M.A., D.Sc., Lecturer in Applied Mathematics in the University of St Andrews. St Salvator's Hall, St Andrews	
1929		* Coull, George, D.Sc., Pharmaceutical Chemist, Smith's Place House, Leith	
1928		* Coutie, Rev. Alexander, B.Sc., Ph.D., South Manse, Fraserburgh, Aberdeenshire	
1914		* Coutts, William Barron, M.A., B.Sc., Senior Lecturer in Range Finding and Optics, Military College of Science, Red Barracks, Woolwich, S.E. 18. 11 Coleraine Road, Blackheath, S.E. 3	
1911		* Cowan, Alexander, Papermaker, Valleyfield, Penicuik, Midlothian	
1931		* Cowan, John Macqueen, M.A., D.Sc. (Edin.), B.A. (Oxon.), F.L.S., Assistant Keeper, Royal Botanic Garden, Edinburgh. 17 Inverleith Place, Edinburgh	
1920		Craib, William Grant, M.A. (Aberdeen), Regius Professor of Botany in the University of Aberdeen	
1916	C.	† Craig, E. H. Cunningham, B.A. (Cambridge), Geologist and Mining Engineer, The Dutch House, Beaconsfield	
1908		Craig, James Ireland, M.A., B.A., Woolwich House, The Drive, Sydenham, London, S.E. 26. (At present—Turf Club, Cairo)	
1925	C. K.	* Craig, Robert Meldrum, M.A., B.Sc., F.G.S., Lecturer in Economic Geology in the University of Edinburgh	
1908		Crawford, Lawrence, M.A., D.Sc., Professor of Pure Mathematics, The University, Cape Town	
1922	C.	* Crew, Francis Albert Eley, M.D., Ch.B., D.Sc., Ph.D. (SECRETARY TO ORDINARY MEETINGS), Professor of Animal Genetics in the University of Edinburgh, and Director of the Institute of Animal Genetics. 10 Salisbury Road, Edinburgh	1928-31. Sec. 1931-
1931		* Crichton, John, M.A., B.Sc. (Edin.), Assistant Superintendent, Meteorological Office, Edinburgh. 10 Laverockbank Road, Edinburgh	
1870		Crichton-Browne, Sir Jas., Kt., M.D., LL.D., D.Sc., F.R.S., Vice-President and Treasurer of the Royal Institution of Great Britain. 45 Hans Place, London, S.W. 1	
1916		* Crombie, James Edward, M.A., LL.D., Millowner, Parkhill House, Dyce, Aberdeenshire	
1929		* Cruickshank, Ernest William Henderson, M.D., D.Sc., Ph.D., Professor of Physiology, Dalhousie University, Halifax, Nova Scotia	
1914		* Cumming, Alexander Charles, O.B.E., D.Sc., Roselands, Crescent Road, Blundell Sands, Liverpool	
1928		* Cumming, William Murdoch, D.Sc. (Glasg.), F.I.C., M.Inst.Chem.E., Senior Lecturer on Organic Chemistry, Royal Technical College, Glasgow. "Bonnieblink," 4 Newlands Road, Newlands, Glasgow	
1917		* Cunningham, Bryson, D.Sc., B.E., M.Inst.C.E., Lecturer on Waterways, Harbours, and Docks, University College, London. 141 Copers Cope Road, Beckenham, Kent	
1930		* Cunningham, John, C.I.E., B.A., M.D., Lt.-Colonel, Indian Medical Service. 2 Murrayfield Avenue, Edinburgh	
1904		Cuthbertson, John, Secretary, West of Scotland Agricultural College, 6 Charles Street, Kilmarnock	
1885		Daniell, Alfred, M.A., LL.B., D.Sc., Advocate, The Athenæum Club, Pall Mall, London	
1924		* Darwin, Charles Galton, M.A., F.R.S. (SECRETARY TO ORDINARY MEETINGS), Tait Professor of Natural Philosophy in the University of Edinburgh. 14 Heriot Row, Edinburgh	1925-28. Sec. 1928-
1921		* Datta, Rasik Lal, D.Sc., Industrial Chemist to the Government of Bengal Department of Industries. 14A Jagadish Nath Roy Lane, Calcutta, India	
1930	C.	* Davies, Lewis Merson, F.G.S., F.R.A.I., Lt.-Colonel, Royal Artillery, Officer Commanding the Scottish Coast Defences. 8 Garscube Terrace, Murrayfield, Edinburgh	
1931		* Dawson, Shepherd, M.A., D.Sc., Principal Lecturer on Psychology, Training College, Glasgow. Hazel Bank, Bearsden, Dumbartonshire	

Date of Election.			Services on Council, etc.
1928		Dawson, Warren Royal, F.R.S.L., Honorary Librarian of Lloyd's, 28 Grange Road, Barnes, London, S.W. 13	
1917		* Day, T. Cuthbert, Partner of the firm of Hislop & Day. 36 Hillside Crescent, Edinburgh	
1923		* Deane, Arthur, M.R.I.A., Curator, Public Art Gallery and Museum, Belfast. Threave, Saintfield Road, Newtownbreda, Belfast	
1894		† Denny, Sir Archibald, Bart., LL.D., 5 St Helen's Place, London, E.C. 4	
1906		† Dewar, Thomas William, M.D., F.R.C.P., Kincairn, Dunblane	
1925		Dey, Alexander John, Managing Director of T. & H. Smith, Ltd., Manufacturing Chemists, Blandfield Works, Edinburgh. Rothiemay, Corstorphine, Edinburgh	
1924		* Dinham, C. H., B.A., Geological Survey, 28 Jermyn Street, London, S.W. 1	
1885	C.	Dixon, James Main, M.A., Litt. Hum. Doctor, Professor of English, University of Southern California. University Avenue, Los Angeles, California, U.S.A.	
1923		* Dixon, Ronald Audley Martineau, of Thearne, F.G.S., F.S.A.Scot., F.R.G.S., Thearne Hall, near Beverley	
1897		Dobbie, James Bell, F.Z.S., 12 South Inverleith Avenue, Edinburgh	
1881	C.	Dobbin, Leonard, Ph.D., formerly Reader in Chemistry in the University of Edinburgh. Faladam, Blackshiels, Midlothian	1904-07, 1913-16.
1918		* Dodd, Alexander Scott, B.Sc., Ph.D., F.I.C., F.C.S., City Analyst for Edinburgh. 20 Stafford Street, Edinburgh	
1925		* Donald, Alexander Graham, M.A., F.F.A., F.S.A.Scot., Secretary of the Scottish Provident Institution, Edinburgh. 18 Carlton Terrace, Edinburgh	
1905		Donaldson, Rev. Wm. Galloway, J.P., F.R.G.S., F.E.I.S., The Manse of Forfar, Forfar	
1882	C.	Dott, David B., F.I.C., Memb. Pharm. Soc., Ravenslea, Musselburgh	
1921	M-B. C.	* Dougall, John, M.A., D.Sc., Publisher's Reader, 47 Airthrey Avenue, Glasgow, W. 4	
1901 & 1918		Douglas, Carstairs Cumming, M.D., D.Sc., Professor of Medical Jurisprudence and Hygiene, Anderson's College, Glasgow. 110 South Brae Drive, Jordanhill, Glasgow	
1910		* Douglas, Loudon MacQueen, Author and Lecturer, Newpark, Mid-Calder, Midlothian	
1923	C.	* Drever, James, M.A., B.Sc., D.Phil., Professor of Psychology, University of Edinburgh. Ivybank, Wardie Road, Edinburgh	1929-
1901		Drinkwater, Thomas W., L.R.C.P.E., L.R.C.S.E., Chemical Laboratory, Surgeons' Hall, Edinburgh	
1917		* Dron, Robert W., M.A., M.Inst.C.E., M.Inst.M.E., Professor of Mining in the University of Glasgow. 17 Blythswood Square, Glasgow	
1928		* Drummond, J. Montagu F., M.A. (Cantab.), Harrison Professor of Botany in the University of Manchester	1928-31.
1925		* Dryerre, Henry, M.R.C.S., Ph.D., Professor of Physiology, Royal (Dick) Veterinary College; Physiological Biochemist, Animal Diseases Research Association. Kenmore, Lasswade	
1921		*Drysdale, Charles Vickery, O.B.E., D.Sc. (Lond.), M.I.E.E., F.Inst.P., Director of Scientific Research and Experiment to the Admiralty, S.R.E. Department, Archway Block N., Admiralty, Whitehall, London, S.W. 1	
1904		Dunlop, William Brown, M.A., 4A St Andrew Square, Edinburgh	
1892	C.	Dunstan, M. J. R., M.A., F.I.C., F.C.S., formerly Principal, Royal Agricultural College, Cirencester. Windyacre, Wrotham, Kent	
1906	C.	Dyson, Sir Frank Watson, K.B.E., M.A., D.Sc., LL.D., F.R.S., Astronomer Royal, Royal Observatory, Greenwich	1907-10.
1925		* Eastwood, George Samuel, B.Sc., Principal Teacher of Mathematics, Beath Secondary School, Cowdenbeath, Fife. Craigie Lea, Cowdenbeath, Fife	
1904		Edwards, John, LL.D., 4 Great Western Terrace, Kelvinside, Glasgow	
1931		* Eggleton, Philip, D.Sc., Lecturer in Biochemistry in the Department of Physiology, University of Edinburgh. 36 Gillespie Crescent, Edinburgh	
1924		* Elliot, Walter Elliot, M.C., M.B., Ch.B., D.Sc., LL.D., M.P., 14 Markham Square, Chelsea, London	
1906	C.	Ellis, David, D.Sc., Ph.D., Professor of Botany and Bacteriology, Royal Technical College, Glasgow	
1924		* Evans, Arthur Humble, M.A., Sc.D., Lecturer in English History (under Special Board), Cheviot House, Crowthorne, Berks	
1924		Evans, William Edgar, B.Sc., Assistant in charge of Herbarium, Royal Botanic Garden, Edinburgh	

Date of Election.			Service on Council, etc.
1879	C. N.	Ewart, James Cossar, M.D., LL.D., F.R.C.S.E., F.R.S., F.Z.S., formerly Regius Professor of Natural History, University of Edinburgh. Craigybield, Penicuik, Midlothian	1882-85, 1904-07. V-P 1907-12.
1902		Ewen, John Taylor, O.B.E., B.Sc., M.I.Mech.E., J.P., H.M. Inspector of Schools (Emeritus), Pitscandly, Forfar	
1878	C.	† Ewing, Sir James Alfred, K.C.B., M.A., D.Sc., LL.D., F.R.S., Hon. Memb. Inst.C.E., J.P., formerly Principal of the University of Edinburgh and Director of Naval Education, Admiralty. 5 Herschel Road, Cambridge	1888-91, 1919-20. V-P 1920-28. P 1924-29.
1900	C.	Eyre, John W. H., M.D., M.S. (Dunelm), D.P.H. (Camb.), Professor of Bacteriology, Guy's Hospital, London, S.E. 1	
1931		* Fairbairn, William Ronald Dodds, M.A., M.D., D.Psych. (Edin.), Consultant Physician and Lecturer in Psychology, University of Edinburgh. 18 Lansdowne Crescent, Edinburgh	
1910	C.	* Fairgrieve, Mungo M'Callum, M.A. (Camb. and Glasg.), Master at the Edinburgh Academy. 37 Queen's Crescent, Edinburgh	
1907	C.	Falconer, John Downie, M.A., D.Sc., F.G.S., Director of the Geological Survey of Nigeria. The Cedars, Hatton Road, Harlington, Middlesex	
1923		* Feldman, William Moses, M.D., B.S., M.R.C.P. (Lond.), M.R.C.S. (Eng.), Physician, St Mary's Hospital for Women and Children, Plaistow. 851 Finchley Road, London, N.W. 11	
1928		* Fenton, Edward Wyllie, M.A., B.Sc. (Aberd.), Head of Biological Department, Edinburgh and East of Scotland College of Agriculture, 13 George Square, Edinburgh	
1899		† Fergus, Andrew Freeland, M.D., LL.D., Fernycrag, Crichton Road, Rothesay	
1907		Fergus, Edward Oswald, c/o Messrs M'Kay & Boyd, Solicitors, 50 Wellington Street, Glasgow	
1904		Ferguson, James Haig, M.D., LL.D., F.R.C.P.E., F.R.C.S.E., 7 Coates Crescent, Edinburgh	
1925	C.	* Ferrar, William Leonard, M.A., Fellow and Tutor of Hertford College, Oxford	
1927	C.	* Finlay, Thomas Matthew, M.A., D.Sc. (Edin.), Lecturer in Palæontology in the University of Edinburgh. 11 Dudley Terrace, Leith	
1911		† Fleming, John Arnold, F.C.S., etc., Pottery Manufacturer, Locksley, Helensburgh, Dumbartonshire	
1906		Fleming, Robert Alexander, M.A., M.D., LL.D., F.R.C.P.E., Consulting Physician, Royal Infirmary. 10 Chester Street, Edinburgh	
1900	C. N.	Flett, Sir John S., K.B.E., M.A., D.Sc., LL.D., F.R.S., Director of the Geological Survey of Great Britain and of the Museum of Practical Geology, London, 28 Jermyn Street, S.W. 1	1916-19.
1872		Forbes, George, M.A., LL.D., F.R.S., M.Inst.C.E., M.Inst.E.E., F.R.A.S., formerly Professor of Natural Philosophy in Anderson's College, Glasgow. 11 Little College Street, Westminster, S.W.	
1892		Ford, John Simpson, F.I.C., 7 Corrennie Drive, Edinburgh	
1921		* Forrest, George Topham, F.R.I.B.A., F.G.S., F.S.Arc., Architect to the London County Council, and Superintending Architect of Metropolitan Buildings, The County Hall, Westminster Bridge, London, S.E. 1	
1928	C.	* Forrest, James, M.A., B.Sc. (Glasg.), D.Sc. (St Andrews), Lecturer in Physics, University College, Dundee. "Cumbrae," Oxford Street, Blackness, Dundee	
1920	C.	* Franklin, Thomas Bedford, B.A. (Camb.), Stancliffe Hall, near Matlock, Derbyshire	
1910		* Fraser, Alexander, Actuary, 15 S. Learmonth Gardens, Edinburgh	
1929		* Fraser, David Kennedy, M.A., B.Sc., Lecturer in charge of course for training of Teachers of mentally defective children under Scottish National Committee, Psychologist to Glasgow Education Authority. Edge o' the Moor, Milngavie, Dumbartonshire	
1928		* Fraser, John, M.C., M.D., Ch.M., F.R.C.S.E., Regius Professor of Clinical Surgery in the University of Edinburgh. 32 Moray Place, Edinburgh	
1915		* Fraser, Rev. Joseph Robert, The New Manse, Kinneff, Inverbervie, Montrose	
1928		* Fraser, Kenneth, M.D. (Edin.), D.P.H. (Camb.), D.T.M. (Edin.), Deputy County Medical Officer of Health, Cumberland. The Croft, Scotby, near Carlisle	
1914		* Fraser, William, Managing Director, Neill & Co., Ltd., Printers, 212 Causewayside, Edinburgh	

Date of Election.			Service on Council, etc.
1896	C.	Fraser-Harris, David Fraser, B.Sc. (Lond.), D.Sc. (Birm.), M.D., formerly Professor of Physiology in the Dalhousie University, Halifax, Nova Scotia. Grove Park Lodge (3), Chiswick, London, W. 4	
1907		Galbraith, Alexander, "Ravenswood," Dalmuir, Glasgow	
1888	C.	Galt, Alexander, D.Sc., formerly Keeper of the Department of Technology, Royal Scottish Museum, Edinburgh. C/o Clydesdale Bank, 1 Melville Place, Edinburgh	
1901		Ganguli, Sanjiban, M.A., Principal, Maharaja's College, and Director of Public Instruction, Jaipur State, Jaipur, India	
1923		* Gardiner, Frederick, M.D., B.Sc., F.R.C.S.E., F.R.S.M., Lecturer in Diseases of the Skin in the University of Edinburgh ; Physician to the Royal Infirmary. 35 Manor Place, Edinburgh	
1926		* Gardner, John Davidson, B.Sc., Assoc.M.Inst.C.E., Chief Assistant to Messrs D. & C. Stevenson, Civil Engineers, Edinburgh. 23 Ivy Terrace, Edinburgh	
1930	C.	* Geddes, Alexander Ebenezer M'Lean, O.B.E., M.A., D.Sc. (Aberd.), Lecturer in Natural Philosophy in the University of Aberdeen. 12 Louisville Avenue, Aberdeen	
1909	C.	* Geddes, Rt. Hon. Sir Auckland C., G.C.M.G., K.C.B., P.C., M.D., D.C.L., formerly British Ambassador to the U.S.A. Athenæum Club, Pall Mall, London	
1880	C.	Geddes, Patrick, formerly Professor of Sociology and Civics, University of Bombay, India. Collége des Écossais, Montpellier, France	
1909		* Gentle, William, B.Sc., Head Master, George Heriot's School. 10 West Savile Road, Edinburgh	
1920	C.	* Ghosh, Sudhamoy, M.Sc. (Cal.), D.Sc. (Edin.), F.C.S., Professor of Chemistry, School of Tropical Medicine and Hygiene, Central Avenue, Calcutta, India	
1914		* Gibb, Sir Alexander, G.B.E., C.B., M.Inst.C.E., Consulting Civil Engineer, Queen Anne's Lodge, Westminster, London, S.W. 1	
1916		* Gibb, Alfred William, M.A., D.Sc., Professor of Geology in the University of Aberdeen. 1 Belvidere Street, Aberdeen	
1910	C.	* Gibb, David, M.A., B.Sc., Lecturer in Mathematics, Edinburgh University. 15 South Lauder Road, Edinburgh	
1917	C.	* Gibson, Alexander, M.B., Ch.B., F.R.C.S. (Eng.), 661 Broadway, Winnipeg, Canada	
1921		* Gibson, Walcot, D.Sc., F.R.S., F.G.S., formerly Assistant Director, H.M. Geological Survey (Scotland), Pathways, Fairlight Road, Hythe, Kent	
1911		Gidney, Lt.-Col. Sir Henry A. J., F.R.C.S., D.O., D.P.H., J.P., M.L.A., I.M.S. (retired), Army Specialist Public Health, c/o The Allahabad Bank, Ltd., Calcutta, India	
1925		* Gillies, William King, M.A., B.A., F.E.I.S., Rector of the Royal High School, Edinburgh. Davaar, 12 Suffolk Road, Edinburgh	
1907		Gilruth, John Anderson, M.R.C.V.S., D.V.Sc. (Melb.), Clowes Street, South Yarra, Melbourne, Australia	
1909		* Gladstone, Hugh Steuart, M.A., M.B.O.U., F.Z.S., Capenoch, Thornhill, Dumfriesshire	
1911		Gladstone, Reginald John, M.D., F.R.C.S. (Eng.), Lecturer and Senior Demonstrator of Anatomy, King's College, University of London. 22 Court Lane Gardens, London, S.E. 21	
1898		Glaister, John, M.D., F.R.F.P.S. (Glas.), D.P.H. (Camb.), LL.D., lately Regius Professor of Forensic Medicine and Public Health in the University of Glasgow. 3 Newton Place, Glasgow	
1925	C.	* Goldie, Archibald Hayman Robertson, M.A., B.A., Superintendent, Meteorological Office, Air Ministry, Edinburgh. 6 Drumsheugh Gardens, Edinburgh	1929–
1910		Goodall, Joseph Strickland, M.B. (Lond.), M.R.C.P., F.R.C.S.Ed., M.S.A. (Eng.), Professor of Physiology and Biology, City of London Hospital. 136 Harley Street, London, W. 1	
1901		Goodwillie, James, M.A., B.Sc., 239 Clifton Road, Aberdeen	
1920	C.	* Gordon, William, B.Sc., Ph.D., M.I.Mech.E., Lecturer in Engineering in the University of Edinburgh. 3 Wellington Street, Edinburgh	
1913	C. M-B.	* Gordon, William Thomas, M.A., D.Sc. (Edin.), M.A. (Cantab.), Professor of Geology, University of London, King's College, Strand, W.C.	
1897		Gordon-Munn, John Gordon, M.D., Croys, Dalbeattie	
1923		* Grabham, George Walter, O.B.E., M.A. (Cantab.), F.G.S., Government Geologist, Anglo-Egyptian Sudan. Box 178, Khartoum	
1924		Graham, Robert James Douglas, M.A., D.Sc., Lecturer in Plant Physiology in the University of Edinburgh. Royal Botanic Garden, Edinburgh	
1931		* Grant, Robert, Publisher (Oliver and Boyd) Edinburgh ; (Gurney and Jackson) London. 6 Kilgraston Road, Edinburgh	

Date of Election.			Service on Council, etc.
1898	C.	Gray, Albert A., M.D., 4 Clairmont Gardens, Glasgow	
1909	C.	* Gray, James Gordon, D.Sc., Professor of Applied Physics in the University of Glasgow. 11 The University, Glasgow	1913–15.
1918		* Gray, Wm. Forbes, F.S.A.Scot., Editor and Author, 8 Mansionhouse Road, Edinburgh	
1927	C.	* Greenwood, Alan William, M.Sc. (Melb.), Ph.D. (Edin.), Lecturer in the Institute of Animal Genetics, University of Edinburgh	
1922		* Greenwood, William Osborne, M.D. (Leeds), B.S. (Lond.), L.S.A., Obstetric Surgeon and Physician, Woodroyd, 19 Ripon Road, Harrogate, Yorks	
1905	C. K.	† Gregory, John Walter, D.Sc., LL.D., F.R.S., formerly Professor of Geology in the University of Glasgow. Bassetts, Little Baddow, Chelmsford	1908–11. V.P 1920–23.
1925		* Greig, David Middleton, M.B., C.M., F.R.C.S.E., Conservator of the Museum of the Royal College of Surgeons of Edinburgh. 12 Abbotsford Cres., Edinburgh	
1906		Greig, Edward David Wilson, C.I.E., M.D., D.Sc., Lt.-Col., H.M. Indian Medical Service, 38 Coates Gardens, Edinburgh	
1931		* Greig, John Russell, Ph.D. (Edin.), Director, The Moredun Institute Animal Diseases Research Association. Wedderlie, Kirkbrae, Liberton	
1905		† Greig, Sir Robert Blyth, M.C., LL.D., Secretary to the Department of Agriculture for Scotland, York Buildings, Queen Street, Edinburgh	1921–24. V.P 1924–27.
1910		* Grimshaw, Percy Hall, Keeper, Natural History Department, The Royal Scottish Museum. 49 Lygon Road, Edinburgh	
1899		† Guest, Edward Graham, M.A., B.Sc., J.P., formerly City Treasurer, Edinburgh. 5 Newbattle Terrace, Edinburgh	
1927		* Gulland, John Masson, M.A. (Oxon.), D.Sc. (Edin.), Ph.D. (St Andrews), Senior Assistant in Biochemistry, The Lister Institute of Preventive Medicine, Chelsea Bridge Road, London, S.W. 1	
1907		Gulliver, Gilbert Henry, D.Sc., A.M.I.Mech.E., 99 Southwark Street, London, S.E.	
1930		* Guthrie, Douglas, M.D., F.R.C.S., Lecturer in Diseases of the Ear, Nose, and Throat, School of Medicine of the Royal Colleges, Edinburgh. 4 Rothesay Place, Edinburgh	
1911		* Guy, William, F.R.C.S., L.R.C.P., L.D.S.Ed., LL.D., (Penn.), Consulting Dental Surgeon, Edinburgh Royal Infirmary; Dean, Edinburgh Dental Hospital and School; Lecturer on Human and Comparative Dental Anatomy and Physiology, 11 Wemyss Place, Edinburgh	
1922		* Hannay, Robert Kerr, M.A., Professor of Ancient History and Palæography in the University of Edinburgh. Historiographer-Royal for Scotland. 5 Royal Terrace, Edinburgh	
1923		* Hanneford-Smith, William, Assoc.Inst.C.E., Hon. A.R.I.B.A., Member of the Institute of Metals, 3 The Avenue, Gravesend, Kent	
1918		* Hardie, Patrick Sinclair, M.A., B.Sc., c/o Mrs Taylor, 151 Bruntsfield Place, Edinburgh	
1928		* Harding, William Gerald, F.R.Hist.S., F.S.A.Scot., F.E.S., St Patrick's Cottage, Little Common, Bexhill, Sussex	
1928	C.	* Harris, Robert Graham, M.A., D.Sc. (Edin.), (Aeronautical) Research Physicist, Lorraine, Manor Road, Farnborough, Hants	
1914		Harrison, Edward Philip, Ph.D., F.Inst.P., Chief Scientist, H.M.S. "Vernon," Portsmouth	
1921		* Harrison, John William Heslop, D.Sc. (Durham), F.R.S., Professor of Botany, Armstrong College, Newcastle. The Avenue, Birtley, Co. Durham	
1926	C.	* Harrower, John Gordon, M.B., Ch.B. (Glas.), F.R.C.S.E., D.Sc. (Edin.), Professor of Anatomy, King Edward VII. Medical College, Singapore	
1926		* Harvey, William Frederick, C.I.E., M.A., M.B., C.M., D.P.H., Lieut.-Col. I.M.S. (retired), Histologist, Research Laboratory, Royal College of Physicians, Edinburgh. 56 Garscube Terrace, Edinburgh	
1893		Hehir, Sir Patrick, K.C.I.E., C.B., C.M.G., F.R.C.P.E., F.R.C.S.E., D.P.H. (Camb.), D.T.M. (Liverpool), Maj.-General I.M.S. (retired), Ennisfarne, Westward Ho, Devon	
1900		Henderson, John, D.Sc., A.Inst.E.E., Kinnoul, Gregory's Road, Beaconsfield, Bucks	
1931		* Henderson, John, F.C.I.I., Manager and Secretary, Edinburgh Assurance Co. Ltd., Member and Past President of the Insurance Society of Edinburgh. Seaforth Cottage, York Road, Trinity, Edinburgh	
1929		* Henderson, Thomas, J.P., F.S.A.Scot., Actuary of the Savings Bank of Glasgow. 5 Belmont Crescent, Glasgow, W.2	
1908		* Henderson, William Dawson, M.A., B.Sc., Ph.D., Lecturer, Zoological Laboratories, University, Bristol	

Date of Election.			Service on Council, etc.
1925		* Heron, Alexander Macmillan, D.Sc., Officiating Superintendent, Geological Survey of India, Calcutta, India	
1916		* Herring, Percy Theodore, M.D., F.R.C.P.E., Professor of Physiology, University of St Andrews. Linton, St Andrews	1917-20. 1931-
1922		Hindle, Edward, M.A., Sc.D. (Camb.), Ph.D., A.R.C.Sc., London School of Hygiene and Tropical Medicine. 32 Belsize Avenue, Hampstead, London, N.W. 3	
1902		Hinxman, Lionel W., B.A., formerly of the Geological Survey of Scotland. 4 Morant Gardens, Ringwood, Hants	
1904		Hobday, Frederick T. G., C.M.G., F.R.C.V.S., Principal, Royal Veterinary College, Camden Town, London, N.W. 1	
1928	C.	* Hobson, Alfred Dennis, M.A. (Camb.), Lecturer in Zoology in the University of Edinburgh. 71 Newington Road, Edinburgh	
1928		* Hodge, William Vallance Douglas, M.A. (Edin.), M.A. (Camb.), Lecturer in Mathematics in the University of Bristol	
1885		Hodgkinson, W. R., C.B.E., M.A., Ph.D., F.I.C., F.C.S., formerly Professor of Chemistry and Physics at the Ordnance College, Woolwich. 89 Shooter's Hill Road, Blackheath, Kent	
1923		* Hogben, Lancelot Thomas, M.A., D.Sc., Professor of Social Biology (London School of Economics), University of London	
1927		Holden, Henry Smith, D.Sc., F.L.S., Professor of Botany, University College, Nottingham	
1930		* Holland, Sir Thomas Henry, K.C.S.I., K.C.I.E., Hon. D.Sc., LL.D., F.R.S., Principal of the University of Edinburgh.	1931-
1911		Holland, William Jacob, LL.D. (St Andrews), Director-Emeritus, Carnegie Institute, Pittsburg, Pa. 5545 Forbes Street, Pittsburg, Pa., U.S.A.	
1929		* Hora, Sunder Lal, D.Sc. (Punjab et Edin.), F.L.S., F.Z.S., F.A.S.B., Senior Assistant Superintendent, Zoological Survey of India. Indian Museum, Calcutta	
1920	C.	* Horne, Alexander Robert, O.B.E., B.Sc., M.I.Mech.E., A.M.I.C.E., Professor of Mechanical Engineering, Heriot-Watt College, Edinburgh. 31 Queen's Crescent, Edinburgh	
1896		Horne, J. Fletcher, M.D., F.R.C.S.E., Shelley Hall, Huddersfield	
1904	C.	Horsburgh, Ellice Martin, M.A., D.Sc., Reader in Technical Mathematics, University of Edinburgh. 11 Granville Terrace, Edinburgh	
1897		Houston, Sir Alex. Cruikshanks, K.B.E., C.V.O., M.B., C.M., D.Sc., F.R.S., 20 Nottingham Place, London, W. 1	
1912	C. M-B.	* Houstoun, Robert Alexander, M.A., Ph.D., D.Sc., Lecturer in Physical Optics, University, Glasgow. 45 Kirklee Road, Glasgow	1929-
1893		Howden, Robert, M.A., M.B., C.M., D.Sc., LL.D., Em. Professor of Anatomy in the University of Durham. Broomfield, Crieff	
1910		Hume, William Fraser, D.Sc. (Lond.), Director, Geological Survey of Egypt, Helwân, Egypt. The Laurels, Rustington, Sussex	
1927		* Hunt, Owen Duke, B.Sc. (Manch.), "Corrofell," Newton Ferrers, South Devon	
1923		* Hunter, Rev. Adam Mitchell, M.A., D.Litt., Librarian of New College, Edinburgh. 3 Suffolk Road, Edinburgh	
1928		* Hunter, Arthur, F.F.A., LL.D. (Edin.), Vice-President and Chief Actuary of the New York Life Insurance Co. 124 Lloyd Road, Montclair, N.J., U.S.A.	
1916		* Hunter, Charles Stewart, L.R.C.P.E., L.R.C.S.E., D.P.H., Cotswold, 36 Streatham Hill, London, S.W. 2	
1911		Hunter, Gilbert Macintyre, M.Inst.C.E., M.Inst.E.S., M.Inst.M.E., formerly Resident Engineer, Nitrate Railways, Iquique, Chile. Kingscroft, Cramond Brig, near Edinburgh	
1887	C.	Hunter, William, C.B., M.D., M.R.C.P.L. and E., M.R.C.S., LL.D., 103 Harley Street, London	
1927		* Hyslop, James, M.A. (Glasg.), Ph.D. (Camb.), Lecturer in Mathematics in the University of Glasgow	
1908		Hyslop, Theophilus Bulkeley, M.D., M.R.C.P.E., 5 Portland Place, London, W. 1	
1923	C.	* Ince, Edward Lindsay, M.A. (Cantab.), D.Sc. (Edin.), Lecturer in Mathematics in the University of Edinburgh. 6 Rutland Gardens, West Ealing, London, W. 13	
1920		* Inglis, James Gall, Publisher and Editor of Educational Works, 36 Blacket Place, Edinburgh	
1927		* Inglis, John Alexander, of Auchindinny and Redhall, K.C., M.A. (Oxon.), LL.B. (Edin.), King's and Lord Treasurer's Remembrancer. 13 Randolph Crescent, Edinburgh	
1912		* Inglis, Robert John Mathieson, M.Inst.C.E., Assistant Chief Engineer, Lond. & N.E. Railway, Liverpool Street Station, London. Dixton, Hadley Common, Barnet	

Date of Election.			Service on Council, etc.
1904	C.	Innes, Robert T. A., formerly Director, Government Observatory, Johannesburg, Transvaal. 91 Becker Street, Johannesburg, South Africa	
1917		* Irvine, Sir James Colquhoun, Kt., C.B.E., Ph.D. (Leipzig), D.Sc. (St Andrews), Hon. D.Sc. (Liverpool, Princeton), Hon. Sc.D. (Cambridge, Yale, Pennsylvania), Hon. LL.D. (Glasgow, Aberdeen), Hon. D.C.L. (Durham), D.L., F.R.S., Hon. Mem. American Chemical Society, Principal of the University of St Andrews	1920-22. V-P 1922-25.
1930	C.	* Jack, David, M.A., B.Sc. (Edin.), Ph.D. (St Andrews), Lecturer in Natural Philosophy in the United College, University of St Andrews. 22 Grange Road, St Andrews	
1923		* Jack, John Louttit, Solicitor, Assistant Secretary, Department of Health for Scotland, 121A Princes Street, Edinburgh	
1889	C.	James, Alexander, M.D., F.R.C.P.E., Huntington, Haddington	
1901		Jardine, Robert, M.D., M.R.C.S., F.R.F.P.S. (Glas.), Wiston Manse, Lamington	
1912	C.	* Jeffrey, George Rutherford, M.D. (Glasg.), F.R.C.P. (Edin.), Bootham Park Private Mental Hospital, York	
1906	C. K.	Jehu, Thomas John, M.A., M.D., F.G.S. (VICE-PRESIDENT), Professor of Geology in the University of Edinburgh. 35 Great King Street, Edinburgh	1917-20, 1923-26. V-P 1929-
1900		† Jerdan, David Smiles, M.A., D.Sc., Ph.D., 26 Avenue du Château d'Eau, Saventhem, Belgium	
1931		* Johnson, Thomas, D.Sc. (Lond.), Emeritus Professor of Botany, Royal College of Science for Ireland. Tomeg, Hillview Drive, Corstorphine	
1925		* Johnston, Christopher Nicholson, The Hon. Lord Sands, Kt., K.C., LL.D., D.D., Senator of the College of Justice. 4 Heriot Row, Edinburgh	1929-
1895		Johnston, Col. Henry Halcro, C.B., C.B.E., D.Sc., M.D., F.L.S., late A.M.S., Stromness Hotel, Stromness, Orkney	
1928		* Johnston-Saint, Percy Johnston, M.A. (Camb.), Secretary, Wellcome Historical Medical Museum, 54A Wigmore Street, London, W. 1. 4 Wyndham Place, Bryanston Square, London, W.	
1928		* Johnstone, Robert William, C.B.E., M.A., M.D. (Edin.), F.R.C.S.E., M.R.C.P.E., Professor of Midwifery and Diseases of Women in the University of Edinburgh. 26 Palmerston Place, Edinburgh	
1927		* Jones, Edward Taylor, D.Sc. (Lond.), Professor of Natural Philosophy in the University of Glasgow	1927-30.
1888		Jones, John Alfred, M.Inst.C.E., Fellow of the University of Madras, Sanitary Engineer to the Government of Madras. (Address not known)	
1930	C.	* Jones, Samuel Griffith, D.Sc. (Univ. Wales), Lecturer in Botany in the University of Glasgow. Cliffe Holme, Thorn Drive, Bearsden, Dumbartonshire	
1928		* Jones, Tudor Jenkyn, M.B., Ch.B. (Glasg.), Lecturer in Anatomy in the University of Liverpool	
1922		* Juritz, Charles Frederick, M.A., D.Sc., F.I.C., Chief of the Union Department of Chemistry, Villa Marina, Three Anchor Bay, Cape Town, South Africa	
1925	C.	* Kemp, Charles Norman, B.Sc., Technical Radiologist, Secretary of the Royal Scottish Society of Arts. Ivy Lodge, Laverockbank Road, Edinburgh	
1929		* Kendall, James, M.A., D.Sc., F.R.S., Professor of Chemistry, University of Edinburgh. 14 Mayfield Gardens, Edinburgh	1931-
1912		† Kennedy, Robert Foster, M.D. (Queen's Univ., Belfast), M.B., B.Ch. (R.U.I.), Assoc. Professor of Neurology, Cornell University, New York. Ninth Floor, 410 East 57th Street, near First Avenue, New York, U.S.A.	
1927		* Kennedy, Walter Phillips, B.Sc., Ph.D. (Edin.) L.R.C.P. and S.E., Beit Memorial Research Fellow, Physiology Department, University, Edinburgh	
1909		Kenwood, Henry Richard, C.M.G., M.B., C.M., Chadwick Em. Professor of Hygiene in the University of London. "Wadhurst," Queen's Road, Finsbury Park, London, N.	
1925	C. M-B.	* Kermack, William Ogilvy, M.A., D.Sc., Chemist to the Research Laboratory of the Royal College of Physicians, 2 Forrest Road, Edinburgh	
1908		* Kerr, Andrew William, F.S.A.Scot., 81 Great King Street, Edinburgh	
1903 & 1923	C. N.	Kerr, John Graham, M.A., F.R.S., F.L.S., F.Z.S., Regius Professor of Zoology, University of Glasgow. 9 The University, Glasgow	1904-07, 1913-16, 1924-27. V-P 1928-31.
1927		* Kerr, John Martin Munro, M.D., F.R.F.P.S. (Glas.), Professor of Midwifery in the University of Glasgow	
1891		Kerr, Joshua Law, M.D., J.P., "Stonehaven," Sheffield, Tasmania	

Date of Election.			Service on Council, etc.
1913		* Kerr, Walter Hume, M.A., B.Sc., formerly Lecturer on Engineering Drawing and Structural Design in the University of Edinburgh. Glenfriars, Jedburgh	
1926		* Khastgir, Satis Ranjan, M.Sc. (Calcutta), D.Sc. (Edin.), Ph.D. (Edin.), Physics Department, University College, Colombo, Ceylon	
1907		King, Archibald, M.A., B.Sc., H.M. Inspector of Schools, 10 Leslie Road, Pollokshields, Glasgow	
1925		* King, Leonard Augustus Lucas, M.A., Professor of Zoology in the West of Scotland Agricultural College, Glasgow. 48 University Avenue, Glasgow, W. 2	
1918		* Kingon, Rev. John Robert Lewis, M.A., D.Sc., F.L.S., 25 Innes Avenue, Bloemfontein, O.F.S., South Africa	
1901		Knight, Rev. G. A. Frank, M.A., D.D., F.S.A.Scot., 10 Hillhead Street, Glasgow	
1907		Knight, James, M.A., D.Sc., F.C.S., F.G.S., J.P., Rector, Queen's Park High School, Langside, Glasgow	
1927		* Lambie, Charles George, M.C., M.D., F.R.C.P.E., Bosch Professor of Medicine in the University of Sydney	
1920	C.	* Lamont, John Charles, Lieut.-Col., I.M.S. (retired), C.I.E., M.B., C.M. (Edin.), M.R.C.S. (Eng.), 7 Merchiston Park, Edinburgh	
1925	C. N.	* Lang, William Henry, M.B., C.M., D.Sc., F.R.S., Barker Professor of Cryptogamic Botany in the University of Manchester	
1931		* Langrishe, John du Plessis, D.S.O., M.B., B.Ch. (Dub.), D.P.H. (R.Cs.P. and S.), Lt.-Col. R.A.M.C. (retired), Lecturer in Public Health, University of Edinburgh. 2 South Gillsland Road, Edinburgh	
1910	C.	* Lauder, Alexander, D.Sc., Head of Chemistry Department, Edinburgh and East of Scotland College of Agriculture, 13 George Square, Edinburgh. Lecturer in Agricultural Chemistry, University of Edinburgh	1917–20. Sec. 1923–28.
1885	C.	Laurie, Arthur Pillans, M.A., D.Sc., LL.D., J.P., formerly Principal of the Heriot-Watt College, Edinburgh. 38 Springfield Road, St John's Wood, London, N.W. 8	1908–11, 1913–16.
1921		* Laurie, The Rev. Albert Ernest, M.C., C.F., D.D., Rector of Old St Paul's, and Canon of St Mary's Cathedral, Edinburgh. Lauder House, Jeffrey Street, Edinburgh	
1894	C.	Laurie, Malcolm, B.A., D.Sc., F.L.S., 4 Wordsworth Road, Harpenden, Herts	
1905		Lawson, David, M.A., M.D., L.R.C.P. and S.E., Druimdarroch, Banchory, Kincardineshire	
1903		Leighton, Gerald Rowley, O.B.E., M.D., D.Sc., Medical Officer, Scottish Board of Health, 121A Princes Street, Edinburgh	
1930		* Lelean, Percy Samuel, C.B., C.M.G., F.R.C.S., L.R.C.P., D.P.H., Professor of Public Health in the University of Edinburgh. 2 Barnton Loan, Davidson's Mains, Edinburgh	
1910		Levie, Alexander, F.R.C.V.S., D.V.S.M., Balmae, Manor Road, Littleover, Derby	
1916	C.	* Levy, Hyman, M.A., D.Sc., Professor of Mathematics, Imperial College of Science and Technology, London, S.W. 7. 62 Kenilworth Avenue, Wimbledon Park, London, S.W. 19	
1914	C. N.	Lewis, Francis John, D.Sc., F.L.S., Professor of Biology, University of Alberta, Edmonton South, Alberta, Canada	
1918		* Lidstone, George James, F.F.A., F.I.A., LL.D., formerly Manager and Actuary of the Scottish Widows' Fund Life Assurance Society. Hermiston House, Hermiston, Currie, Midlothian	1919–22.
1905		Lightbody, Forrest Hay, 53 Queen Street, Edinburgh	
1931		* Lightfoot, Nicholas Morpeth Hutchinson, M.A. (Camb.), Lecturer in Mathematics, Heriot-Watt College, Edinburgh. Tayinloan, Loanhead, Midlothian	
1923		* Lim, Robert Kho Seng, M.B., Ch.B., D.Sc., Ph.D., Peking Union Medical College, Department of Physiology, Peking, China	
1912		* Lindsay, John George, M.A., B.Sc. (Edin.), Rector of Dunfermline High School	
1920	C.	* Lindsay, Thomas A., M.A. (Hons.), B.Sc., Head Master, Higher Grade School, Bucksburn, Aberdeenshire	
1912		* Linlithgow, The Most Honourable the Marquis of, K.T., Hopetoun House, South Queensferry	
1903		† Liston, William Glen, C.I.E., M.D., Lt.-Col. Indian Medical Service (retired), Milburn Tower, Gogar, Corstorphine, Edinburgh	
1929		* Little, John Robert, F.C.I.I., F.C.I.S., General Manager and Secretary of the Century Insurance Co., Ltd. 5 Dalrymple Crescent, Edinburgh	
1926		* Lorraine, Norman Stanley Rees, M.D., D.P.H. (Edin. and Glasg.), Medical Officer of Health, Shoeburyness Urban District, and Assistant County Medical Officer of Health, County of Essex. 1 Burlescoombe Leas, Burlescoombe Road, Thorpe Bay, Southend-on-Sea	
1898		Lothian, Alexander Veitch, M.A., B.Sc., Glendoune, 996 Great Western Road, Glasgow	

Date of Election.			Service on Council, etc.
1930		* Low, James Wotherspoon, B.Sc. (Edin.), Ph.D. (Bristol), 5 Palmerston Road, Edinburgh	
1923	C.	* Ludlam, Ernest Bowman, M.A., D.Sc., Lecturer in Chemistry in the University of Edinburgh	
1900		† Lusk, Graham, M.A., Ph.D., LL.D., Professor of Physiology, Cornell University Medical College, New York, N.Y., U.S.A.	
1923		* Lyford-Pike, James, M.A., B.Sc., Lecturer in Forestry in the University of Edinburgh. Rosetta, Liberton, Edinburgh	
1924		* Lyon, David Murray, M.D., F.R.C.P.E., D.P.H., D.Sc., Professor of Therapeutics and Clinical Medicine in the University of Edinburgh. Druim, Colinton	
1894		Mabbott, Walter John, M.A., formerly Rector of County High School, Duns, Berwickshire. The Hawthorn, Farnham Lane, Haslemere, Surrey	
1917		* Macalister, Sir Donald, Bt., K.C.B., M.D., M.A., B.Sc., Chancellor of the University of Glasgow. Barrmore, Lady Margaret Road, Cambridge	
1929		* M'Arthur, Donald Neil, D.Sc., Ph.D., F.I.C., Professor of Agricultural Chemistry, West of Scotland Agricultural College, Glasgow, C.2. 35 Kersland Street, Glasgow, W. 2	
1921		* M'Arthur, Neil, M.A., B.Sc., Lecturer in Mathematics, Glasgow University. 1 Holyrood Crescent, Glasgow	
1926		* M'Bride, James Alexander, B.A. (Roy. Univ., Ireland), B.Sc. (Lond.), formerly Rector of Queen's Park Secondary School, Glasgow. Scottish Liberal Club, Princes Street, Edinburgh	
1883		M'Bride, Peter, M.D., F.R.C.P.E., 3 St Peter's Grove, York	
1931	C.	* McCallien, William John, D.Sc. (Glas.), Lecturer in Geology, University of Glasgow. Glenorchy, Tarbert, Argyll	
1930		* M'Candlish, Andrew Corrie, B.Sc. (Glasg.), M.S.A., Ph.D. (Iowa), Advisory Officer in Milk Production, West of Scotland Agricultural College. Claunch, Sorbie, Wigtownshire	
1923		* M'Cracken, William, J.P., F.S.I., F.H.A.S., Land Agent and Technical Adviser, Englesea House, Crewe	
1931	C.	* M'Crea, William Hunter, M.A., Ph.D. (Camb.), B.Sc. (Lond.), F.R.A.S., Lecturer in Mathematics, University of Edinburgh. Mathematical Institute, 16 Chambers Street, Edinburgh	
1918		* M'Culloch, Rev. James David, 43 Brougham Street, Greenock	
1905		Macdonald, Hector Munro, O.B.E., M.A., F.R.S., Professor of Mathematics, University of Aberdeen. 52 College Bounds, Aberdeen	1908-11.
1897	C.	Macdonald, James A., M.A., B.Sc., formerly H.M. Inspector of Schools. "Rothes," Frankscroft, Peebles	
1920		* M'Donald, Stuart, M.A., M.D., F.R.C.P.E., Professor of Pathology, University of Durham. College of Medicine, Newcastle-on-Tyne	
1928		* MacDonald, Thomas Logie, M.A., B.Sc. (Glasg.), F.R.A.S., 9 Colebrooke Terrace, Glasgow, W. 2	
1904		Macdonald, William, M.S.Agr., Sc.D., Ph.D., D.Sc., Editor, *Agricultural Journal* of South Africa. Rand Club, Johannesburg, Transvaal	
1886		Macdonald, William J., M.A., LL.D., 15 Comiston Drive, Edinburgh	
1931		* M'Dougall, John Bowes, M.D. (Glas.), F.R.F.P.S.G., F.R.C.P. (Edin.), Medical Director, British Legion Village, Preston Hall, Kent. Preston Hall, Aylesford, Kent	
1901	C.	MacDougall, R. Stewart, M.A., D.Sc., Professor of Biology, Royal Veterinary College, Edinburgh. Ivy Lodge, Gullane, East Lothian	1914-17.
1910		Macewen, Hugh Allen, O.B.E., M.B., Ch.B., D.P.H. (Lond. and Camb.), Local Government Board, Ministry of Health, Whitehall, London, S.W.	
1888	C.	M'Fadyean, Sir John, Kt., M.B., B.Sc., LL.D., formerly Principal and Professor of Comparative Pathology in the Royal Veterinary College, Camden Town, London. Highlands House, Leatherhead	
1885	C.	† Macfarlane, J. M., D.Sc., LL.D., Em. Professor of Botany. 427 West Hansberry Street, Germantown, Pa., U.S.A.	
1897		MacGillivray, Angus, M.D., C.M., D.Sc., F.S.A.Scot., 23 South Tay Street, Dundee	
1878		M'Gowan, George, F.I.C., Ph.D., 21 Montpelier Road, Ealing, London, W. 5	
1923		* Macgregor, Murray, M.A., D.Sc., Assistant Director (Scotland), H.M. Geological Survey. 19 Grange Terrace, Edinburgh	1930-
1903		M'Intosh, Donald C., M.A., D.Sc., Education Offices, Elgin	
1911		M'Intosh, John William, M.R.C.V.S., Dollis Hill Farm, Cricklewood, London, N.W. 2	
1927	C.	* M'Intyre, Donald, M.B.E., M.D. (Glas.), F.R.C.S.E., Assistant Physician, Glasgow Royal Maternity and Women's Hospital, and Glasgow Samaritan Hospital. 9 Park Circus, Glasgow, C. 3	

Date of Election.			Service on Council, etc.
1912	C.	M'Kendrick, Anderson Gray, M.B., D.Sc., F.R.C.P.E., Lt.-Col., Indian Medical Service (retired), Superintendent, Research Laboratory, Royal College of Physicians, 2 Forrest Road, Edinburgh	1924-27.
1914		* M'Kendrick, Archibald, F.R.C.S.E., D.P.H., L.D.S., 12 Rothesay Place, Edinburgh	
1900	C.	M'Kendrick, John Souttar, M.D., F.R.F.P.S. (Glas.), 2 Buckingham Terrace, Hillhead, Glasgow	
1910	C.	* Mackenzie, Alister, M.A., M.D., D.P.H., Principal Medical Officer and Lecturer in Hygiene under the Glasgow Provincial Committee, Training Centre, Jordanhill, Glasgow. 22 Queen's Gate, Dowanhill, Glasgow	
1916	C.	* Mackenzie, John E., D.Sc., Reader in Chemistry, University of Edinburgh. 2 Ramsay Garden, Edinburgh	
1905		Mackenzie, Sir Wm. Colin, K.B., M.D., F.R.C.S., Director Australian Institute of Anatomy, Canberra, F.C.T., Australia	
1904	C.	Mackenzie, Sir W. Leslie, M.A., M.D., D.P.H., LL.D., F.R.C.P. (Edin.), formerly Medical Member of the Scottish Board of Health, 14 Belgrave Place, Edinburgh	
1926		* MacKichan, The Very Rev. Dugald, M.A., D.D., LL.D., formerly Principal of Wilson College, Bombay. 13 Learmonth Terrace, Edinburgh	
1929	C.	* Mackie, John, M.A., D.Sc., Mathematical Master, Leith Academy. 19 Beresford Avenue, Trinity, Leith	
1928		* Mackie, Thomas Jones, M.D., M.R.C.P. (Edin.), Professor of Bacteriology in the University of Edinburgh. 22 Mortonhall Road, Edinburgh	
1918		* Mackie, Wm., M.A., M.D., LL.D., D.P.H., 40 Clouston Street, Glasgow, N.W.	
1910		* MacKinnon, James, M.A., Ph.D., LL.D., formerly Professor of Ecclesiastical History, Edinburgh University. 12 Lygon Road, Edinburgh	
1904		Mackintosh, Donald James, C.B., M.V.O., D.L., M.B., C.M., LL.D., Supt. Western Infirmary, Glasgow	
1899		Maclean, Sir Ewan John, M.D., M.R.C.P. (Lond.), J.P., Professor of Obstetrics and Gynæcology, Welsh National School of Medicine, 12 Park Place, Cardiff	
1888	C.	Maclean, Magnus, M.A., D.Sc., LL.D., M.Inst.C.E., M.I.E.E., formerly Professor of Electrical Engineering in the Royal Technical College. 108 University Avenue, Glasgow, W. 2	1916-19.
1913		* M'Lellan, Dugald, M.Inst.C.E., Divisional Engineer, L.M. and S. Railway, Buchanan Street Station, Glasgow. 1 Queen's Gate, Dowanhill, Glasgow	
1916	C.	* M'Lintock, W. F. P., D.Sc. (Edin.), Museum of Practical Geology, 28 Jermyn Street, London, S.W. 1	
1923		* Macmillan, Rt. Hon. Lord, LL.D., 44 Grosvenor Road, Westminster, S.W. 1	
1917		* Macpherson, Rev. Hector Copland, M.A., Ph.D., F.R.A.S., Guthrie Memorial U.F. Church. 7 Wardie Crescent, Edinburgh	
1921		* M'Quistan, Dougald Black, M.A., B.Sc., Associate-Professor of Natural Philosophy, Royal Technical College, Glasgow. 29 Viewpark Drive, Rutherglen	
1921	C.	* MacRobert, Thomas Murray, M.A., D.Sc., Professor of Mathematics in the University of Glasgow. 10 The University, Glasgow	1931–
1921	C.	* M'Whan, John, M.A. (Glasgow), Ph.D. (Gött.), Lecturer in Mathematics in the University of Glasgow. 84 Munro Road, Jordanhill, Glasgow, W. 3	
1927		* Madwar, Mohamed Reda, B.Sc., Ph.D. (Edin.), Assistant, Helwân Observatory, Egypt	
1898	C.	† Mahâlanobis, S. C., B.Sc., (Edin.), Professor of Physiology, University of Calcutta, formerly Professor of Physiology, and sometime Dean, Presidency College, Calcutta. P. 45 New Park Street, Calcutta	
1913		Majumdar, Tarak Nath, D.P.H. (Cal.), L.M.S., F.C.S., Health Officer, Calcutta, IV, Calcutta, India. P. 235 Russa Road, P.O. Tollygunge	
1917		* Malcolm, L. W. Gordon, M.Sc. (Cantab.), Conservator, Wellcome Historical Medical Museum, 54 Wigmore Street, London, W. 1	
1908		Mallik, Devendranath, Sc.D., B.A., Principal, Carmichael College, Rungpur, Bengal, India	
1925		* Malloch, James, M.A., J.P., F.S.A.Scot., Executive Officer to the National Committee for the Training of Teachers. Earleville, Camperdown Street, Broughty Ferry	
1912		Maloney, William Joseph, M.B.E., M.C., M.D. (Edin.), LL.D., formerly Professor of Neurology at Fordham University. 420 Park Avenue, New York City, U.S.A.	
1913		Marchant, Rev. Sir James, K.B.E., LL.D., F.R.A.S., F.L.S., Director, National Council for Promotion of Race-Regeneration. Rhondda House, 60 Gower Street, London, W.C. 1	
1882	C.	Marshall, D. H., M.A., Em. Professor of Physics, Queen's University. Elmtree House, Union Street, W., Kingston, Ontario, Canada	
1901	C.	Marshall, Francis Hugh Adam, Sc.D., F.R.S., Reader in Agricultural Physiology in the University of Cambridge, Christ's College, Cambridge	

Date of Election.			Service on Council, etc.
1920	C.	* Marshall, John, M.A., D.Sc. (St Andrews), B.A. (Cantab.), University Reader in Mathematics, Bedford College, Regent's Park, London, N.W. 1. Logan House, Torrington Park, North Finchley, London, N. 12	
1931		* Mason, John Huxley, F.R.C.V.S., Government Veterinary Laboratory, Onderstepoort, Pretoria, South Africa	
1918		Masson, George Henry, M.D., D.Sc., F.R.C.P.E., Port of Spain, Trinidad, British West Indies	
1885	C.	Masson, Sir David Orme, K.B.E., M.A., D.Sc., LL.D., F.R.S., Em. Professor of Chemistry in the University of Melbourne	
1898	C. M-B.	Masterman, Arthur Thomas, M.A., D.Sc., F.R.S., formerly Superintending Inspector, H.M. Board of Agriculture and Fisheries. Royal Auto Club, Pall Mall, London, S.W. 3 Kedale Road, Seaford	1902-04.
1911		Mathews, Gregory Macalister, M.B.O.U., Meadway, St Cross, Winchester, Hants	
1921		* Mathieson, John, F.R.S.G.S., late Division Superintendent, Ordnance Survey (retired), 42 East Claremont Street, Edinburgh	
1906		Mathieson, Robert, F.C.S., St Serf's, Innerleithen	
1928		* Matthai, George, M.A. (Camb.), F.Z.S., F.L.S., Professor of Zoology, The Government College, Lahore, India	
1924		* Matthews, James Robert, M.A., F.L.S., Professor of Botany in the University of Reading	
1917		* Maylard, A. Ernest, M.B., B.Sc. (Lond.), F.R.F.P.S. (Glasgow), Kingsmuir, Peebles	
1922		* Meakins, Jonathan Campbell, M.D., LL.D., F.R.C.P.Ed., Professor of Medicine and Director of the Department of Medicine, M'Gill University, Montreal, Canada	
1931		* Mears, Frank Charles, F.R.I.B.A., Architect, 8 Forres Street, Edinburgh. 14 Ramsay Garden, Edinburgh	
1926		* Mekie, David Clark Thomson, M.A., Ph.D., Head Master, Bonnington Road Public School. 11 Minto Street, Edinburgh	
1901	C.	Menzies, Alan W. C., M.A., B.Sc., Ph.D., F.C.S., Professor of Chemistry, Princeton University, Princeton, New Jersey, U.S.A.	
1927		* Menzies, Frederick Norton Kay, M.D., F.R.C.P. (Edin.), D.P.H. (Lond.), Medical Officer of Health and School Medical Officer, Administrative County of London, County Hall, London, S.E.	
1929		* Mercer, Walter, M.B., Ch.B., F.R.C.S.E., Lecturer in Clinical Surgery, University of Edinburgh. 3 Rothesay Place, Edinburgh	
1917		* Merson, George Fowlie, Manufacturing Technical Chemist, St John's Hill Works, Edinburgh	
1902	C.	† Metzler, William H., A.B., Ph.D., Corresponding Fellow of the Royal Society of Canada, Dean of the New York State College for Teachers, Albany, N.Y., U.S.A.	
1885	C. M-B.	Mill, Hugh Robert, D.Sc., LL.D., Hill Crest, Dormans Park, E. Grinstead	
1910		* Miller, John, M.A., D.Sc., Professor of Mathematics, Royal Technical College, 2 Northbank Terrace, North Kelvinside, Glasgow	
1930		* Miller, William Christopher, M.R.C.V.S., Lecturer, Institute of Animal Genetics (Sheep Section), King's Buildings, West Mains Road, Edinburgh. Scotiswoode, Alnwickhill Road, Liberton, Edinburgh	
1905		Milne, Archibald, M.A., D.Sc., Deputy Director of Studies, Edinburgh Provincial Training College. 38 Morningside Grove, Edinburgh	
1905		Milne, C. H., M.A., D.Litt., Head Master, Daniel Stewart's College. 19 Merchiston Gardens, Edinburgh	
1904	C.	Milne, James Robert, D.Sc., Lecturer in Natural Philosophy, University of Edinburgh. 7 Grosvenor Crescent, Edinburgh	
1886		Milne, William, M.A., B.Sc., 70 Beechgrove Terrace, Aberdeen	
1899		Milroy, Thomas Hugh, M.D., B.Sc., LL.D., Professor of Physiology in Queen's University, Belfast	
1889	C.	Mitchell, A. Crichton, D.Sc., Hon. Doc. Sc. (Genève), formerly Director of Public Instruction in Travancore, India. 246 Ferry Road, Edinburgh. (Society's Representative on Governing Body of Heriot-Watt College)	1915-16. 1930– Cnr. 1916-26. V-P 1926-29.
1897		† Mitchell, George Arthur, M.A., 9 Lowther Terrace, Kelvinside, Glasgow	
1900		Mitchell, James, M.A., B.Sc., Islay Lodge, Lochgilphead, Argyll	
1911		Modi, Edalji Manekji, D.Sc., LL.D., Litt.D., F.C.S., etc., Proprietor and Director of Arthur Road Chemical Works, Meher Buildings, Tardeo, Bombay, India	

Date of Election.			Service on Council, etc.
1906	C.	† Moffat, Rev. Alexander, M.A., LL.D., formerly Professor of Physics, Christian College, Madras. Park Hotel, 33 Royal Terrace, Edinburgh	
1929		Moir, Henry, F.F.A., F.I.A., President, United States Life Insurance Co., in the City of New York, 156 Fifth Avenue, New York City. Upper Montclair, New Jersey	
1890	C.	† Mond, Robert Ludwig, M.A. Cantab., LL.D., F.C.S., 9 Cavendish Sq., London, W. 1	
1887	C.	Moos, N. A. F., D.Sc., L.C.E., J.P., Director of Bombay and Alibag Observatories (retired). Red Leaf, Pedder Road, Bombay, India	
1896		Morgan, Alexander, O.B.E., M.A., D.Sc., formerly Principal, Edinburgh Provincial Training College. 1 Midmar Gardens, Edinburgh	
1930		* Morison, John Miller Woodburn, M.D., F.R.C.P.E., D.M.R. and E., Professor of Radiology in the University of London, and Director of the Radiological Department of the Cancer Hospital, Fulham Road, London, S.W. 3	
1926		* Morris, James Archibald, R.S.A., F.S.A. Scot., Savoy Croft, Ayr	
1919		* Morris, Robert Owen, M.A., M.D., C.M. (Edin.), D.P.H. (Liverpool), Tuberculosis Institute, Aberdovey, N. Wales	
1892	C.	Morrison, J. T., M.A., B.Sc., Professor of Mathematical Physics, University, Stellenbosch, Cape Colony	
1914		Mort, Spencer, M.Ch., M.D. (Glas.), F.R.C.S. (Edin.), F.C.S., Medical Superintendent and Surgical Director, North Middlesex Hospital, Edmonton, London, N. 18. Director, County of Middlesex Radium Service.	
1930		* Morton, James, LL.D., Chairman and Governing Director of the Scottish Dyes, Ltd. Craigiehall, Cramond Bridge, West Lothian	
1901		Moses, O. St John, M.D., D.Sc., F.R.C.S., Lt.-Col. I.M.S. (retired), formerly Professor of Medical Jurisprudence, Medical College, Calcutta. 18 Manstone Road, Cricklewood, London, N.W. 2	
1892	C. K.	Mossman, Robert Cockburn, Lacar 4332, Villa Devoto F.C.P., Buenos Aires, Argentina	
1916		* Muir, Robert, M.A., M.D., Sc.D., LL.D., F.R.S., F.R.C.P. (Edin.), Professor of Pathology, University of Glasgow. 30 Victoria Crescent Road, Glasgow, W. 2	
1874	C. K. V. J.	Muir, Sir Thomas, C.M.G., M.A., D.Sc., LL.D., F.R.S., formerly Superintendent-General of Education for Cape Colony, Elmcote, Sandown Road, Rondebosch, South Africa	1885-88. V-P 1888-91.
1907		Muirhead, James M. P., J.P., F.R.S.L., F.S.S., "Carmel," Rouwkoop Road, Rondebosch, South Africa	
1907		† Musgrove, James, M.D., F.R.C.S. (Edin. and Eng.), LL.D., Em. Professor of Anatomy, University of St Andrews. The Swallowgate, St Andrews	
1888		Napier, A. D. Leith, M.D., C.M., M.R.C.P., Hampton Street, Hawthorne, Unley, S. Australia	
1931		* Nelson, Alexander, B.Sc. (Glas.), Ph.D. (Edin.), N.D.A., Lecturer in Agricultural Botany, University of Edinburgh. 11 Wardie Avenue, Edinburgh	
1924		* Nelson, Philip, M.A., M.D., Ph.D., F.S.A., Beechwood, Calderstones, Liverpool	
1898		Newman, Sir George, K.C.B., M.D., D.C.L., LL.D., F.R.C.P., Chief Medical Officer of the Ministry of Health and the Board of Education, Whitehall, London, S.W. 1	
1928		* Nichols, James Edward, M.Sc. (Dunelm), Ph.D. (Edin.), British Research Association for the Woollen and Worsted Industries, Torridon, Headingley, Leeds. Flat XIV, 3 Moorland Terrace, Leeds	
1927		* Noble, Thomas Paterson, M.D. (Edin.), F.R.C.S. (Eng.), Phya Thai Palace, Bangkok, Siam	
1925		* Novar, The Right Hon. The Viscount, P.C., G.C.M.G., LL.D., Raith, Kirkcaldy	
1928	C.	* O'Donoghue, Charles Henry, D.Sc. (Lond.), Reader in Zoology in the University of Edinburgh. 24 Marchhall Crescent, Edinburgh	
1925		* Ogg, William Gammie, M.A., B.Sc., Ph.D., Director, Macaulay Institute for Soil Research, Craigiebuckler, Aberdeen	
1923	C.	* Ogilvie, Alan G., M.A., B.Sc. (Oxon.), Professor of Geography in the University of Edinburgh. 40 Fountainhall Road, Edinburgh	
1929		* Ogilvie, Frederick Wolff, M.A., Professor of Political Economy, University of Edinburgh. 20 Murrayfield Gardens, Edinburgh	
1886		Oliver, James, M.D., F.L.S., Physician to the London Hospital for Women, 123 Harley Street, London, W.	
1895	C.	† Oliver, Sir Thomas, Kt., D.L., M.D., LL.D., F.R.C.P., Em. Professor of Medicine in the University of Durham, 7 Ellison Place, Newcastle-upon-Tyne	

Date of Election.			Service on Council, etc.
1930		* Oliver, William, B.Sc., A.M.I.C.E., Professor of Organisation of Industry and Commerce, University of Edinburgh, and Director of Mitchell Graham & Sons, Ltd., Electrical and Mechanical Engineers. 70 Netherby Road, Trinity, Edinburgh	
1930		* O'Riordan, George Francis, B.Sc.(Eng.), M.Inst.Mech.E., Principal of Battersea Polytechnic, London, S.W. 7. Hesslewood, Cambalt Road, Putney Hill, London, S.W. 15	
1924		* Orr, John Boyd, D.S.O., M.C., M.A., D.Sc., M.D., Director of Rowett Research Institute for Research in Animal Nutrition. Research Lecturer in Physiology of Nutrition in the University of Aberdeen	
1915		* Orr, Lewis P., F.F.A., General Manager of the Scottish Life Assurance Co., 19 St Andrew Square, Edinburgh. 8 Belgrave Place, Edinburgh	
1927		* Owen, William John, Memb. Roy. Soc. Victoria. c/o Australian Institute of Anatomy, Canberra, F.C.T., Australia	
1908		Page, William Davidge. (Address not known)	
1905		Pallin, Lt.-Col. William Alfred, C.B.E., D.S.O., F.R.C.V.S., (retired), 5 Tower Gardens, Hythe, Kent	
1924		* Parker, Joseph, D.Sc., Principal, Fife Mining School, Cowdenbeath. 128 Stenhouse Street, Cowdenbeath	
1901		Paterson, David, F.C.S., Leewood, Rosslyn Castle, Midlothian	
1918		* Paterson, Rev. William Paterson, D.D., LL.D., Professor of Divinity, University, Edinburgh. 39 George Square, Edinburgh	
1927		* Patterson, Charles, M.Inst. Marine Engineers; Lecturer in Mechanical Engineering Design and Theory of Machines, University of Edinburgh. 22 Dudley Terrace, Trinity, Edinburgh	
1919	C.	* Patterson, Thomas Stewart, D.Sc. (London and Glasgow), Ph.D. (Heidelberg), Professor of Organic Chemistry in the University of Glasgow. 89 Oakfield Avenue, Hillhead, Glasgow, W.2	
1926		* Patton, Donald, M.A., B.Sc., Ph.D., Lecturer in Botany, Glasgow Provincial College for the Training of Teachers. 15 Jordanhill Drive, Glasgow, W. 3	
1928	C.	* Peacock, Alexander David, D.Sc., Professor of Zoology, University College, Dundee	
1907		Pearce, John Thomson, B.A., B.Sc., Bolton Manse, Haddington, East Lothian	
1914		Pearson, Joseph, D.Sc., F.L.S., Director of the Colombo Museum, and Marine Biologist to the Ceylon Government, Colombo Museum, Ceylon	
1904		Peck, James Wallace, C.B., M.A., Second Secretary, Scottish Education Department, Dover House, Whitehall, London, S.W. 1	1926-28.
1887	C. M-B.	Peddie, Wm., D.Sc., Professor of Natural Philosophy in University College, Dundee. The Weisha, Ninewells, Dundee	1904-07, 1908-11. V-P 1919-22.
1925		* Penman, David, D.Sc., M.Inst.M.E., Principal Dhanbad School of Mines, India. Mines Department, Dhanbad, East Indian Railway, India	
1928		* Percival, George Hector, M.B., M.R.C.P. (Edin.), Ph.D., Assistant Physician, Skin Department, Royal Infirmary, Edinburgh. 17 Atholl Crescent, Edinburgh	
1893		Perkin, Em. Professor Arthur George, D.Sc., F.R.S., F.I.C., Grosvenor Lodge, Hyde Park, Leeds	
1931	C.	* Phemister, James, M.A., D.Sc. (Glas.), Senior Geologist, H.M. Geological Survey. Everland, 2 Denham Green Terrace, Edinburgh	
1918	C.	† Philip, Alexander, M.A., LL.B., Writer, The Mary Acre, Brechin	
1889		† Philip, Sir Robert William, Kt., M.A., M.D., LL.D., F.R.C.P.E., Professor of Tuberculosis, University of Edinburgh. 45 Charlotte Square, Edinburgh	V-P 1927-30.
1907	C.	† Phillips, Major Charles E. S., O.B.E., Castle House, Shooters Hill, Woolwich, S.E. 18	
1929	C.	* Phillips, John Frederick Vicars, D.Sc., F.L.S., Professor of Botany, The University of the Witwatersrand, Johannesburg, Union of South Africa	
1928		* Pilcher, Robert Stuart, General Manager and Engineer, Manchester Corporation Tramways and Motors. 55 Piccadilly, Manchester	
1914		* Pilkington, Basil Alexander, "Kambla," Davidson's Mains	
1908	C.	* Pirie, James Hunter Harvey, B.Sc., M.D., F.R.C.P.E., Research Pathologist and Bacteriologist, The South African Institute for Medical Research. P.O. Box 1038, Johannesburg, South Africa	
1911		* Pirie, James Simpson, M.Inst.C.E., 25 Grange Road, Edinburgh	
1906		Pitchford, Herbert Watkins, C.M.G., F.R.C.V.S., Lt.-Col., Victoria Club, Pietermaritzburg, South Africa	

Date of Election.			Service on Council, etc.
1924		* Ponder, Eric, M.D., D.Sc., Professor of General Physiology, Washington Square College, New York University, New York, U.S.A.	
1919		* Porritt, B. D., M.Sc. (Lond.), F.I.C., Director of Research, Research Association of British Rubber Manufacturers, 105–7 Lansdowne Road, Croydon, Surrey	
1888		† Prain, Sir David, Lt.-Col., Indian Medical Service (retired), C.M.G., C.I.E., M.A., M.B., LL.D., F.R.S., F.L.S., For. Memb. K. Svensk. Vetensk. Akad.; Hon. Memb. Soc. Lett. ed Arti d. Zelanti, Acireale; Pharm. Soc. Gt. Britain; Corr. Memb. Bayer. Akad. Wiss., etc.; formerly Director, Royal Botanic Gardens, Kew, Surrey. The Well Farm, Warlingham, Surrey	
1926	C.	* Prashad, Baini, D.Sc., Superintendent, Zoological Survey of India, Indian Museum, Calcutta	
1892	C.	Pressland, Arthur J., M.A. (Camb.), 28 Carlyle Road, Cambridge	
1928		Price, Charles Edward, J.P., formerly M.P. for Central Edinburgh, Hon. Freeman of the City of Edinburgh. 16 Rothesay Terrace, Edinburgh	
1915		† Price, Frederick William, M.D., M.R.C.P. (Edin.), Physician to the Royal Northern Hospital, London. 133 Harley Street, London, W.	
1911		Purdy, John Smith, D.S.O., M.D., C.M. (Aberd.), D.P.H. (Camb.), F.R.G.S., Town Hall, Sydney, N.S.W., Australia	
1920	C.	* Purser, George Leslie, M.A. (Cantab.), F.Z.S., Lecturer in Embryology, University of Aberdeen	
1898		Purves, John Archibald, D.Sc., Chilliswood, Trull, Taunton	
1899	C.	Ramage, Alexander G., Lochcote, Linlithgowshire	
1904		Ratcliffe, Joseph Riley, M.B., C.M., c/o The Librarian, The University, Birmingham	
1900		Raw, Nathan, C.M.G., M.D., 30 Clarendon Court, Maida Vale, London, W.9	
1927	C.	* Read Herbert Harold, D.Sc. (Lond.), A.R.C.Sc., F.G.S., George Herdman Professor of Geology, The University of Liverpool	
1929		* Read, Selwyn, B.A., Schoolmaster, Edinburgh Academy. 2 Oxford Terrace, Edinburgh	
1902		Rees-Roberts, John Vernon, M.D., D.Sc., D.P.H., 90 Fitzjohns Avenue, Hampstead, London, N.W. 3	
1913		Reid, Harry Avery, O.B.E., F.R.C.V.S., D.V.H., Bacteriologist and Pathologist, Department of Agriculture, Wellington, New Zealand, c/o Bank of New Zealand, 1 Queen Victoria Street, London, E.C.	
1924		* Reid, William Carstairs, Civil Engineer, 23 Saxe-Coburg Place, Edinburgh	
1914		Renshaw, Graham, M.D., M.R.C.S., L.R.C.P., L.S.A., Editor of the *Avicultural Magazine*, Sale Bridge House, Sale, Manchester	
1918		* Richardson, Major Harry, M.Inst.E.E., M.Inst.M.E., 16 Stratford Place, London, W. 1	
1908		Richardson, Linsdall, F.G.S., 10 Oxford Parade, Cheltenham, Glos.	
1875		Richardson, Ralph, W.S., 29 Eglinton Crescent, Edinburgh	
1927		* Richey, James Ernest, B.A., B.A.I., Trinity College, Dublin, F.G.S., District Geologist, H.M. Geological Survey, Scotland. 19 Grange Terrace, Edinburgh	
1930		* Ritchie, Allan Watt, Chief Sanitary Inspector of the City of Edinburgh. 2 Queensferry Terrace, Edinburgh	
1916	C.	* Ritchie, James, M.A., D.Sc. (VICE-PRESIDENT), Regius Professor of Natural History in the University of Aberdeen	1921–24. 1926–28. Sec. 1928–31. V-P 1931–
1914	C.	* Ritchie, James Bonnyman, D.Sc., Head Master, Academy, Forres	
1906	C.	Ritchie, William Thomas, M.D., F.R.C.P.E., Professor of Medicine in the University of Edinburgh. 10 Douglas Crescent, Edinburgh	
1929		* Robb, Richard Alexander, M.A., B.Sc., M.Sc., Lecturer in Mathematics, University of Glasgow. 19 Seyton Avenue, Langside, Glasgow, S.1	
1931		* Robb, William, N.D.A., Director of Research, Scottish Society for Research in Plant Breeding. Craigs House, Corstorphine, Midlothian	
1898	C.	Roberts, Hon. Alexander William, D.Sc., F.R.A.S., Lovedale, South Africa	
1919		* Roberts, Alfred Henry, O.B.E., M.Inst.C.E., Superintendent and Engineer, Leith Docks. 40 Buckingham Terrace, Edinburgh	
1926		* Roberts, John Alexander Fraser, M.A. (Cantab.), B.Sc., Institute of Animal Genetics, University of Edinburgh	
1928	C.	* Roberts, Owen Fiennes Temple, M.C., M.A. (Camb.), Lecturer in Astronomy and Meteorology in the University of Aberdeen. 20 Belgrave Terrace, Aberdeen	
1902	C.	Robertson, Robert A., M.A., B.Sc., Professor of Botany in the University of St Andrews	

Date of Election.			Service on Council, etc.
1919		* Robertson, William Alexander, F.F.A., Century Insurance Co., Ltd., 18 Charlotte Square. Kilwarlin, Barnshot Road, Colinton	
1896	C.	Robertson, W. G. Aitchison, D.Sc., M.D., F.R.C.P.E., Barrister-at-law, Lincoln's Inn. St Margarets, St Valerie Road, Bournemouth	
1926		* Romanis, William Hugh Cowie, M.A., M.B., M.C. (Cantab.), F.R.C.S. (Eng.), Surgeon to St Thomas's Hospital, London, etc. 120 Harley Street, London, W.1	
1916		* Ronald, David, M.Inst.C.E., Chief Engineer, Scottish Board of Health, 125 George Street, Edinburgh	
1909	C.	* Ross, Alex. David, M.A., D.Sc., F.Inst.P., F.R.A.S., Professor of Physics, University of Western Australia, Perth, Western Australia	
1921		* Ross, Edward Burns, M.A. (Edin. and Camb.), Professor of Mathematics in the Madras Christian College, Madras	
1931		* Ruse, Harold Stanley, B.A. (Oxon.), Lecturer in Mathematics, University of Edinburgh. The Mathematical Institute, 16 Chambers Street, Edinburgh	
1906		Russell, Alexander Durie, B.Sc., Mathematical Master, Falkirk High School. 14 Heugh Street, Falkirk	
1930		Russell, David, LL.D., Paper Manufacturer. Silverburn, Leven, Fife	
1902	C. K.	Russell, James, 22 Glenorchy Terrace, Edinburgh	
1925	C.	* Saddler, William, M.A., B.A., Professor of Mathematics, Canterbury College, Christchurch, N.Z.	
1906		Saleeby, Caleb Williams, M.D., 13 Greville Place, Hampstead, London, N.W.6	
1916	C.	* Salvesen, The Rt. Hon. Lord, P.C., K.C., LL.D., Judge of the Court of Session (retired), Dean Park House, Edinburgh	1920-22. V-P 1922-25.
1914		* Salvesen, Theodore Emile, 37 Inverleith Place, Edinburgh	
1912	C. K.	* Sampson, Ralph Allen, M.A., D.Sc., LL.D. F.R.S. (General Secretary), Astronomer Royal for Scotland, Professor of Astronomy, University, Edinburgh. Royal Observatory, Edinburgh	1912-15, 1919-21. V-P 1915-18. Sec. 1922-23. Gen. Sec. 1923-
1908		Samuel, Sir John S., K.B.E., D.L., J.P., F.S.A.Scot., 13 Park Circus, Glasgow, W.	
1927	C.	* Sandeman, Ian, M.A., B.Sc., Ph.D. (St Andrews), Lakeview, Beach Road, Jaffra, North Ceylon	
1930		* Sansome, Frederick Whalley, B.Sc.(Agr.), Ph.D., F.L.S., Assistant, John Innes Horticultural Institution, Merton, London. Old Garden, Church Lane, Merton Park	
1922		* Sarker, Bijali Behari, M Sc., D.Sc. (Edin.), Post Graduate Lecturer in Physiology, University, Calcutta. 33/8 Lansdowne Road, Calcutta	
1903		Sarolea, Charles, Ph.D., D.Litt., formerly Professor of French, University of Edinburgh. 21 Royal Terrace, Edinburgh	
1927		* Schlapp, Robert, M.A. (Edin.), Ph.D. (Camb.), Lecturer in Applied Mathematics in the University of Edinburgh. 1 Peel Terrace, Edinburgh	
1885	C.	† Scott, Alexander, M.A., D.Sc., F.R.S., 34 Upper Hamilton Terrace, London, N.W.8	
1919		* Scott, Alexander, M.A., D.Sc., Holmwood, Mill House Lane, Walstanton, Stoke-on-Trent	
1919		* Scott, Alexander Ritchie, B.Sc. (Edin.), D.Sc. (Lond.), Principal London County Council, Beaufoy Institute, Prince's Road, Vauxhall Street, London, S.E.11	
1917		* Scott, Henry Harold, M.D., F.R.C.P. (London), M.R.C.S. (Eng.), D.P.H., Medical Secretary, Colonial Medical Research Committee, Colonial Office, Downing Street, London, S.W.1. "Bailleul," Albemarle Road, Beckenham, Kent	
1928		* Senior-White, Ronald, F.E.S., Malariologist, Bengal-Nagpur Railway, Kidderpore, P.O., Calcutta, India	
1926		* Seton, Colonel Sir Bruce Gordon, Bart., C.B., M.R.C.S., L.R.C.P., Indian Medical Service (retired). 12 Grosvenor Crescent, Edinburgh	
1930		* Shankland, Ernest Claud, F.R.Met.S., River Superintendent to the Port of London Authority. Mariners, Balfour Gardens, Folkestone	

Date of Election.			Service on Council, etc.
1900	C. N.	Sharpey-Schafer, Sir Edward Albert, M.D., D.Sc., LL.D., F.R.S. (PRESIDENT), Corresponding Member of the French Academy of Medicine, Professor of Physiology in the University of Edinburgh	1900–03, 1906–09, 1918–19. V-P 1913–17. P 1929–
1927		* Sharpley, Forbes Wilmot, B.Sc. (Eng.) (Lond.), A.M.I.E.E., Professor of Electrical and Mechanical Engineering, Indian School of Mines, Dhanbad, Bihar and Orissa, India	
1931		* Shaw, John James M'Intosh, M.A., M.D., F.R.C.S., Lecturer in Surgery and Clinical Surgery, University of Edinburgh. Greenaway, Kinnear Road, Edinburgh	
1927		* Shearer, Ernest, M.A., B.Sc. (Edin.), Professor of Agriculture and Rural Economy, Edinburgh University, and Principal of the Edinburgh and East of Scotland College of Agriculture, 13 George Square, Edinburgh	
1931		* Shearer, James Fleming, M.A. (Glas.), Lecturer in Natural Philosophy, University of Glasgow. Balmanno, Flloyd Street, Coatbridge	
1908		* Simpson, George Freeland Barbour, M.D., F.R.C.P.E., F.R.C.S.E., J.P., 43 Manor Place, Edinburgh	
1900	C.	† Simpson, James Young, M.A., D.Sc., Professor of Natural Science in the New College, Edinburgh. 25 Chester Street, Edinburgh	1922–25.
1900		Sinhjee, Sir Bhagvat, G.C.I.E., M.D., LL.D. (Edin.), H.H. the Thakur Sahib of Gondal, Gondal, Kathiawar, Bombay, India	
1903		† Skinner, Robert Taylor, M.A., J.P., Head Master, Donaldson's Hospital, Edinburgh	
1930	C.	* Slater, Robert Henry, B.Sc., Ph.D. (Edin.), Research Chemist, 4 Kingsburgh Road, Murrayfield, Edinburgh	
1929		* Smail, James Cameron, Assoc.Inst.E.E., Principal, Heriot-Watt College, Edinburgh. 1 Grange Terrace, Edinburgh	
1926		* Small, James, D.Sc., Ph.C., Professor of Botany, Queen's University, Belfast. Ardcolm, Knock, Belfast	
1901		Smart, Edward, B.A., B.Sc., Tillyloss, Tullylumb Terrace, Perth	
1920		* Smellie, William Robert, M.A., D.Sc., Geologist on the Staff of the Anglo-Persian Oil Company. Ardene, Mossend, Lanarkshire	
1928		Smith, Alick Drummond Buchanan, M.A., B.Sc. (Agric.) (Aberd.), M.S.A. (Iowa), Lecturer, Institute of Animal Genetics, University of Edinburgh	
1921		* Smith, Norman Kemp, M.A., D.Phil., D.Litt., LL.D., Professor of Logic and Metaphysics, University of Edinburgh. Ellerton, Grange Loan, Edinburgh	
1923		* Smith, Percy James Lancelot, M.A. (Oxon.), F.I.C., F.C.S., Science Master, Loretto School. 47 Dalrymple Loan, Musselburgh	
1911		* Smith, Stephen, B.Sc., Engineer, 31 Grange Loan, Edinburgh	
1929		* Smith, Sydney, M.D., F.R.C.S., Professor of Forensic Medicine, University of Edinburgh. 10 Oswald Road, Edinburgh	
1907	C.	† Smith, William Ramsay, D.Sc., M.D., C.M., Permanent Head of the Health Department, South Australia, Belair, South Australia	
1880		Smith, Sir William (Robert), M.D., D.Sc., LL.D., Principal of The Royal Institute of Public Health, Em. Professor of Forensic Medicine and Toxicology in King's College, University of London. 36 Russell Square, London, W.C. 1	
1919		* Smith, William Wright, M.A., D. ès Sc., Regius Professor of Botany, University of Edinburgh, Regius Keeper of the Royal Botanic Garden, and King's Botanist in Scotland. Inverleith House, Edinburgh	Sec. 1923–28. V-P 1928–31.
1899		Snell, Ernest Hugh, M.D., B.Sc., D.P.H. Camb., Barrister-at-law, Middle Temple, late Medical Officer of Health, Coventry. 8 Eaton Road, Coventry	
1880		Sollas, William Johnson, M.A., D.Sc., LL.D., F.R.S., Fellow of University College, Oxford, and Professor of Geology and Palæontology in the University of Oxford	
1910		* Somerville, Robert, B.Sc., Lindrum, Queensferry Road, Dunfermline	
1889		† Somerville, Sir Wm., K.B.E., M.A., D.Sc., D.Oec., formerly Sibthorpian Professor of Rural Economy and Fellow of St John's College in the University of Oxford. Rye House, Foxcombe Hill, near Oxford	
1911	C.	* Sommerville, Duncan M'Laren Young, M.A., D.Sc., Professor of Pure and Applied Mathematics, Victoria College, Wellington, New Zealand	
1929		* Southwell, Thomas, D.Sc., A.R.C.S., Lecturer in Helminthology, School of Tropical Medicine, Liverpool. 8 Waverley Road, Sefton Park, Liverpool	
1925		* Staig, Robert Arnot, M.A., Lecturer in Zoology in the University of Glasgow. Glenlea, Lasswade, Midlothian	

Date of Election.			Service on Council, etc.
1891		Stanfield, Richard, A.R.S.M., M.Inst.C.E., formerly Professor of Mechanics and Engineering in the Heriot-Watt College, Edinburgh. 24 Mayfield Gardens, Edinburgh	1926-29.
1923		* Stebbing, Edward Percy, M.A., Professor of Forestry in the University of Edinburgh	
1885 & 1915		* Steggall, John Edward Aloysius, M.A., Hon. A.R.I.B.A., Professor of Mathematics at University College, Dundee (St Andrews University). Woodend, Perth Road, Dundee	
1923		* Stenhouse, Andrew G., F.G.S., 191 Newhaven Road, Leith	
1929	C.	* Stephen, Alexander Charles, B.Sc., Assistant, Natural History Department, Royal Scottish Museum, Edinburgh. "Eastcroft," Cramond Bridge, Edinburgh	
1912	C. K.	† Stephenson, John, C.I.E., M.B., D.Sc. (Lond.), F.R.S., Lt.-Col. I.M.S. (retired), British Museum (Natural History), Cromwell Road, London, S.W. 7. 42 Orsett Terrace, London, W. 2	
1910		* Stephenson, Thomas, D.Sc., F.C.S., Editor of the *Prescriber*, 13 Glencairn Crescent, Edinburgh	
1931		* Steven, George Alexander, B.Sc. (Edin.), Assistant Naturalist at the Plymouth Laboratory, Marine Biological Association of the United Kingdom. 28 Lincoln Avenue, Plymouth, Devon	
1925		* Stevens, Alexander, M.A., B.Sc., Lecturer in Geography in the University of Glasgow	
1886	C.	Stevenson, Charles A., B.Sc., M.Inst.C.E., 28 Douglas Crescent, Edinburgh	
1884		† Stevenson, David Alan, B.Sc., M.Inst.C.E., Troqueer, Kingsknowe, Colinton, Midlothian	1928-31.
1919		* Stevenson, David Alan, B.Sc., M.Inst.C.E., 22 Glencairn Crescent, Edinburgh	
1931		* Stewart, Corbet Page, Ph.D. (Edin.), Lecturer in General Biochemistry, University of Edinburgh. 150 Craigleith Road, Edinburgh	
1925		* Stewart, David Smith, B.Sc., Ph.D., Assoc.M.Inst.C.E., Lecturer on Structural Engineering Drawing in the University of Edinburgh. 12 Lasswade Road, Liberton, Edinburgh	
1924		* Stiles, Sir Harold Jalland, K.B.E., M.B., F.R.C.S.E., LL.D., formerly Professor of Clinical Surgery in the University of Edinburgh. Whatton Lodge, Gullane, E. Lothian	
1902		Stockdale, Herbert Fitton, LL.D., Director of the Royal Technical College, Glasgow. Clairinch, Upper Helensburgh, Dumbartonshire	
1889	C.	Stockman, Ralph, M.D., LL.D., F.R.C.P.E., F.F.P.S.G., Professor of Materia Medica and Therapeutics in the University of Glasgow	1903-05.
1926		* Stokoe William Norman, B.Sc., Ph.D. (Lond.), Chief Chemist and Works Manager, Craigmillar Creamery Co., Ltd. 8 Cobden Road, Edinburgh	
1906		Story, Fraser, formerly Professor of Forestry, University College, Bangor, North Wales. 4K Artillery Mansions, Victoria Street, London, S.W. 1	
1907		Strong, John, C.B.E., M.A., LL.D., Professor of Education in the University of Leeds. Devonshire Hall, Headingley, Leeds	
1930		* Struthers, John William, M.B., Ch.B., F.R.C.S.E., Secretary and Treasurer, Royal College of Surgeons, Edinburgh, Surgeon to the Royal Infirmary, Edinburgh. 15 Ainslie Place, Edinburgh	
1930	C.	* Stump, Claude Witherington, M.D., D.Sc., Professor of Embryology and Histology in the University of Sydney	
1903		Sutherland, David W., C.I.E., M.D., M.R.C.P., Lt.-Col. I.M.S. (retired), Braeside, Belhaven, Dunbar	
1930		* Sutherland, John Donald, C.B.E., LL.D., F.S.I., Chevalier of the Legion of Honour (France), Assistant Commissioner, Forestry, Scotland. 11 Inverleith Row, Edinburgh	
1925		Sutton, Richard L., M.D., D.Sc., LL.D., Professor of Diseases of the Skin in the University of Kansas School of Medicine, U.S.A.	
1917	C. N.	* Tait, John, D.Sc., M.D., Professor of Physiology, M'Gill University, Montreal, Canada	
1904		Tait, John W., B.Sc., formerly Rector of Leith Academy, Netherby, Pitkeathly, Bridge of Earn	
1895		† Talmage, James Edward, D.Sc., LL.D., F.R.M.S., F.G.S., F.G.S.A., formerly President and Professor of Geology, University of Utah. 47 East S. Temple Street, Salt Lake City, Utah, U.S.A.	
1890	C.	† Tanakadate, Aikitu, Hon. Professor of Natural Philosophy in the Imperial University of Japan. Koisikawa, Zôsigayamati, 144, Tokyo, Japan	
1870		Tatlock, Robert R., F.C.S., City Analyst's Office, 156 Bath Street, Glasgow	
1899		Taylor, James, M.A., formerly Mathematical Master in the Edinburgh Academy. 18 Hillview, Blackhall, Edinburgh	

Date of Election.			Service on Council, etc.
1917	C.	* Taylor, William White, M.A., D.Sc., formerly Lecturer on Chemical Physiology, University, Edinburgh. Park Villa, Liberton, Edinburgh	
1892		Thackwell, J. B., M.B., C.M., D.P.H., 423A Battersea Park Road, London, S.W. 11	
1885	C.	Thompson, D'Arcy Wentworth, C.B., M.A., D.Sc., D.Litt., F.R.S., Foreign Hon. Member Amer. Acad. Arts and Sciences (CURATOR OF LIBRARY AND MUSEUM), Professor of Natural History, University, St Andrews. 44 South Street, St Andrews	1892-95, 1896-99, 1907-10, 1912-15, 1922-25. V.P 1916-19. Curator 1926-
1917	C. N.	* Thompson, John M'Lean, M.A., D.Sc., F.L.S., Professor of Botany, University of Liverpool	
1931		* Thomson, David Cleghorn, M.A. (Edin.), B.A. (Oxon.), Scottish Regional Director, British Broadcasting Corporation. 11 York Place, Edinburgh	
1896		Thomson, George Ritchie, C.M.G., M.B., C.M., LL.D., formerly Professor of Surgery, University of the Witwatersrand, Johannesburg, Transvaal. Hordle Grange, Hordle, Hants	
1903		Thomson, George S., 31 Tooley Street, London, S.E. 1	
1906		Thomson, Gilbert, M.A., M.Inst.C.E., 164 Bath Street, Glasgow, C. 2	
1926		* Thomson, Godfrey Hilton, D.Sc., Ph.D., Professor of the Theory, History, and Practice of Education in the University of Edinburgh	1931-
1887	C.	Thomson, Sir J. Arthur, M.A., LL.D., Em. Regius Professor of Natural History in the University of Aberdeen. St Mary's Lodge, Limpsfield, Surrey	1906-08, 1920-23.
1906	C.	Thomson, James Stuart, M.Sc., Ph.D., 5 Chesterton Terrace, Cirencester, Gloucestershire	
1926	C.	* Thomson, John, M.A., B.Sc., Ph.D. (Glasg.), Lecturer in Plant Physiology in the University of Glasgow. 2 Chartwell Terrace, Bearsden, Glasgow	
1880		Thomson, John Millar, LL.D., F.R.S., Hon. Fellow King's College and Queen's College, London. 6 Douro Place, Kensington, London, W. 8	
1899		Thomson, R. Tatlock, F.C.S., 156 Bath Street, Glasgow	
1912	C.	Thomson, Robert Black, M.B. (Edin.), Aliwal North, Cape Province, S.A.	
1882		Thomson, Sir William, Kt., M.A., B.Sc., LL.D., formerly Principal, University of the Witwatersrand. Dunedin, Glencairn, Simonstown, South Africa	
1917		* Thorneycroft, Wallace, J.P., Strete Ralegh, Whimple, Exeter, Devon	
1920		* Todd, John Barber, B.Sc., Ph.D., M.I.Mech.E., Lecturer in Engineering in the University of Edinburgh. 98 Findhorn Place, Edinburgh	
1917		* Tovey, Donald Francis, B.A.(Oxon.), M.Mus.(Hon.), Birmingham, Professor of Music, University, Edinburgh. 18 Buccleuch Place, Edinburgh	
1914		† Tredgold, Alfred Frank, M.D. (Durham), F.R.C.P. (Lond.), Lecturer on Mental Deficiency at London University, and Bethlem Royal Hospital, "St Martins," Guildford	
1915		* Trotter, George Clark, M.D. (Edin.), D.P.H. (Aberdeen), Medical Officer of Health, Metropolitan Borough, Islington. Braemar, 17 Haslemere Road, Crouch End, London, N. 8	
1922	C. K.	* Turnbull, Herbert Westren, M.A., Professor of Mathematics in the University of St Andrews. 2 Queens Terrace, St Andrews	1928-31.
1905		Turner, Arthur Logan, M.D., LL.D., F.R.C.S.E., (VICE-PRESIDENT), 27 Walker Street, Edinburgh	1926-29. V.P 1930-
1925		* Turner, Harry Moreton Stanley, M.B.E., M.D., M.R.C.S., L.R.C.P., D.T.M. and H., Chevalier de l'Ordre Royale du Sauveur de Grèce. Wing-Commander, R.A.F. (retired). 65 Stert Street, Abingdon-on-Thames, near Oxford	
1924		* Turner, Richard, O.B.E., M.B., C.M., Hotel Hydropathic, Peebles	
1895		Turton, Albert H., M.I.M.M., 233 George Road, Erdington, Birmingham	
1918	C.	* Tyrrell, G. W., A.R.C.Sc., Ph.D., F.G.S., Chief Assistant and Lecturer in Petrology, Geological Department, University, Glasgow	1926-29.
1910		Vincent, Swale, M.D. (Lond.), D.Sc. (Edin.), Professor of Physiology in the University of London. 15 Fishpool Street, St Albans, Herts	
1930	C.	* Voge, Cecil Innes Bothwell, B.Sc., Ph.D. (Edin.), Research Chemist. Eden Lodge, Eden Lane, Edinburgh	
1926		* Wakeley, Cecil Pembrey Grey, F.R.C.S. (Eng.), Surgeon to King's College Hospital and the West End Hospital for Nervous Diseases, Lecturer in Anatomy, King's College, London. 24 Queen Anne Street, Cavendish Square, London, W. 1	

Date of Election.			Service on Council, etc.
1925	C.	* Walker, Frederick, M.A., Ph.D., D.Sc., Lecturer in Geology, University, St Andrews	
1891	C. M-B.	Walker, Sir James, Kt., D.Sc., Ph.D., LL.D., F.R.S., formerly Professor of Chemistry in the University of Edinburgh. 5 Wester Coates Road, Edinburgh	1903–05, 1910–13, 1922–25, 1928–31. V-P 1916–19.
1931		* Walker, William James, Ph.D. (Edin.), Research Chemist, H.M. Fuel Research Station, East Greenwich, London, S.E. 10. C/o Harrison, 64 Sandtoft Road, Charlton, London, S.E. 7	
1902		Wallace, Alexander G., M.A., 66 Fonthill Road, Aberdeen	
1886	C.	Wallace, Robert, M.A., LL.D., F.L.S., Em. Professor of Agriculture and Rural Economy in the University of Edinburgh. C/o Mrs M'Call, 11 Bruntsfield Crescent, Edinburgh	
1898		Wallace, Wm., M.A., Campsie, Alta, Canada	
1920		* Walmsley, Thomas, M.D. (Glasg.), Professor of Anatomy, Queen's University, Belfast	
1931	C.	* Walton, John, M.A.(Camb.), D.Sc.(Manchester), Regius Professor of Botany, University of Glasgow. 4 Doune Gardens, Glasgow, N.W.	
1927	C.	* Wardlaw, Claude Wilson, Ph.D., D.Sc. (Glasg.), Imperial College of Tropical Agriculture, Trinidad, B.W.I.	
1923		* Warren, John Alexander, M.Inst.C.E., M.Cons.E. (Westminster). 74 Balshagray Avenue, Partick	
1901	C.	. Waterston, David, M.A., M.D., F.R.C.S.E., Professor of Anatomy, University, St Andrews	1916–19. 1925–28.
1927		* Watson, Charles Brodie Boog, F.S.A.Scot., 24 Garscube Terrace, Edinburgh	
1923		* Watson, H. Ferguson, M.D., F.R.F.P.S., Ph.D., D.P.H. (Glasg.), H.M. Senior Deputy Commissioner, General Board of Control for Scotland. 25 Palmerston Place, Edinburgh	
1923	C.	* Watson, William, M.A. (Edin.), B.Sc. (Edin.), Lecturer in Physics, Heriot-Watt College, Edinburgh. 17 Braidburn Crescent, Edinburgh	
1911		† Watt, James, W.S., F.F.A., LL.D. (Treasurer), Craiglockhart House, Craiglockhart Avenue, Edinburgh, W.	1924–26. Treasurer 1926–
1911		* Watt, Rev. Lauchlan Maclean, M.A., D.D., Minister of Glasgow Cathedral. 1 Athole Gardens, Glasgow	
1928		* Watters, Alexander Marshall, M.A., B.Sc. (Glasg.), Rector of Hawick High School. High School House, Hawick	
1896		† Webster, John Clarence, B.A., M.D., F.R.C.P.E., Professor of Obstetrics and Gynæcology, Rush Medical College, Shediac, N.B., Canada	
1907	M-B. C.	† Wedderburn, Ernest Maclagan, M.A., LL.B., W.S., D.Sc., Professor of Conveyancing in the University of Edinburgh. 6 Succoth Gardens, Edinburgh	1913–16, 1921–24.
1908	M-B. C.	† Wedderburn, J. H. Maclagan, M.A., D.Sc., P.O. Box 53, Princeton, N.J., U.S.A.	
1904		Wedderspoon, William Gibson, M.A., LL.D., Indian Educational Service, Senior Inspector of Schools, Burma. The Education Office, Rangoon, Burma	
1930		* White, Adam Cairns, M.B., Ch.B., Ph.D., Assistant Pharmacologist, Wellcome Physiological Research Laboratory, Beckenham, Kent	
1931		* Whitson, Rt. Hon. Thomas Barnby, C.A., LL.D., The Lord Provost of the City of Edinburgh. 27 Eglinton Crescent, Edinburgh	
1911		* Whittaker, Charles Richard, F.R.C.S. (Edin.), F.S.A.Scot., Lynwood, Hatton Place, Edinburgh	
1912	C. V. J. B-P.	* Whittaker, Edmund Taylor, M.A., Hon. Sc.D. (Dubl.), LL.D., F.R.S., Foreign Member of the R. Accademia dei Lincei, Rome, Professor of Mathematics in the University of Edinburgh. 48 George Square, Edinburgh	1912–15, 1922–25. Sec. 1916–22. V-P 1925–28.
1928	C.	* Whittaker, John Macnaghten, M.A. (Edin.), B.A. (Camb.), D.Sc., Fellow and Lecturer of Pembroke College, Cambridge	
1918		* Whyte, Rev. Charles, M.A., LL.D., F.R.A.S., U.F. Church Manse, Kingswells, Aberdeen	
1929	C.	* Wiesner, Bertold Paul, Ph.D., Lecturer in Sex Physiology, Institute of Animal Genetics, University of Edinburgh	
1918		* Wight, John Thomas, M.I.Mech.E., General Manager, Messrs MacTaggart, Scott & Co., Ltd., Station Iron Works, Loanhead. Calderwood Villa, Lasswade	

Date of Election.			Service on Council, etc.
1925		* Wilkie, David Percival Dalbreck, O.B.E., M.D., Ch.M., F.R.C.S., Professor of Surgery in the University of Edinburgh. 9 Ainslie Place, Edinburgh	
1926	C.	* Williams, Samuel, M.Sc., Ph.D., Lecturer in Plant Morphology in the University of Glasgow. 27 Lindsay Place, Kelvindale, Glasgow	
1924		* Williams, William Arthur, F.I.C., 1 Lennox Street, Edinburgh	
1908		* Williamson, Henry Charles, M.A., D.Sc., formerly Naturalist to the Fishery Board for Scotland, Marine Laboratory, Aberdeen. 11 St Mary's Road, Downfield, Dundee	
1928	C.	* Williamson, John, M.A. (Edin.), Ph.D. (Chicago), Associate Professor of Mathematics in Johns Hopkins University, Baltimore, U.S.A.	
1910	C.	* Williamson, William, F.L.S., 7 Ventnor Terrace, Edinburgh	
1927	C.	* Williamson, William Turner Horace, B.Sc. (Aberd.), Ph.D. (Edin.), Chief Chemist, Egyptian Ministry of Agriculture, Cotton Research Board, Giza, Cairo, Egypt	
1900		Wilson, Alfred C., Bloomfield House, Sadberge, near Darlington	
1911		* Wilson, Andrew, M.Inst.C.E., 66 Netherby Road, Trinity, Edinburgh	
1902	V. J.	† Wilson, Charles T. R., M.A., LL.D., F.R.S., Nobel Prize, Physics, 1927, Jacksonian Professor of Natural Philosophy in the University of Cambridge. Glencorse, Storey's Way, Cambridge	
1922		* Wilson, John, F.R.I.B.A., Fellow of the Inst. of Scottish Architects. Chief Architect, Scottish Department of Health. 20 Lomond Road, Edinburgh	
1920	C.	* Wilson, Malcolm, D.Sc. (London), A.R.C.Sc., F.L.S., Reader in Mycology and Bacteriology in the University of Edinburgh. Brent Knoll, Kinnear Road, Edinburgh	1931–
1924		* Wilson, William, M.A., LL.B., Advocate, Regius Professor of Public Law in the University of Edinburgh. 38 Moray Place, Edinburgh	
1895		Wilson-Barker, Sir David, Kt., R.D., R.N.R., F.R.G.S., formerly Captain-Superintendent Thames Nautical Training College, H.M.S. "Worcester." 22 Redcliffe Gardens, London, S.W. 10	
1931	C.	* Wishart, John, M.A., B.Sc. (Edin.), D.Sc. (Lond.), School of Agriculture, Cambridge. Astræa, 18 Storey's Way, Cambridge	
1922	C. B.	* Wordie, James Mann, M.A. (Camb.), B.Sc. (Glasg.), 52 Montgomery Drive, Glasgow, and St John's College, Cambridge	
1890		Wright, Johnstone Christie, Conservative Club, Edinburgh	
1896		† Wright, Sir Robert Patrick, LL.D., formerly Chairman of the Board of Agriculture for Scotland. The Heugh, North Berwick, East Lothian	
1911	C.	* Wrigley, Ruric Whitehead, M.A. (Cantab.), Assistant Astronomer, Royal Observatory, Edinburgh	
1882		Young, Frank W., C.B.E., F.C.S., H.M. Inspector of Schools (Emeritus). 35 Pentland Terrace, Edinburgh	
1904		Young, R. B., M.A., D.Sc., F.G.S., Professor of Geology and Mineralogy in the South African School of Mines and Technology, Johannesburg, Transvaal	

Number of Ordinary Fellows, 712.

LIST OF HONORARY FELLOWS OF THE SOCIETY.

(*At 26th October* 1931.)

HIS MOST EXCELLENT MAJESTY THE KING.

HIS ROYAL HIGHNESS THE PRINCE OF WALES.

FOREIGNERS (LIMITED TO THIRTY-SIX BY LAW I).

Elected

1916 Charles Eugène Barrois, formerly Professor of Geology and Mineralogy, Université, Lille, France: 37, rue Pascal, Lille.
1923 Henri Bergson, Honorary Professor, College of France, Paris.
1930 Vilhelm Frimann Koren Bjerknes, Professor of Physics, Geophysical Institute, Bergen.
1927 Niels Bohr, Nobel Laureate, Physics, 1922, Professor of Physics, University of Copenhagen.
1927 Jules Bordet, Nobel Laureate, Medicine, 1919, Professor of Bacteriology, University of Brussels.
1923 Marcellin Boule, Professor at the National Museum of Natural History, Laboratory of Palæontology, 3 Place Valhubert, Paris 5e.
1905 Waldemar Christofer Brögger, Professor of Mineralogy and Geology, K. Frederiks Universitet, Oslo, Norway.
1916 Douglas Houghton Campbell, Em. Professor of Botany, Leland Stanford Junior University, California, U.S.A.
1920 William Wallace Campbell, President-Emeritus of the University of California, Berkeley, and Director-Emeritus of the Lick Observatory, Mt. Hamilton, California, U.S.A.
1930 Walter Bradford Cannon, Professor of Physiology, Harvard University, Cambridge, U.S.A.
1930 Maurice Caullery, Professor of Zoology in the University of Paris. Evolution des Etres Organises Laboratoire, 105 Bould. Raspail, Paris, VIe.
1921 Reginald Aldworth Daly, Professor of Geology, Harvard University, Cambridge, Mass.
1910 Hugo de Vries, Professor of Plant Anatomy and Physiology, Lunteren, Holland.
1927 Albert Einstein, Nobel Laureate, Physics, 1921, Professor of Mathematical Physics, University of Berlin.
1930 Giulio Fano, Professor of Physiology in the Royal University of Rome.
1910 Karl Ritter von Goebel, Professor of Botany, University, München, Germany.
1913 George Ellery Hale, Honorary Director of Mount Wilson Observatory (Carnegie Institution of Washington), Pasadena, California, U.S.A.
1921 Johan Hjort, Professor of Marine Biology, University, Oslo.
1923 Arnold Frederik Holleman, Professor of Organic Chemistry, University, Amsterdam.
1923 Tullio Levi-Civita, Professor of Mathematics (Higher Analysis), University, Rome.
1927 Hans Horst Meyer, Emeritus Professor of Pharmacology, University of Vienna.
1923 Arthur Amos Noyes, Institute of California, Pasadena, U.S.A.
1908 Henry Fairfield Osborn, Research Professor of Zoology, Columbia University, and President, American Museum of Natural History, New York, U.S.A., Senior Geologist, U.S.A. Geological Survey.
1908 Ivan Petrovitch Pavlov, Em. Professor of Physiology, Inst. Exper. Med., Leningrad, Nobel Laureate, Physiology and Medicine, 1904: 7, Linia, No. 2, Vassilievsky, Ostrov, Leningrad, Russia.
1920 Ch. Emile Picard, Perpetual Secretary, Academy of Sciences, Paris.
1921 Salvatore Pincherle, Professor of Mathematics in the University of Bologna.
1913 Santiago Ramón y Cajal, Nobel Laureate, Medicine, 1906, formerly Professor of Histology and Pathological Anatomy, University, Madrid, Spain.
1920 Charles Richet, Professor of Physiology, Faculty of Medicine, Paris, Nobel Laureate, Medicine, 1913.
1927 Johannes Schmidt, A Director of the Carlsberg Laboratorium, Copenhagen.
1930 Erik Helge Oswald Stensiö, Professor, Royal Natural History Museum, Stockholm.
1913 Vito Volterra, Professor of Mathematical Physics, Regia Università, Rome, Italy.
1927 Richard Willstätter, Professor of Chemistry, University of Munich, Nobel Laureate, Chemistry, 1915. Munich 27, Moehlstrasse 29.
1923 Edmund Beecher Wilson, Professor of Zoology, Columbia University, New York, U.S.A.

Total, 33.

BRITISH SUBJECTS (LIMITED TO TWENTY BY LAW I).

Elected

1927 Sir William Henry Bragg, O.M., K.B.E., M.A., D.Sc., LL.D., F.R.S., Fullerian Professor of Chemistry, Royal Institution, London. Nobel Laureate, Physics, 1915.

1927 Sir David Bruce, K.C.B., M.B., C.M., Hon.D.Sc., LL.D., F.R.S., Major-General and Colonel-Commandant, A.M.S. Lister Institute, Chelsea Gardens, London, S.W. 1.

1930 Sir Arthur Stanley Eddington, M.A., Hon. D.Sc., F.R.S., Plumian Professor of Astronomy and Experimental Philosophy in the University of Cambridge.

1927 Sir John Bretland Farmer, Kt., M.A., D.Sc., LL.D., F.R.S., *formerly Professor of Botany,* Imperial College of Science and Technology, London.

1900 Andrew Russell Forsyth, M.A., Sc.D., LL.D., Hon. Math.D., F.R.S., Em. Professor of Mathematics in the Imperial College of Science and Technology, London; formerly Sadleirian Professor of Pure Mathematics in the University of Cambridge. Imperial College of Science and Technology, London, S.W. 7.

1910 Sir James George Frazer, O.M., Kt., D.C.L., LL.D., Litt.D., F.R.S., Commandeur de la Légion d'Honneur, Fellow of Trinity College, Cambridge.

1930 Sir William Bate Hardy, Kt., M.A., F.R.S., Director of Food Investigation, Department of Scientific and Industrial Research, 5 Grange Road, Cambridge.

1927 Sir Frederick Gowland Hopkins, Kt., M.A., M.B., D.Sc., LL.D., F.R.S., Pres. R.S., Joint Nobel Laureate, Medicine, 1929, Professor of Bio-Chemistry, University of Cambridge.

1930 Sir Arthur Keith, Kt., M.D., LL.D., F.R.S., Hunterian Professor and Conservator of the Museum of the Royal College of Surgeons, London.

1913 Sir Horace Lamb, M.A., Sc.D., D.Sc., LL.D., F.R.S., formerly Professor of Mathematics in the University of Manchester. 6 Selwyn Gardens, Cambridge.

1910 Sir Joseph Larmor, Kt., M.A., D.Sc., LL.D., D.C.L., F.R.S., Lucasian Professor of Mathematics in the University of Cambridge. St John's College, Cambridge.

1930 John Edward Marr, Sc.D., F.R.S., Fellow of St John's College, and Em. Professor of Geology, Cambridge. 126 Huntingdon Road, Cambridge.

1930 Robert Robinson, D.Sc., F.R.S., Waynflete Professor of Chemistry in the University of Oxford. The Dyson Perrins Laboratory, South Parks Road, Oxford.

1921 Sir Ronald Ross, K.C.B., K.C.M.G., F.R.S., Nobel Laureate, Physiology and Medicine, 1902, Director in Chief, The Ross Institute and Hospital for Tropical Diseases, Putney Heath, London, S.W. 15. *Hon. Consultant in Malaria, Ministry of Pensions, London.*

1921 Rt. Hon. Lord Rutherford of Nelson, O.M., M.A., D.Sc., B.A., LL.D., Past Pres. R.S., Nobel Laureate, Chemistry, 1908, Cavendish Professor of Experimental Physics in the University of Cambridge.

1916 Sir Arthur Schuster, Kt., Ph.D., D.Sc., LL.D., Dr ès Sc. Geneva, Honorary Professor of Physics in the University of Manchester. Yeldall, Twyford, Berks.

1930 Dukinfield Henry Scott, M.A., D.Sc., LL.D., Ph.D., F.R.S., formerly Honorary Keeper of the Jodrell Laboratory, Royal Botanic Gardens, Kew. East Oakley House, Basingstoke, Hants.

1908 Sir Charles Scott Sherrington, O.M., G.B.E., M.A., D.Sc., M.D., LL.D., Past Pres. R.S., Waynflete Professor of Physiology in the University of Oxford. Physiological Laboratory, Oxford.

1905 Sir Joseph John Thomson, O.M., Kt., D.Sc., LL.D., Past Pres. R.S., Nobel Laureate, Physics, 1906, formerly Cavendish Professor of Experimental Physics, University of Cambridge, Master of Trinity College, Cambridge.

Total, 19.

CHANGES IN FELLOWSHIP DURING SESSION 1930–1931.

ORDINARY FELLOWS OF THE SOCIETY ELECTED.

WILLIAM ALEXANDER BAIN.
WILLIAM MACDONALD BAIRD.
THOMAS PURVES BLACK.
JOHN ANTHONY CARROLL.
JOHN MACQUEEN COWAN.
JOHN CRICHTON.
SHEPHERD DAWSON.
PHILIP EGGLETON.
WILLIAM RONALD DODDS FAIRBAIRN.
ROBERT GRANT.
JOHN RUSSELL GREIG.
JOHN HENDERSON.
THOMAS JOHNSON.
JOHN DU PLESSIS LANGRISHE.
NICHOLAS MORPETH HUTCHINSON LIGHTFOOT.
WILLIAM JOHN McCALLIEN.
WILLIAM HUNTER M'CREA.
JOHN BOWES M'DOUGALL.
JOHN HUXLEY MASON.
FRANK CHARLES MEARS.
ALEXANDER NELSON.
JAMES PHEMISTER.
WILLIAM ROBB.
HAROLD STANLEY RUSE.
JOHN JAMES M'INTOSH SHAW.
JAMES FLEMING SHEARER.
GEORGE ALEXANDER STEVEN.
CORBET PAGE STEWART.
DAVID CLEGHORN THOMSON.
WILLIAM JAMES WALKER.
JOHN WALTON.
THOMAS BARNBY WHITSON.
JOHN WISHART.

ORDINARY FELLOWS DECEASED.

ARCHIBALD BARR.
SIR BYROM BRAMWELL.
RAYMOND KEILLER BUTCHART.
JAMES CURRIE.
J. D. HAMILTON DICKSON.
JOHN DUNSTAN.
WILLIAM ELDER.
RICHARD TAUNTON FRANCIS.
JOHN EDWARD GEMMELL.
CHARLES ROBERT GIBSON.
DAVID HEPBURN.
DAVID THOMAS JONES.
JOHN ANGUS MACDONALD.
WM. CARMICHAEL M'INTOSH.
ERNEST ROMNEY MATTHEWS.
HON. ALFRED GEORGE NASH.
SIR FRANCIS GRANT OGILVIE.
SIR DAVID PAULIN.
PETER PINKERTON.
JAMES LORRAIN SMITH.
SPENCER C. THOMSON.
THOMAS P. WATSON.

FOREIGN HONORARY FELLOWS DECEASED.

MARCEL EUGÈNE EMILE GLEY.
ALBERT ABRAHAM MICHELSON.

ORDINARY FELLOWS RESIGNED.

RT. HON. LORD MURRAY.
ARTHUR ROBINSON.
JAMES A. S. WATSON.

LAWS OF THE SOCIETY.

Adopted July 3, 1916; amended December 18, 1916.

(Laws VIII, IX, and XIII amended May 3, 1920. Law VI amended February 7, 1921. Law XIX amended June 16, 1924. Law VI amended July 2, 1928.)

I.

THE ROYAL SOCIETY OF EDINBURGH, which was instituted by Royal Charter in 1783 for the promotion of Science and Literature, shall consist of Ordinary Fellows (hereinafter to be termed Fellows) and Honorary Fellows. The number of Honorary Fellows shall not exceed fifty-six, of whom not more than twenty may be British subjects, and not more than thirty-six subjects of Foreign States.

Fellows only shall be eligible to hold office or to vote at any Meeting of the Society.

ELECTION OF FELLOWS.

II.

Each Candidate for admission as a Fellow shall be proposed by at least four Fellows, two of whom must certify from personal knowledge. The Official Certificate shall specify the name, rank, profession, place of residence, and the qualifications of the Candidate. The Certificate shall be delivered to the General Secretary before the 30th of November, and, subject to the approval of the Council, shall be exhibited in the Society's House during the month of January following. All Certificates so exhibited shall be considered by the Council at its first meeting in February, and a list of the Candidates approved by the Council for election shall be issued to the Fellows not later than the 21st of February.

III.

The election of Fellows shall be by Ballot, and shall take place at the first Ordinary Meeting in March. Only Candidates approved by the Council shall be eligible for election. A Candidate shall be held not elected, unless he is supported by a majority of two-thirds of the Fellows present and voting.

IV.

On the day of election of Fellows two scrutineers, nominated by the President, shall examine the votes and hand their report to the President, who shall declare the result.

V.

Each Fellow, after his election, is expected to attend an Ordinary Meeting, and sign the Roll of Fellows, he having first made the payments required by Law VI. He shall be introduced to the President, who shall address him in these words:

In the name and by the authority of THE ROYAL SOCIETY OF EDINBURGH, I admit you a Fellow thereof.

PAYMENTS BY FELLOWS.

VI.

Each Fellow shall, before he is admitted to the privileges of Fellowship, pay an admission fee of three guineas, and a subscription of three guineas for the year of election. He shall continue to pay a subscription of three guineas at the beginning of each session so long as he remains a Fellow.

Each Fellow who was elected subsequent to December 1916 and previous to December 1920 shall also pay a subscription of three guineas at the beginning of each session so long as he remains a Fellow.

Each Fellow who was elected previous to December 1916, and who has not completed his twenty-five annual payments, shall, at the beginning of each session, pay three guineas until his twenty-five annual payments are made. Each Fellow who has completed or shall complete his payments shall be invited to contribute one guinea per annum or to pay a single sum of ten guineas.

A Fellow may compound for the annual subscriptions by a single payment of fifty guineas, or on such other terms as the Council may from time to time fix.

VII.

A Fellow who, after application made by the Treasurer, fails to pay any contribution due by him, shall be reported to the Council, and, if the Council see fit, shall be declared no longer a Fellow. Notwithstanding such declaration all arrears of contributions shall remain exigible.

ELECTION OF HONORARY FELLOWS.

VIII.

Honorary Fellows shall be persons eminently distinguished in Science or Literature. They shall not be liable to contribute to the Society's Funds. Personages of the Blood Royal may be elected Honorary Fellows at any time on the nomination of the Council, and without regard to the limitation of numbers specified in Law I.

IX.

Honorary Fellows shall be proposed by the Council. The nominations shall be announced from the Chair at the First Ordinary Meeting after their selection. The names shall be printed in the circular for the last Ordinary Meeting of the Session, when the election shall be by Ballot, after the manner prescribed in Laws III and IV for the Election of Fellows.

EXPULSION OF FELLOWS.

X.

If, in the opinion of the Council, the conduct of any Fellow is injurious to the character or interests of the Society, the Council may, by registered letter, request him to resign. If he fail to do so within one month of such request, the Council shall call a Special Meeting of the Society to consider the matter. If a majority consisting of not less than two-thirds of the Fellows present and voting decide for expulsion, he shall be expelled by declaration from the Chair, his name shall be erased from the Roll, and he shall forfeit all right or claim in or to the property of the Society.

XI.

It shall be competent for the Council to remove any person from the Roll of Honorary Fellows if, in their opinion, his remaining on the Roll would be injurious to the character or interests of the Society. Reasonable notice of such proposal shall be given to each member of the Council, and, if possible, to the Honorary Fellow himself. Thereafter the decision on the question shall not be taken until the matter has been discussed at two Meetings of Council, separated by an interval of not less than fourteen days. A majority of two-thirds of the members present and voting shall be required for such removal.

MEETINGS OF THE SOCIETY.

XII.

A Statutory Meeting for the election of Council and Office-Bearers, for the presentation of the Annual Reports, and for such other business as may be arranged by the Council, shall be held on the fourth Monday of October. Each Session of the Society shall begin at the date of the Statutory Meeting.

XIII.

Meetings for reading and discussing communications and for general business, herein termed Ordinary Meetings, shall be held, when convenient, on the first and third Mondays of each month from November to July inclusive, with the exception that in January the meetings shall be held on the second and fourth Mondays.

XIV.

A Special Meeting of the Society may be called at any time by direction of the Council, or on a requisition to the Council signed by not fewer than six Fellows. The date and hour of such Meeting shall be determined by the Council, who shall give not less than seven days' notice of such Meeting. The notice shall state the purpose for which the Special Meeting is summoned; no other business shall be transacted.

PUBLICATION OF PAPERS.

XV.

The Society shall publish Transactions and Proceedings. The consideration of the acceptance, reading, and publication of papers is vested in the Council, whose decision shall be final. Acceptance for reading shall not necessarily imply acceptance for publication.

DISTRIBUTION OF PUBLICATIONS.

XVI.

Fellows who are not in arrear with their Annual Subscriptions and all Honorary Fellows shall be entitled gratis to copies of the Parts of the Transactions and the Proceedings published subsequently to their admission.

Copies of the Parts of the Proceedings shall be distributed by post or otherwise, as soon as may be convenient after publication; copies of the Transactions or Parts thereof shall be obtainable upon application, either personally or

by an authorised agent, to the Librarian, provided the application is made within five years after the date of publication.

CONSTITUTION OF COUNCIL.

XVII.

The Council shall consist of a President, six Vice-Presidents, a Treasurer, a General Secretary, two Secretaries to the Ordinary Meetings (the one representing the Biological group and the other the Physical group of Sciences),* a Curator of the Library and Museum, and twelve ordinary members of Council.

ELECTION OF COUNCIL.

XVIII.

The election of the Council and Office-Bearers for the ensuing Session shall be held at the Statutory Meeting on the fourth Monday of October. The list of the names recommended by the Council shall be issued to the Fellows not less than one week before the Meeting. The election shall be by Ballot, and shall be determined by a majority of the Fellows present and voting. Scrutineers shall be nominated as in Law IV.

XIX.

The President may hold office for a period not exceeding five consecutive years; the Vice-Presidents, not exceeding three; the Secretaries to the Ordinary Meetings, not exceeding five; the General Secretary, the Treasurer, and the Curator of the Library and Museum, not exceeding ten; and ordinary members of Council, not exceeding three consecutive years; provided that the Treasurer may be re-elected for more than ten successive years in cases where the Council declares to the Society that an emergency exists.

XX.

In the event of a vacancy arising in the Council or in any of the offices enumerated in Law XVII, the Council shall proceed, as soon as convenient, to elect a Fellow to fill such vacancy for the period up to the next Statutory Meeting.

* The Biological group includes Anatomy, Anthropology, Botany, Geology, Pathology, Physiology, Zoology; the Physical group includes Astronomy, Chemistry, Mathematics, Metallurgy, Meteorology, Physics.

POWERS OF THE COUNCIL.

XXI.

The Council shall have the following powers:—(1) To manage all business concerning the affairs of the Society. (2) To decide what papers shall be accepted for communication to the Society, and what papers shall be printed in whole or in part in the Transactions and Proceedings. (3) To appoint Committees. (4) To appoint employees and determine their remuneration. (5) To award the various prizes vested in the Society, in accordance with the terms of the respective deeds of gift, provided that no member of the existing Council shall be eligible for any such award. (6) To make from time to time Standing Orders for the regulation of the affairs of the Society. (7) To control the investment or expenditure of the Funds of the Society.

At Meetings of the Council the President or Chairman shall have a casting as well as a deliberative vote.

DUTIES OF PRESIDENT AND VICE-PRESIDENTS.

XXII.

The President shall take the Chair at Meetings of Council and of the Fellows. It shall be his duty to see that the business is conducted in accordance with the Charter and Laws of the Society. When unable to be present at any Meetings or attend to current business, he shall give notice to the General Secretary, in order that his place may be supplied. In the absence of the President his duties shall be discharged by one of the Vice-Presidents.

DUTIES OF THE TREASURER.

XXIII.

The Treasurer shall receive the monies due to the Society and shall make payments authorised by the Council. He shall lay before the Council a list of arrears in accordance with Rule VII. He shall keep accounts of all receipts and payments, and at the Statutory Meeting shall present the accounts for the preceding Session, balanced to the 30th of September, and audited by a professional accountant appointed annually by the Society.

DUTIES OF THE GENERAL SECRETARY.

XXIV.

The General Secretary shall be responsible to the Council for the conduct of the Society's correspondence, publications, and all other business except that which relates to finance. He shall keep Minutes of the Statutory and Special

Meetings of the Society and Minutes of the Meetings of Council. He shall superintend, with the aid of the Assistant Secretary, the publication of the Transactions and Proceedings. He shall supervise the employees in the discharge of their duties.

DUTIES OF SECRETARIES TO ORDINARY MEETINGS.

XXV.

The Secretaries to Ordinary Meetings shall keep Minutes of the Ordinary Meetings. They shall assist the General Secretary, when necessary, in superintending the publication of the Transactions and Proceedings. In his absence, one of them shall perform his duties.

DUTIES OF CURATOR OF LIBRARY AND MUSEUM.

XXVI.

The Curator of the Library and Museum shall have charge of the Books, Manuscripts, Maps, and other articles belonging to the Society. He shall keep the Card Catalogue up to date. He shall purchase Books sanctioned by the Council.

ASSISTANT-SECRETARY AND LIBRARIAN.

XXVII.

The Council shall appoint an Assistant-Secretary and Librarian, who shall hold office during the pleasure of the Council. He shall give all his time, during prescribed hours, to the work of the Society, and shall be paid according to the determination of the Council. When necessary he shall act under the Treasurer in receiving subscriptions, giving out receipts, and paying employees.

ALTERATION OF LAWS.

XXVIII.

Any proposed alteration in the Laws shall be considered by the Council, due notice having been given to each member of Council. Such alteration, if approved by the Council, shall be proposed from the Chair at the next Ordinary Meeting of the Society, and, in accordance with the Charter, shall be considered and voted upon at a Meeting held at least one month after that at which the motion for alteration shall have been proposed.

Additions to the Library—Presentations, etc.—1930-1931.

Academia Sinica (National Research Institute of China):—

Bulletin of the National Research Institute of History and Philology. Vol. I, No. – . 8vo. Canton, Peiping, 1928– .

The Historical Material Series. (In Chinese.) 8vo.

Memoir of the Institute of Astronomy. No. 1– . 8vo. Nanking, 1929– .

Memoir of the Institute of Chemistry. No. 1– . 8vo. 1930– .

Memoir of the Institute of Geology. No. 1– . 8vo. Shanghai, 1928– .

Memoir of the Institute of Meteorology. No. 1– . 8vo. Nanking, 1929– .

Memoirs of the Institute of Social Sciences. No. 1– . 8vo. Shanghai, 1929– .

Monograph of the Institute of Social Sciences. No. 1– . 8vo. Shanghai, 1929– .

Monographs of the Institute of History and Philology. (In Chinese.) 8vo.

Preliminary Reports of Excavations at Anyang. Part 1– . 8vo. Peiping, 1929– .

Report of the National Ceramic Laboratory. No. 1– . 8vo. 1930– .

Scientific Papers of the National Research Institute of Physics. No. 1– . 8vo. Shanghai, 1930– .

(*Exchange, Academia Sinica.*)

Baltic Geodetic Commission Special Publication. No. 1. 8vo. Helsingfors, 1930. (*Presented.*)

Bulletin de l'Observatoire Géophysique Central. No. 1– . 4to. Leningrad, 1930– . (*Exchange.*)

Bulletin: Florida Geological Survey. Nos. 3 and 4. 8vo. Tallahassee, 1930. (*Presented.*)

Bulletins of the National Research Council: "Physics of the Earth":—

77–1: Volcanology.

78–2: The Figure of the Earth.

79–3: Meteorology.

80–4: Age of the Earth.

8vo. Washington, 1931. (*Exchange.*)

Carnegie Institution of Washington: Publications:—

No. 338. Stock, L. F. Proceedings and Debates of British Parliaments respecting North America. Vol. III. 1702–1727. 8vo. Washington, 1930.

No. 404. Contributions to Palæontology. 8vo. Washington, 1930.

No. 407. Contributions to Embryology. Vol. XXI, Nos. 118–125. 4to. Washington, 1930.

No. 408. Goranson, Roy W. Thermodynamic Relations in Multicomponent Systems. 8vo. Washington, 1930.

No. 409. Donnan, E. Documents Illustrative of the History of the Slave Trade to America. Vol. I. 1441–1700. 8vo. Washington, 1930.

No. 411. M'Murrich, J. Playfair. Leonardo da Vinci. The Anatomist (1452–1519). 8vo. Washington, 1930.

(*By Exchange.*)

Centenaire de Marcelin Berthelot, 1827–1927. Fol. Paris, 1929. (*Presented by the Centenary Committee.*)

The Codex Alexandrinus in reduced photographic facsimile. Old Testament. Parts 1 and 2. 1915, 1930. (*Presented by the British Museum.*)

Collection of Czechslovak Chemical Communications. Published by Regia Societas Scientiarum Bohemica. Vol. I– . 8vo. Prague, 1929. (*Exchange.*)

"Discovery" Reports. Vol. I– . 4to. London, 1929– . (*Presented by "Discovery" Committee, Colonial Office, London.*)

Family Council Law in Europe. 8vo. London. (*Presented by the Eugenics Society.*)

Fauna of British India, etc. :—

Southwell, T. Cestoda. Vol. II. 8vo. London, 1930.

Cameron, M. Coleoptera : *Staphylinidæ*. Vol. II. 8vo. London, 1931.

(*Presented by India Office.*)

The Gibraltar Society Annual Journal. Vol. I, 1930. La. 8vo. Gibraltar, 1931. (*Presented.*)

Glaister, John, jun. A Study of Hairs and Wools belonging to the Mammalian Group of Animals, including a special study of Human Hair. (Egyptian University : Faculty of Medicine. Publication No. 2.) 4to. Cairo, 1931. (*Presented by the Author.*)

Greig, J. Y. T. David Hume. 8vo. London, 1931. (*Presented by the Author.*)

Gunther, R. T. Early Science in Oxford. Vols. VI, VII. Life and Work of Robert Hooke. 8vo. Oxford, 1930.

Vol. VIII. Hooke's Philosophical Collections. 8vo. Oxford, 1931.

(*Purchased.*)

Indian Journal of Physics. Vol. 4. Parts I– . Proceedings of the Indian Association for the Cultivation of Science. Vol. 13. Part I– . 8vo. Calcutta, 1929. (*Exchange.*)

M'Adie, Alexander. Clouds. 4to. Cambridge, 1930. (*From Blue Hill Observatory, Cambridge, Mass., U.S.A.*)

Miscellaneous Publications of the Bureau of Entomology of Chekiang Province. No. 1. 8vo. Westlake, Hangchow, China, 1930. (*Presented.*)

Morris, E. H., Jean Charlot and A. A. Morris. The Temple of the Warriors at Chichen Itzá, Yucatan. Carnegie Publication, No. 406. 2 vols. 4to. Washington, 1931.

Muir, Sir Thomas. Contributions to the History of Determinants 1900–1920. 8vo. London, 1930. (*Presented.*)

Poggendorff's biographisch-literarisches Handwörterbuch. Vol. 5. Abt. I and II, 1904–1922. 8vo. Leipzig and Berlin, 1926. (*Purchased.*)

Publicacions Facultad de Ciencias Exactas, Fisicas y Naturalis. No. 1– . 8vo. Buenos Aires, 1928– .

Publications de l'Observatoire Astronomique de l'Université de Belgrade. Tome I– , Annuaire pour l'An 1929– . 4to. Belgrade, 1928– .

Publications of the South African Institute for Medical Research. Nos. 1–19, 21, 24, and 26. 8vo. Johannesburg, 1913– . (*Presented by Dr J. H. Harvey Pirie.*)

Quarterly Journal of Mathematics: Oxford Series. Vol. I, No. 1– . 8vo. Oxford, 1930– . (*Purchased.*)

Report on the Scientific Results of the "Michael Sars" North Atlantic Deep-sea Expedition, 1910. 4to. Bergen, 1930. (*Purchased.*)

Royal Scottish Museum, Edinburgh:—

(*On Permanent Loan.*)

Bulletin of Miscellaneous Information: Royal Botanic Gardens, Kew. Vols. I–XXXVI. (1887–1925) 8vo. London, 1887–1925.
Additional Series I–XI (No. III missing). 8vo. London, 1898–1920.

Curtis's Botanical Magazine. Third Series. Vols. XL–LX. 8vo. London, 1884–1904.

—— Fourth Series. Vols. I–XII. 8vo. London, 1905–1916.

—— —— General Index. Vols. I–CVII. (1787–1880.) 8vo. London, 1883.

—— Index. 1787–1904. 8vo. London, 1906.

The Gardener's Chronicle: A Weekly Illustrated Journal of Horticulture and Allied Subjects. New Series. Vols. IX–LXVIII. 1879–1920. 4to. London, 1879–1920.

(*Presented by Royal Scottish Museum.*)

Annales de Biologie Lacustre. Tomes 1–8, No. 2. 8vo. Bruxelles, 1906–1916.

Bulletin of the Illinois State Laboratory of Natural History. Vol. VI, No. 2, and Vol. VIII, No. 1. C. A. Kofoid. The Plankton of the Illinois River. 1894–1899. Parts 1 and 2. 8vo. Urbana, 1903 and 1908.

Danish Freshwater Biological Laboratory. Op. 5. C. Wesenberg-Lund. Plankton Investigations of the Danish Lakes. General Part. Text and Plates. 4to. Copenhagen, 1908.

—— —— Studier over de Danske søers Plankton. Specielle Del. Text and Plates. 4to. Copenhagen, 1904.

Delebecque, André. Les Lacs Français. 4to. Paris, 1898.

Behring Sea Arbitration: Report of the Behring Sea Commission, and Report of British Commissioners of June 21, 1892. Fol. London, 1893.

Bourcart, Felix-Ernest. Les Lacs Alpins Suisses étude chimique et physique. 4to. Genève, 1906.

Universidad de Buenos Aires:—

Boletin del Seminario Matematico Argentino. No. 1– . 8vo. Buenos Aires, 1928.

Trabajos del Seminario Matematico Argentino. Tomo I, No. 4– . 8vo. Buenos Aires, 1930– .

(*By Exchange.*)

Walsh, John W. T. The Indexing of Books and Periodicals. 8vo. London, 1930. (*Purchased.*)

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Index of Papers published in the "Transactions" during Session 1930–31.

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